Mon: HW by 5pm

Tues: Warm Up 2 (D2L)

Thurs: Discussion / quiz

* Ex 22, 24, ~~25~~ 26, 28, 30, 32, 37, 40
* Procedure: * attempt / complete before class
  * do not turn in
  * discuss in class in groups
  * 10 min question at end of class.

Counts (5pt) (eventual overall total 600 pts)

* Just as important for learning as turn in.

Warm Up2 from Friday

Calculating velocity

Given x as a function of t, calculus gives

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \sim \text{"derivative of } x \text{ with respect to } t\text{"}$$

Calculus gives rules for computing derivatives.

If $x = at^n$ where a, n are constants, then $\frac{dx}{dt} = nat^{n-1}$

23 Velocity as a derivative, 1

Suppose that the position of an object is

$$x = (5\,\mathrm{m/s^2})\,t^2 + (3\,\mathrm{m/s})\,t$$

Determine the velocity of the object at $t = 3\,\mathrm{s}$. *(131Sp2023)*

Answer: v = derivative of x w.r.t. t

$= \text{deriv of } (5t^2) + \text{deriv}(3t)$

$= 2 \cdot 5\, t^{2-1} + 1 \cdot 3\, t^{1-1}$

$= 10\,\mathrm{m/s^2}\, t + 3\,\mathrm{m/s}$

At 3s $v = 10\,\mathrm{m/s^2} \times 3\,\mathrm{s} + 3\,\mathrm{m/s} = 33\,\mathrm{m/s}$

Velocity Schematic

The flow of ideas about velocity is:

Conceptual Idea → Velocity ~ rate of change of position

↓

Mathematical definition

Observe change in position Δx over time interval Δt. Velocity is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Speed = absolute value of velocity

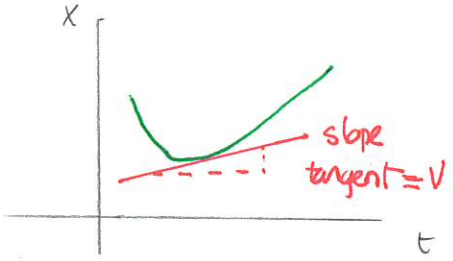
$$s = |v|$$

↓ calculation / computation

Given position information get velocity via

1) calculus ~ differentiation
2) graphically

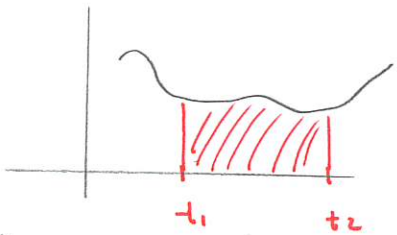
v = slope of tangent to x vs t



Given velocity information get position via

1) calculus ~ integration
2) graphically

Δx = area between graph of v vs t and axis

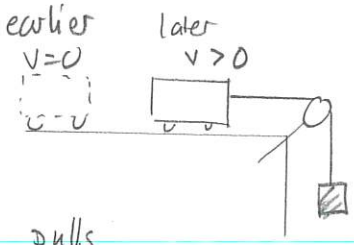


3) $x_2 = x_1 + \Delta x$ gives position but requires x_1

Acceleration

In classical physics it will emerge that interactions between objects do not immediately describe velocity but rather the change in velocity

One can consider this via a cart on a track connected to a suspended object. The cart is released from rest and speeds up as the string pulls.



The crucial concept will then be

> acceleration ~ rate at which velocity changes

We usually associate this with a change in speed but there are other ways for an object to accelerate.

