* Intro
* Syllabus - contact details
* Course website
* Course structure - lectures T Th
  - HW usually T, F by 5pm
  - Exam dates → in syllabus.
* This week - Thurs covers
  - Fri: HW 1 due
* Diagnostic test -

Quantum theory: overview

Quantum theory is a fundamental theory of physics that can successfully describe a wide range of phenomena. Examples include:

1) atoms + molecules - can we predict the nature of electromagnetic radiation that is absorbed or emitted by atoms + molecules?
   - can one predict how atoms bond to form molecules?

2) solids - can we predict thermal properties (e.g. heat capacities) of solids starting with basic physics
   - can we predict electrical properties (e.g. conduction) of solids?

3) light - how does light behave at the single photon level?
   - how do lasers work?

4) elementary particles - how do elementary particles interact?
   - what are the possible elementary particles?

5) quantum foundations and information – what are the fundamental differences between quantum and classical systems?

– how can quantum systems enhance information processing tasks?

Phys 321 will provide

1) much of the basic framework used to describe any quantum system

2) techniques used to elaborate + apply the framework to:

* spin particles
* harmonic oscillators
* hydrogen atoms.

Quantum theory vs classical physics

Quantum theory uses concepts that are often very different from those in classical physics. In general, there is no method for "deriving" quantum physics from classical physics. We can sometimes use suggestive analogies

Classical oscillator

+

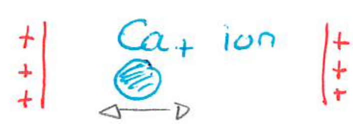
known initial state ($t=0$)

⇓

can predict position, velocity at any later time with certainty.

Deterministic

Quantum oscillator



+

known initial state ($t=0$)

⇓

Can make statistical predictions about measurement outcomes at later times

Probabilistic

No direct derivation

Read Ch 1 my notes

Mathematics + quantum physics

The mathematical structure that underlies quantum physics is:

states of system described by complex number <u>vectors</u>

evolution of the state described by complex number operators (<u>matrices</u>)

LINEAR ALGEBRA

Sometimes turn into <u>differential equations</u>

Probability Theory

Measurements described via <u>probabilities</u> of outcomes

Spin-1/2 Particles

We will present the basic framework, concepts and physics in terms of a particular type of particle called a spin-1/2 particle. This refers to the angular momentum of the particle. It has the advantages that:

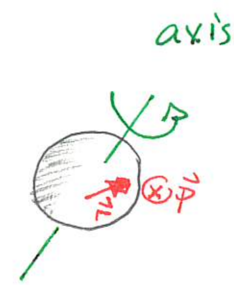
* most states of the particle are easy to describe mathematically
* all conceivable measurements can readily be described in terms of particular physical measurements.
* a large class of possible evolutions of the particle's state can be described easily via particular physical processes.

Furthermore such particles do occur:

* electrons
* protons
* certain atomic nuclei (e.g. carbon-13).
* in NMR and MRI

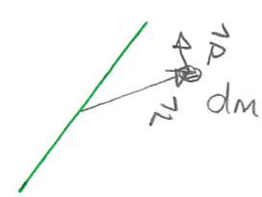
Before presenting the quantum formalism for describing spin-1/2 particles we will develop a classical analogy.

Classical spinning charged object



Consider a classical rigid object that spins about an axis through its center-of-mass. Then each segment of mass orbits and will have an angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$



Then the angular momentum of the entire object would be the sum over all such pieces.

For a rotating object this is called spin angular momentum $\vec{S}$.

For a classical rigid object important facts are:

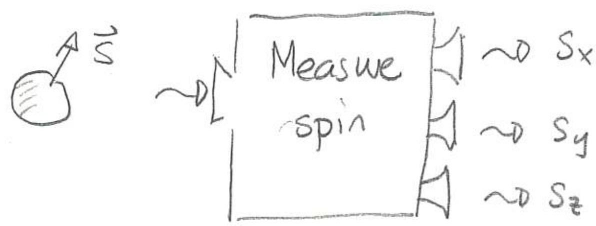
1) the angular momentum is a vector

$$\vec{S} = S_x \hat{x} + S_y \hat{y} + S_z \hat{z}$$

with three components S_x, S_y, S_z

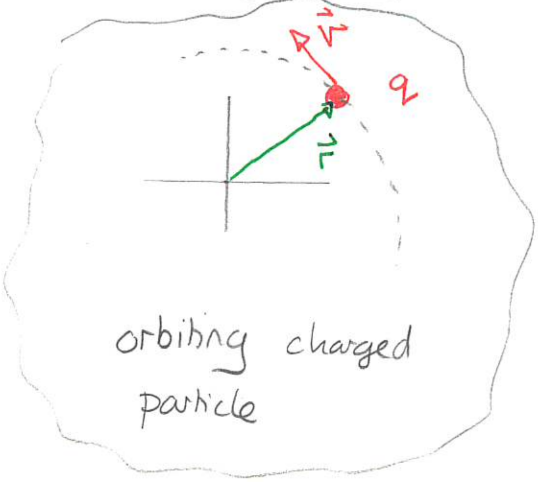
2) the angular momentum vector points along the axis of rotation using the r.h.rule

