Quantum Theory I: Homework 1

Due: 27 January 2023

1 Stern-Gerlach Experiment

Suppose that a beam of particles, each with mass M and velocity $\boldsymbol{v} = v_x \hat{\mathbf{x}}$, enters a region in which the magnetic field is

$$\boldsymbol{B} = (B_0 z) \, \hat{\mathbf{z}}$$

where $B_0 = 600$ Tesla/m. This region extends in the x direction for distance L_B . A detector is placed L_D beyond the end of the magnetic field region. The setup is illustrated in Fig. 1.



Figure 1: Question 1

- a) Suppose that prior to entering the magnetic field, the z component of each particle's magnetic dipole moment has the same value, μ_z . Find an expression, in terms of M and μ_z , for the acceleration of the particles while they are in the region with non-zero magnetic field. Ignore all forces on any particle except that exerted by the magnetic field.
- b) Assume that the particles follow trajectories governed by classical mechanics. Find an expression for the total deflection in the z direction in terms of M, μ_z, v_x, L_B and L_D . Verify that it is proportional to μ_z .

2 Spin-1/2 quantum states

An ensemble of 100 spin-1/2 particles are all prepared in exactly the same state and are subjected to the indicated Stern-Gerlach measurement.



- a) Suppose that the particles were initially in the state $|+\hat{y}\rangle$. Where will they emerge?
- b) Suppose that for some but not all of the particles the measurement outcome is $S_y = -\hbar/2$. What could you conclude with absolute certainty about the state of the particles prior to measurement? Explain your answer.
- c) Suppose that for all of the particles the measurement outcome is $S_y = -\hbar/2$. What could you conclude with absolute certainty about the state of the particles prior to measurement? Explain your answer.
- d) Suppose that the outcome of the measurement is unknown. However, after the first measurement the emerging particles are subjected to an additional SG \hat{y} apparatus and they all emerge with $S_y = +\hbar/2$. What can you conclude about the outcome of the first measurement?
- e) Suppose that the outcome of the measurement is unknown. However, after the first measurement the emerging particles are subjected to an additional SG \hat{y} apparatus and some emerge with $S_y = +\hbar/2$ while others emerge with $S_y = -\hbar/2$. Is it possible that the result of the first measurement was only one outcome for S_y ? Explain your answer.