Demo: PhET Moving Man → Charts Tab

* Initial $x_0 = 0$
  $v_0 = -6$
  $a = 2$

* observe – apparent motion
  – graph v vs. t

* three phases – start to ~~turn at left~~ just before turn at left
  – ~~turn at left~~ just before/after turn at left to just after
  – after turn at left to later

The animation shows that the velocity is constantly changing. It shows that non-zero acceleration can occur if

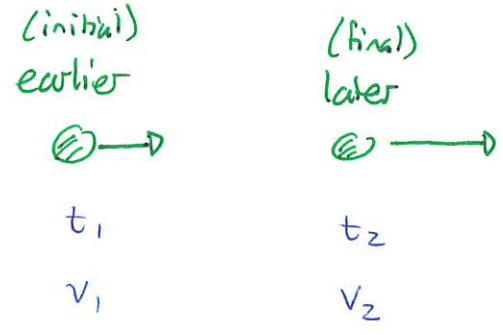
1) the object's speed changes (either increases or decreases)
2) the object's direction of motion changes.

We would like a single definition that captures all of these.

A preliminary definition is the <u>average acceleration</u>

Consider an object at two instants

Then the average acceleration of the object over the interval from t_1 to t_2 is



$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Units: m/s^2

Quiz 1 90% → 95% ≷ 95%

Quiz 2 90% → 95% ≷ 90%

t	v
0.0s	-6.0m/s
1.0s	-4.0m/s
2.0s	-2.0m/s
3.0s	0.0m/s
4.0s	2.0m/s
5.0s	4.0m/s

For the data of these two questions we see:

In every second the velocity increases by exactly +2.0m/s

⇒ acceleration is +2.0m/s per second

⇒ acceleration is $+2.0\,m/s^2$.

Important points:

1) acceleration does not describe velocity, e.g.

~ moving man demo → same acceleration but velocity varies with time.

Quiz 3 → 85% → 95% ≷ 95% →

2) there is no general correlation between acceleration and velocity.

~ comparing two objects, one can have a larger acceleration but a smaller speed

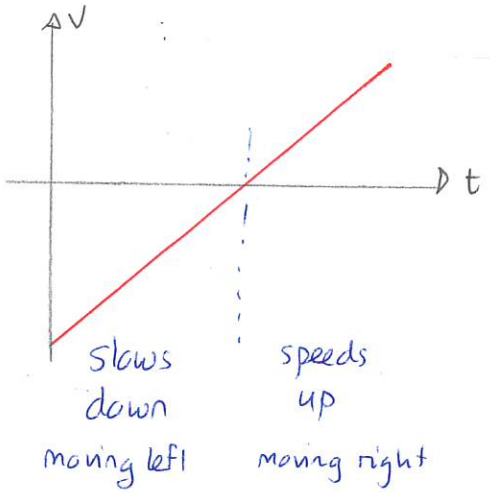
Quiz 4 20% – 90% } 30% – 50%

3) acceleration is not immediately connected to speed. An object's speed may not change between two instants but it can have non-zero acceleration if its direction of motion changes.

4) acceleration can be positive or negative.

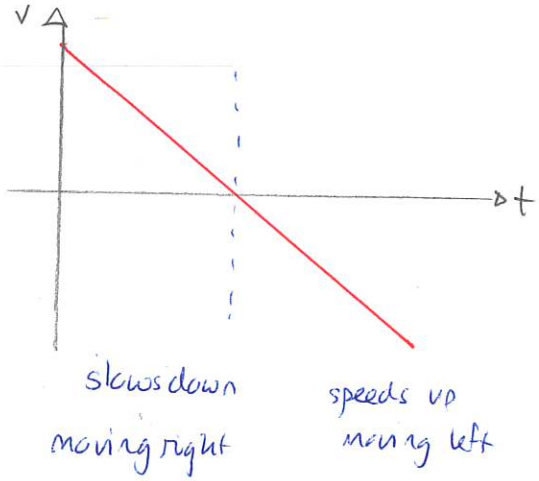
Positive acceleration

Velocity (number) increases



Negative acceleration.

Velocity (number) decreases.



Quiz #5 80% – 95% } 95%

Quiz 6 50% – 90% } 40%

In general

Acceleration describes how velocity (number) changes over time