3) we can measure all three components of the spin with arbitrary precision



all with no uncertainty/error

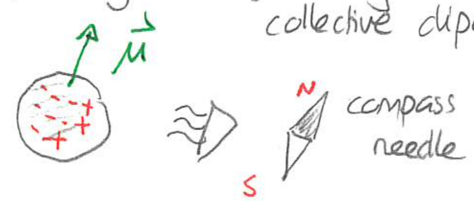
How could this be done for atomic scale particles? If the particle is charged then we can use the rotating charge and its electromagnetic interactions to measure spin. The intermediary here is an electromagnetic quantity called magnetic dipole moment. Classical electromagnetism gives:



Magnetic dipole moment vector

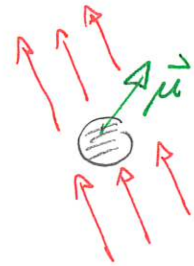
$$\vec{\mu} = \frac{1}{2} \vec{r} \times q\vec{v}$$

Spinning charged object has collective dipole moment



When placed in an external magnetic field $\vec{B}$ a magnetic dipole has potential

$$V = -\vec{\mu}\cdot\vec{B}$$

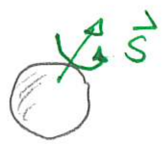


and this gives rise to a force

$$\vec{F} = -\vec{\nabla} V$$

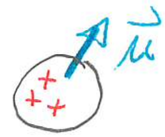
But if both charge and mass are rotating then the object has both spin angular momentum and a magnetic dipole moment. So

Rotating mass



spin angular momentum

Rotating charge



magnetic dipole moment

Related by

$$\vec{\mu} = \frac{g}{2}\frac{Q}{M}\vec{S}$$

where Q = total charge
M = total mass
g = "g-factor"

$\Longrightarrow$

By measuring magnetic dipole moment and knowing the other terms we can determine spin $\vec{S}$

1 Measuring spin via magnetic dipole moment

Consider a particle with mass M, charge $Q > 0$ and g-factor $g > 0$.

a) Suppose that you measured the magnetic dipole moment $\boldsymbol{\mu}$. Determine an expression for the spin $\mathbf{S}$.

Suppose that the particle is placed in an external magnetic field which has the form $\mathbf{B} = B(z)\hat{\mathbf{z}}$ where $B(z) > 0$.

b) What orientation of the magnetic dipole results in the highest energy?|

c) What orientation of the magnetic dipole results in the lowest energy?

d) Determine an expression for the force exerted on the dipole.

e) If the dipole enters the field traveling along the x direction, how will it respond to this field? How does the response depend on the dipole moment?

f) Use these ideas to describe how you could measure the z-component of the spin of the particle. How could you measure the other components?

Answer: a) $\vec{\mu} = \frac{g}{2}\frac{Q}{M}\vec{S} \Rightarrow \vec{S} = \frac{2M}{gQ}\vec{\mu}$

b) V = energy = $-\vec{\mu}\cdot\vec{B}$ is largest when $\vec{\mu}\cdot\vec{B}$ is most negative $\Rightarrow$ $\vec{\mu}$ opposite to $\vec{B}$ $\quad \downarrow\vec{\mu}$

c) V = energy = $-\vec{\mu}\cdot\vec{B}$ is smallest when $\vec{\mu}\cdot\vec{B}$ is most positive

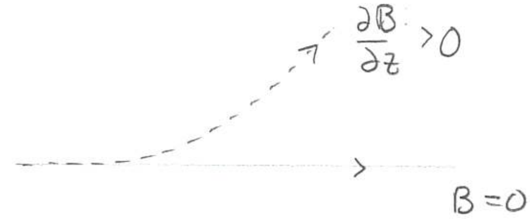
$\Rightarrow$ $\vec{\mu}$ same direction as $\vec{B}$ $\quad \uparrow\vec{\mu}$

d) $\vec{F} = -\vec{\nabla}\cdot V = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z}$ $\qquad V = -\vec{\mu}\cdot\vec{B}$

$= -\mu_z B(z)$

$\Rightarrow \vec{F} = -\left(-\mu_z \frac{\partial B}{\partial z}\right)\hat{z} \Rightarrow \vec{F} = \mu_z \frac{\partial B}{\partial z}\hat{z}$

e) There will be a force in the z-direction
This will deflect the particle
in a curved path.



