Fundamental Mechanics: Class Exam 3

11 November 2022

Name: SOLUTION **Total:** /70

Instructions

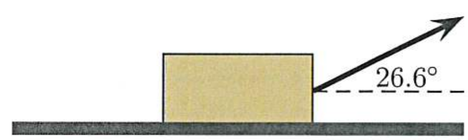
- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$g = 9.81\,\text{m/s}^2$$

Question 1

A crate is dragged along a rough horizontal surface by a rope and moves 4.0 m during a certain period of observation. The rope exerts a 100 N force along the illustrated direction.



a) Determine the work done by the rope during the period of observation.

$W_{rope} = \overbrace{F\,\Delta r \cos\theta}$ +1

$= \underbrace{100\,\text{N} \times 4.0\,\text{m} \times \cos 26.6^\circ = 358\,\text{J}}$ +2

b) During the period of observation, the cart moves with constant speed. Determine the work done the kinetic friction force.

$\overbrace{W_{net} = \Delta K}$ +1

+2

$W_{rope} + W_{fric} + \cancel{W_n + W_{grav}} = \Delta K = 0$

perpendicular $= 0$

$W_{fric} = -W_{rope} = -358\,\text{J}$

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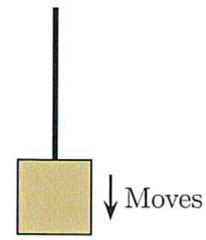
Question 2

A crate is suspended by a rope and is lowered vertically. While this happens, which of the following (choose one) is true about the work done by the rope, W?



i) $W > 0$ if the crate speeds up and $W < 0$ if it slows down.
ii) $W < 0$ if the crate speeds up and $W > 0$ if it slows down.
iii) $W > 0$ regardless of the speed.
iv) $W < 0$ regardless of the speed.

$$W = \vec{T} \cdot \Delta\vec{r} = T\,\Delta r \cos 180^\circ$$
$$= -T\Delta r < 0$$

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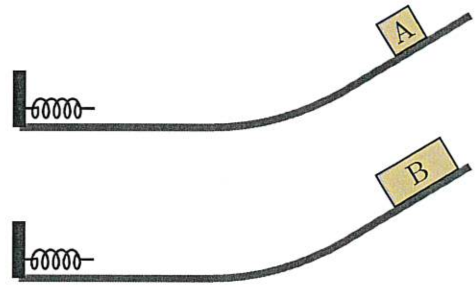
Question 3

Two boxes are released from rest at the same height on identical frictionless surfaces. The mass of box B is four times that of box A. They collide with and compress identical springs. Which of the following (choose one) is true regarding the maximum compression of each spring (Δx_A for that with box A, Δx_B for that with box B)?



i) $\Delta x_B = 4\Delta x_A$.
ii) $\Delta x_B = 2\Delta x_A$.
iii) $\Delta x_B = \Delta x_A$.
iv) $\Delta x_B = \frac{1}{2}\Delta x_A$.
v) $\Delta x_B = \frac{1}{4}\Delta x_A$.

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$$E_f = E_i$$

$$\cancel{K_f} + \cancel{U_{gf}} + U_{spf} = \cancel{K_i} + U_{gi} + \cancel{U_{spi}}$$

$$\Rightarrow \frac{1}{2}k(\Delta x)^2 = mgy_i$$

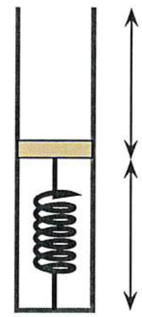
$$\Rightarrow (\Delta x)^2 = \frac{2mgy_i}{k} \Rightarrow \Delta x = \sqrt{\frac{2mgy_i}{k}} = \underbrace{\sqrt{m}}_{\sqrt{\text{four times for } B}} \underbrace{\sqrt{\frac{2gy_i}{k}}}_{\text{same}}$$

$= 2$ times for B

$$\Rightarrow \Delta x_B = 2\Delta x_A$$

Question 4

A 2.0 kg block can move along a vertical cylinder. The block is held at rest against a spring with spring constant 400 N/m, compressing it by 0.25 m. At this instant the base of the block is 0.25 m beneath the top of the cylinder. The block is released and when the base reaches the top of the cylinder, it leaves the spring, moving with speed 2.0 m/s. While it moves up the cylinder a constant kinetic friction force acts on the block. For purposes of gravitational potential energy, let $y = 0$ correspond to the situation where the bottom of the block would lie on the base of the cylinder.



