Quantum Theory I: Homework 6

Due: 14 February 2023

1 Bra vectors and measurements

a) Suppose that a particle in the state

$$|+\hat{\boldsymbol{n}}\rangle = \frac{1}{\sqrt{2}}\,|+\hat{\boldsymbol{z}}\rangle + \frac{3-4i}{5\sqrt{2}}\,|-\hat{\boldsymbol{z}}\rangle$$

is subjected to an SG $\hat{\boldsymbol{y}}$ measurement. Use

$$|+\hat{\boldsymbol{y}}\rangle = \frac{1}{\sqrt{2}}\,|+\hat{\boldsymbol{z}}\rangle + i\frac{1}{\sqrt{2}}\,|-\hat{\boldsymbol{z}}\rangle$$

to compute $\langle +\hat{\mathbf{y}}|$ and use this to compute the probability with which $S_y = +\hbar/2$.

b) Suppose that $\hat{\mathbf{m}} = (\hat{\mathbf{y}} + \hat{\mathbf{z}})/\sqrt{2}$ and a particle emerges from an SG$\hat{\mathbf{m}}$ measurement with $S_m = +\hbar/2$. This particle is then subjected to an SG$\hat{\mathbf{n}}$ measuring device where $\hat{\mathbf{n}} = (\hat{\mathbf{x}} - \hat{\mathbf{y}})/\sqrt{2}$. Determine expressions for $\langle +\hat{\mathbf{n}}|$ and $\langle -\hat{\mathbf{n}}|$ and use these to compute the probabilities with which the outcomes $S_n = +\hbar/2$, and $S_n = -\hbar/2$ occur.

2 Matrices and operators on kets and bras

Let

$$\hat{A} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

in the $\{|+\hat{\boldsymbol{z}}\rangle, |-\hat{\boldsymbol{z}}\rangle\}$ basis.

a) Use matrix operations to determine each of:

$$\hat{A}\,|+\hat{\boldsymbol{x}}\rangle \qquad \hat{A}\,|-\hat{\boldsymbol{x}}\rangle \qquad \hat{A}\,|+\hat{\boldsymbol{y}}\rangle \qquad \hat{A}\,|-\hat{\boldsymbol{y}}\rangle \qquad \hat{A}\,|+\hat{\boldsymbol{z}}\rangle \qquad \hat{A}\,|-\hat{\boldsymbol{z}}\rangle$$

and express the results in terms of $\{|+\hat{\boldsymbol{z}}\rangle, |-\hat{\boldsymbol{z}}\rangle\}$.

b) Use matrix operations to determine each of:

$$\langle +\hat{\boldsymbol{x}}|\,\hat{A} \qquad \langle -\hat{\boldsymbol{x}}|\,\hat{A} \qquad \langle +\hat{\boldsymbol{y}}|\,\hat{A} \qquad \langle -\hat{\boldsymbol{y}}|\,\hat{A} \qquad \langle +\hat{\boldsymbol{z}}|\,\hat{A} \qquad \langle -\hat{\boldsymbol{z}}|\,\hat{A}$$

and express the results in terms of $\{\langle +\hat{\boldsymbol{z}}|, \langle -\hat{\boldsymbol{z}}|\}$.

c) Evaluate

$$\langle +\hat{\boldsymbol{z}}|\,\hat{A}\,|+\hat{\boldsymbol{y}}\rangle \qquad \langle -\hat{\boldsymbol{z}}|\,\hat{A}\,|+\hat{\boldsymbol{y}}\rangle \qquad \langle +\hat{\boldsymbol{y}}|\,\hat{A}\,|+\hat{\boldsymbol{z}}\rangle \qquad \langle -\hat{\boldsymbol{y}}|\,\hat{A}\,|+\hat{\boldsymbol{z}}\rangle$$

d) This operator actually corresponds to a physical operation that can be performed on the spin-1/2 particle. The operation can be described via the following: a spin-1/2 particle emerges from a first Stern-Gerlach measurements, then undergoes the operation described by $\hat{A}$ and is subsequently subjected to a second Stern-Gerlach measurement with a direction chosen such that it gives the $+\hbar/2$ outcome with certainty. Using the various combinations calculated in the previous part, identify directions of the two SG measurements (they will vary from one case to another) so that this process will occur. Your statements should be of the form:

"A particle emerges from SG $\hat{\boldsymbol{m}}$ with outcome $+\hbar/2$, then undergoes the operation described by A, and after that is subjected to SG $\hat{\boldsymbol{n}}$ giving outcome $+\hbar/2$ with certainty."

You should identify various combinations for $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ that make this true in each case. Can you see how $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ are related?

3 Operators constructed from kets and bras

Let

$$\hat{A} = \left|+\hat{\boldsymbol{x}}\right\rangle \left\langle-\hat{\boldsymbol{z}}\right|$$
$$\hat{B} = \left|-\hat{\boldsymbol{x}}\right\rangle \left\langle+\hat{\boldsymbol{y}}\right|$$

a) Determine the matrices representing $\hat{A}$ and $\hat{B}$ in the $\{\left|+\hat{\boldsymbol{z}}\right\rangle, \left|-\hat{\boldsymbol{z}}\right\rangle\}$ basis.

b) Determine expressions for

$$\hat{A}\left|+\hat{\boldsymbol{z}}\right\rangle \qquad \hat{A}\left|-\hat{\boldsymbol{z}}\right\rangle \qquad \hat{B}\left|+\hat{\boldsymbol{z}}\right\rangle \qquad \hat{B}\left|+\hat{\boldsymbol{y}}\right\rangle.$$