The acceleration will be
proportional to μ_z

$\Rightarrow$ deflection proportional to μ_z.

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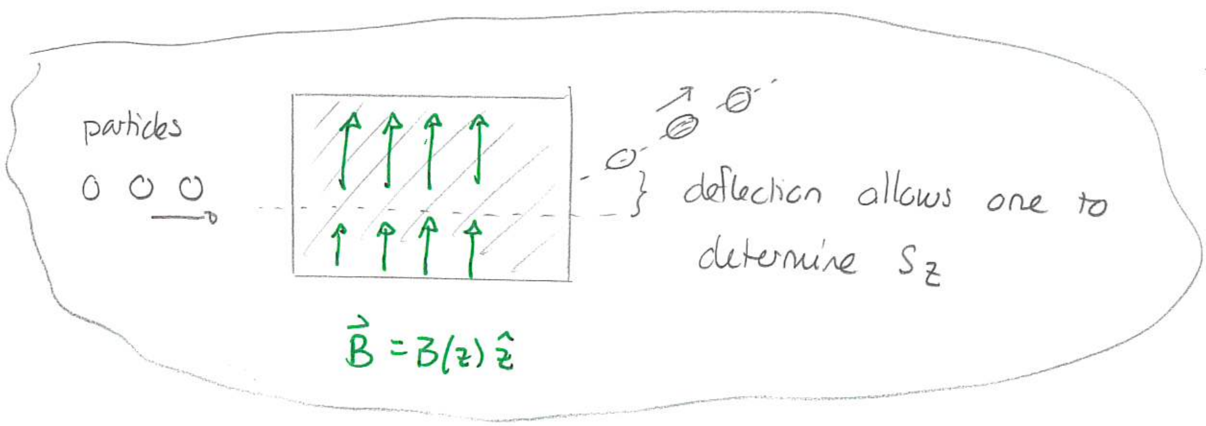
f) * Fire the particle along $\hat{x}$ into a magnetic field with a gradient in the z-direction.

* measure the particle deflection $\rightarrow$ proportional to μ_z
* calculate μ_z from deflection.
* $S_z = \frac{2M}{gG} \mu_z$

To measure say y component, create a magnetic field $\vec{B} = B(y)\hat{y}$ ...

Thus we have

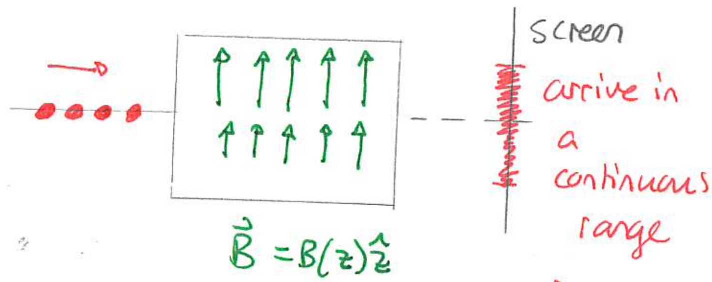


Stern-Gerlach experiment

The method of subjecting particles to a transverse magnetic field allows us to determine one component of the spin angular momentum. It, in effect, measures that component of the spin So we could ask:

> Given a collection of particles, which are randomly prepared, what are the possible outcomes for a measurement of S_z?

In classical physics we expect a continuous distribution of outcomes because the orientations of the magnetic dipole moments and spins would be random.



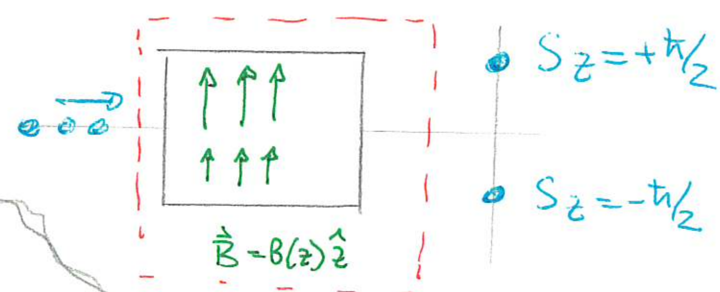
$\Rightarrow$ continuous range of S_z

Demo: Tontest quantique video

This experiment was first done with atomic particles by Otto Stern and Walter Gerlach in Frankfurt in 1922.