$v_f = 2.0\,\text{m/s}$

$y_f = 0.50\,\text{m}$

$\Delta x_f = 0\,\text{m}$

a) Determine the energy of the system at the moment that the block is released.

$$E_i = K_i + U_{gi} + U_{spi}$$

$$= \underbrace{\cancel{\tfrac{1}{2} m v_i^2}}_{+1} + \underbrace{m g y_i}_{+1} + \underbrace{\tfrac{1}{2} k (\Delta x_i)^2}_{+1}$$

$v_i = 0\,\text{m/s}$

$\Delta x_i = 0.25\,\text{m}$

$y_i = 0.25\,\text{m}$

$$= 2.0\,\text{kg} \times 9.8\,\text{m/s}^2 \times 0.25\,\text{m} + \tfrac{1}{2}\, 400\,\text{N/m}\, \underbrace{(0.25\,\text{m})^2}_{+1} = 17.4\,\text{J} \quad +2$$

b) Determine the work done by the friction force and use this to determine the magnitude of the friction force.

$\Delta E = W_{nc}$ and $W_{nc} = W_{fric}$ +1

$\Rightarrow E_f = E_i + W_{fric}$ +1

$K_f + U_{gf} + U_{spf} = 17.4\,\text{J} + W_{fric}$

+2 $\tfrac{1}{2} m v_f^2 + m g y_f + \cancel{\tfrac{1}{2} k (\Delta x_f)^2} = 17.4\,\text{J} + W_{fric}$

$\Delta \vec{r}$ ↑, $\vec{f}_k$ ↓

$W_{fric} = \underbrace{f_k \Delta r \cos 180°}_{+1}$

$= - f_k \Delta r$

$= - f_k\, 0.25\,\text{m}$ +1

+4

$\tfrac{1}{2} \times 2.0\,\text{kg} \times (2.0\,\text{m/s})^2 + 2.0\,\text{kg} \times 9.8\,\text{m/s}^2 \times 0.50\,\text{m} = 17.4\,\text{J} + W_{fric}$

$13.8\,\text{J} = 17.4\,\text{J} + W_{fric}$

$\Rightarrow W_{fric} = 13.8\,\text{J} - 17.4\,\text{J}$

$= -3.6\,\text{J} \Rightarrow - f_k\, 0.25\,\text{m} = -3.6\,\text{J}$

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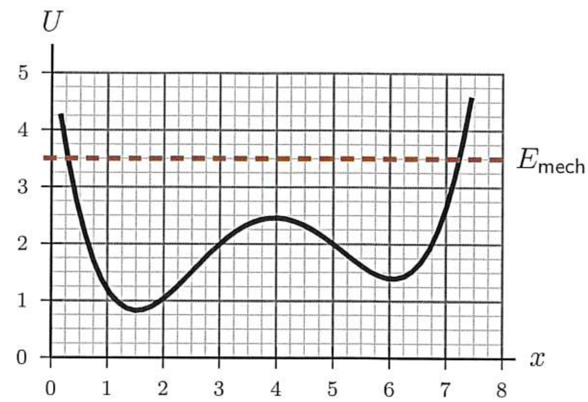
$\Rightarrow f_k = \dfrac{3.6\,\text{J}}{0.25\,\text{m}} = 14.4\,\text{N}$

Question 5

A particle with the illustrated total mechanical energy moves subject to the illustrated potential. Indicate all locations at which the **speed is a maximum** and **the force on the particle is zero**. Explain your answer.



$E = K + U$

$\Rightarrow K = E - U$

+3

$\Rightarrow$ Speed is max where K is maximum

$\Rightarrow$ Speed is maximum where U is minimum $\Rightarrow$ speed max at 1.5m

$\underbrace{F_x = -\frac{dU}{dx} = 0}_{+2}$ when slope $= 0$

$\underbrace{\Rightarrow \text{at } x = 1.5\text{m}, 4.0\text{m}, 6.0\text{m}}_{+3}$

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Question 6

A particle can move horizontally along the x axis. The potential energy of the particle is $U = ax^2 + bx^4$ where $a = -6.0\,\mathrm{J/m^2}$ and $b = 3.0\,\mathrm{J/m^4}$. Determine the force on the particle, including direction, at $x = 2.0\,\mathrm{m}$.

$F = -\frac{dU}{dx} = -\left[2ax + 4bx^3\right]$

$= -\left[-12\,\mathrm{J/m^2}\,x + 12\,\mathrm{J/m^4}\,x^3\right]$

$= -\left[-12\,\mathrm{J/m^2} \times 2.0\,\mathrm{m} + \frac{12\,\mathrm{J}}{\mathrm{m^4}}\,8\,\mathrm{m^3}\right]$

$= -72\,\mathrm{N}$ left.

