Phys 131 2023

Lecture 30

Thurs: Discussion quiz

Ex 246a, 249, 251ab
Ex 253, 254, 255, 256
ensure do some from both

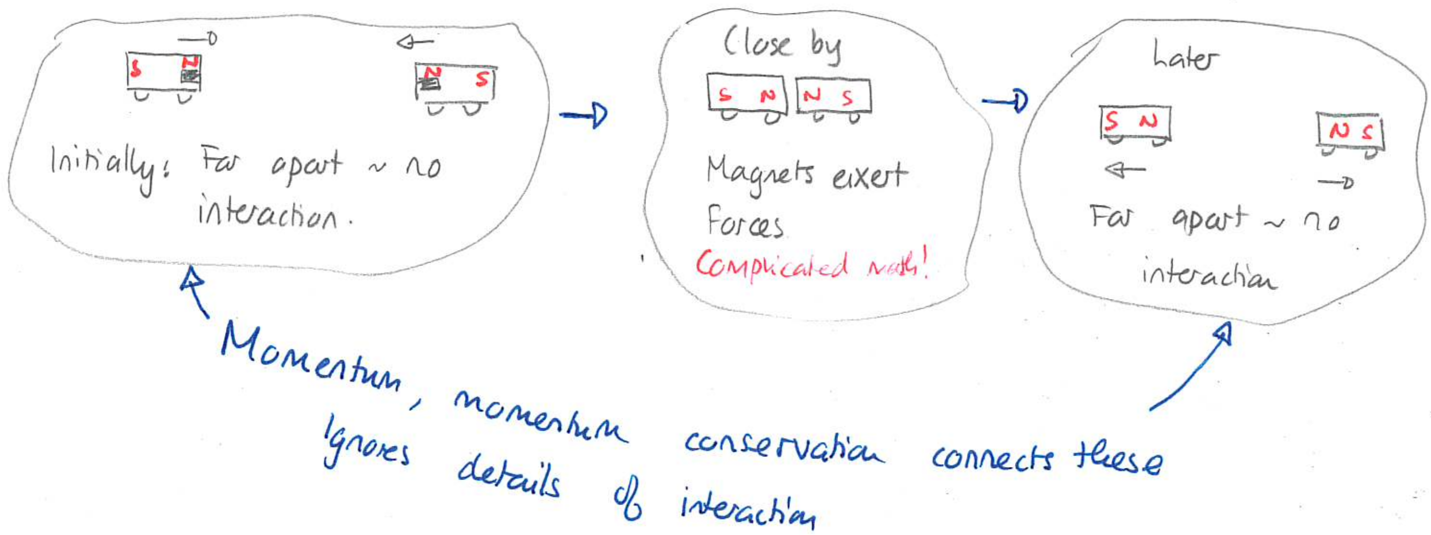
Fri: Group Ex

Interactions in isolated systems

There are physical systems that consist of objects that interact with each other but are effectively isolated from their surroundings.

Demo: Track + cart collisions.

Consider two carts that interact via magnets



In such interactions acceleration will not be constant and it could be very difficult to apply Newton's laws directly to relate the motion at a later time to earlier. Energy is also not conserved since non-conservative forces do non-zero work. This analysis will be useful for collisions and explosions

Demo: * Ship collision video
* Particle collisions

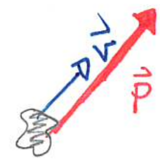
Momentum:

The crucial concept for describing these situations is momentum

The momentum of an object with mass m and velocity $\vec{v}$ is

$$\vec{p} = m\vec{v}$$

Units : kgm/s



mass m

Notes:

1) momentum is a vector
2) the direction of the momentum vector is the same as the direction of the velocity vector
3) objects with the same velocities can have different momenta
4) objects " " same masses " " " "

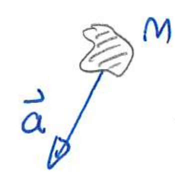
Warm Up!

Momentum and Newton's Second Law

Consider a single object with mass m. Then Newton's Second Law states.

$$\vec{F}_{net} = m\vec{a}$$

$$= m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$



$$\Rightarrow \vec{F}_{net} = \frac{d\vec{p}}{dt}$$

So force describes the rate at which momentum changes. In many situations this reformulation is useful.