Demo: Physics Today Article

The result was that there are only two possible outcomes for a measurement of any spin component. This is not true of all subatomic particles but it is in direct conflict with classical mechanics. Thus



Measures S_z

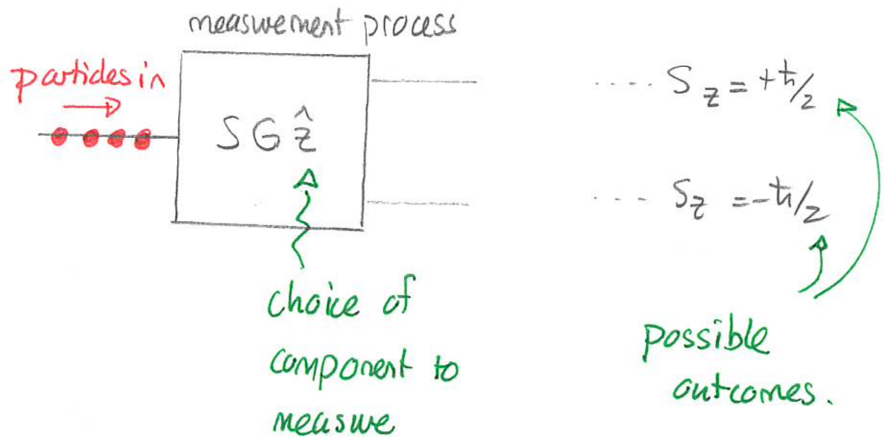
> A spin-$\frac{1}{2}$ particle is such that a measurement of any single component of spin yields <u>one</u> of <u>two</u> possible outcomes $+\hbar/2$ and $-\hbar/2$

Here $\hbar = \frac{h}{2\pi}$ is Plancks' constant

Thus we have that:

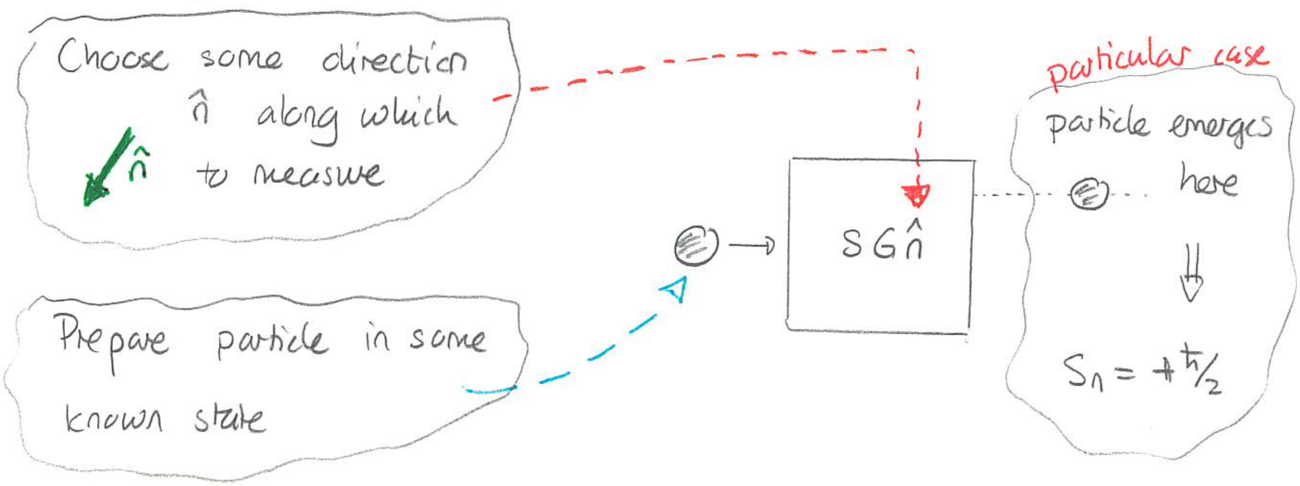
$$\boxed{\begin{array}{l}\text{For a spin-}1/2\text{ particle a measurement of } S_z \\ \text{will yield } \underline{\text{one}} \text{ of} \\ \qquad S_z = +\hbar/2 \quad \text{or} \quad S_z = -\hbar/2\end{array}}$$

We represent this schematically as:



We can extend this to measuring the component along any direction $\hat{n}$ (e.g. $\hat{n} = \hat{x}$ or $\hat{n} = \hat{y}$ or $\hat{n} = \frac{1}{\sqrt{2}}(\hat{x}+\hat{z})$) where $\hat{n}$ is a unit vector.

So we get



The key question is:

Given: * information about the preparation of the particle
* a choice of component to measure, S_n

then * what outcome will we obtain

$$S_n = ??$$

Demo: SG experiment animation Do with

- Spin up electron
- Spin down electron
- Electron in superposition

In general:

Quantum theory will not give a method for predicting the outcome of a measurement with certainty. It will give a method for predicting the probability with which various outcomes occur.