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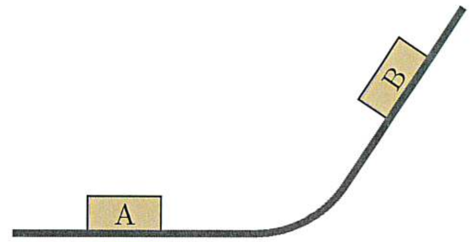
Question 7

Two ~~identical~~ blocks are initially at rest on a frictionless track as illustrated. Block B, with mass 2.0 kg is released from rest at an unknown height above the horizontal surface. It descends and collides with block A, with mass 4.0 kg. They stick together and subsequently move with speed 2.5 m/s.



a) Determine the speed of block B immediately before the collision.

Net external force is zero $\Rightarrow$ $p_{tot\,f} = p_{tot\,i}$ (+1)

$\boxed{A}$ $\leftarrow$ $\boxed{B}$

$V_{Ai} = 0$ $\quad V_{Bi} = ?$

$$M_A V_{Af} + M_B V_f = \cancel{M_A V_{Ai}}^{\,0} + M_B V_{Bi} \quad (+2)$$

$\boxed{A|B}$

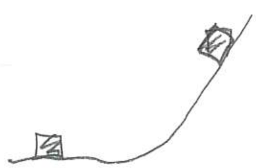
$V_{Af} = V_{Bf} = -2.5\,m/s$

$$\Rightarrow \underbrace{(2.0kg + 4.0kg)}_{(+1)}(-2.5m/s) = \underbrace{2.0kg}_{(+1)} V_{Bi}$$

$$\Rightarrow V_{Bi} = -7.5\,m/s \quad \leftarrow (+2)$$

b) Determine the height above the horizontal surface from which block B was released.

Consider B sliding down. Then $W_{nc} = 0 \Rightarrow \Delta E = 0$



$V_i = 0m/s$

$y_i = ?$

$$\Rightarrow E_f = E_i \quad (+1)$$

$$\Rightarrow K_f + U_{gf} = K_i + U_{gi}$$

$V_f = 7.5m/s$

$y_f = 0m$

$$\Rightarrow \tfrac{1}{2} m v_f^2 + \cancel{m g y_f} = \tfrac{1}{2} \cancel{m v_i^2} + m g y_i \quad (+1)\ (+1)$$

$$\Rightarrow \tfrac{1}{2} \cancel{m} v_f^2 = \cancel{m} g y_i \quad \text{correct masses } (+3)$$

$$\Rightarrow y_i = \frac{v_f^2}{2g} = \frac{(7.5m/s)^2}{2 \times 9.8m/s^2} \quad +3$$

$$\Rightarrow y_i = 2.9m$$

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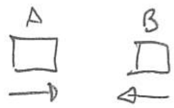
Question 8

Two ice skaters, Alice with mass 60 kg and Bob with mass 90 kg, slide toward each other on a sheet of ice. Alice moves right with speed 10 m/s and Bob moves left with speed 5.0 m/s. They collide and Bob subsequently moves right with speed 1.5 m/s. Ignore friction and air resistance.

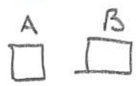
a) Determine ~~their~~ Alice's speed after the collision.

Before



$V_{Ai} = 10\,m/s$ (+1) $\quad V_{Bi} = \underline{-5.0\,m/s}$ (+2)

After



$V_{Af} = 0$ $\quad V_{Bf} = 1.5\,m/s$

Net external force = 0

$\Rightarrow \quad p_{tot\,f} = p_{tot\,i}$ (+1)

$m_A V_{Af} + m_B V_{Bf} = m_A V_{Ai} + m_B V_{Bi}$ (+2)

$60kg\, V_{Af} + 90kg \times 1.5\,m/s$

$= 60kg \times 10\,m/s - 90kg \times 5.0\,m/s$

$= 150\,kg\,m/s$ (+1)

$\Rightarrow \quad 60kg\, V_{Af} = 150\,kg\,m/s - 135\,kg\,m/s = 15\,kg\,m/s$

$\Rightarrow \quad V_{Af} = 0.25\,m/s$

b) Explain whether the total mechanical energy defined as $E = K + U_{\text{grav}}$ is conserved in this collision.

$U_g = 0$ so we need kinetic energies. (+3)

initially $\quad K_A + K_B = \frac{1}{2} M_A V_{Ai}^2 + \frac{1}{2} M_B V_{Bi}^2$

$= \frac{1}{2}\, 60kg \times (10\,m/s)^2 + \frac{1}{2}\, 90kg \times (5.0\,m/s)^2 = 4125\,J$

finally $\quad K_A + K_B = \frac{1}{2}\, 60kg \times (0.25\,m/s)^2 + \frac{1}{2}\, 90kg \times (1.5\,m/s)^2 = 103\,J$

Not conserved!

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