Conservation of momentum

Consider a single isolated object that does not interact with its surroundings. Then $\vec{F}_{net} = 0$ and thus

$$\text{isolated object:} \quad \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} \text{ is constant}$$

This can be extended to a system of objects using a combination of Newton's Second and Third Laws. First we define:

The net / total momentum of a system is

$$\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$$

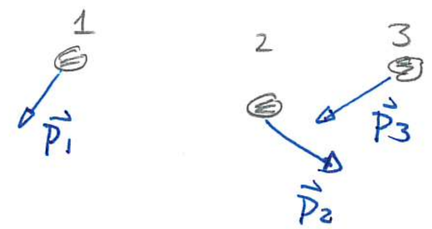
where

$$\vec{p}_i = m_i \vec{v}_i$$

is the momentum of object i. Here

m_i = mass object i

$\vec{v}_i$ = velocity " i



The total momentum is a vector sum.

Quiz 1 95% {80% – 95%

Then Newton's Laws give:

Consider a system of objects. The net external force on the system is the sum of all the forces exerted by objects outside the system on objects in the system. Then

$$\text{net external force} = 0 \Leftrightarrow \frac{d\vec{p}_{tot}}{dt} = 0 \Leftrightarrow \text{momentum is constant.}$$

This is the conservation of momentum.

Quiz 2 50% – better.. {80% – 95%

Warm Up 2

Quiz 3 not done

Proof: (Two objects)

Consider two objects that interact with each other but not their surroundings. Then:

$$\frac{d\vec{p}_{tot}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt}$$

$$= \vec{F}_{net\,1} + \vec{F}_{net\,2}$$

But there is only one force on each object. Thus

$$\vec{F}_{net\,1} = \vec{F}_{2on1}$$

$$\vec{F}_{net\,2} = \vec{F}_{1on2}$$

$$\Rightarrow \quad \frac{d\vec{p}_{tot}}{dt} = \vec{F}_{2on1} + \vec{F}_{1on2}$$

Newton's Third Law states: $\vec{F}_{2\,on\,1} = -\vec{F}_{1on2}$. Thus

$$\frac{d\vec{p}_{tot}}{dt} = 0$$

∎

This extends in the same fashion to more than two objects.

261 Space collision

A 100 kg astronaut is at rest in space. A 0.0050 kg fleck of paint moves toward the astronaut with speed 15×10^3 m/s. It collides with and sticks to the astronaut. *(131Sp2023)*

a) Write an expression for the total momentum of the system before the collision in terms of the masses and speeds of the astronaut and paint. Determine the total momentum of the system before the collision (assume that the paint fleck initially moves along the positive x axis).

b) Write an expression for the total momentum of the system after the collision in terms of the masses and speeds (after collision) of the astronaut and paint.

c) Use momentum conservation to determine the speed of the astronaut after the collision.

d) Now suppose that the paint fleck bounced off the astronaut and reverses direction with speed 8.0×10^3 m/s. Determine the speed of the astronaut after the collision.

Ans:

Before

$V_{pi} = 15\times10^3 m/s$

$V_{ai} = 0 m/s$

After

$V_{pf} = V_{af} = V_f = ?$

We only need horizontal components of momentum

a) $P_{tot\,i} = M_p V_{pi} + M_a V_{ai} = 0.0050 kg \times 15\times10^3 m/s + 0 \, kg m/s = 75 \, kg m/s.$

b) $P_{tot\,f} = M_p V_{pf} + M_a V_{af} = M_p V_f + M_a V_f = (M_p + M_a) V_f$

c) $P_{tot\,f} = P_{tot\,i} \Rightarrow (M_p + M_a) V_f = 75 kg m/s$

$\Rightarrow (100 kg + 0.0050 kg) V_f = 75 kg m/s$

$\Rightarrow V_f = 0.75 m/s$

d) $P_{tot\,f} = P_{tot\,i}$

$\Rightarrow M_p V_{pf} + M_a V_{af} = M_p V_{pi} + \cancel{M_a V_{ai}}$

$\Rightarrow M_p(-8.0\times10^3 m/s) + M_a V_{af} = 75 kg m/s$

$\Rightarrow -5.0\times10^{-3} kg \times 8.0\times10^3 m/s + M_a V_{af} = 75 kg m/s$

$\Rightarrow M_a V_{af} = 115 kg m/s$

$\Rightarrow 100 kg \, V_{af} = 115 kg m/s$

$\Rightarrow V_{af} = 1.2 m/s.$