Phys 111 2023

Lecture 36

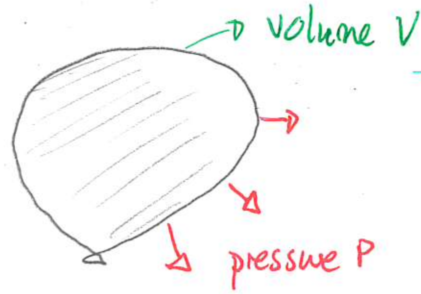
Weds: Warm Up 14

Thurs: Discussion ~~Rex~~ / Quiz

Ex:

Ideal gas processes

Recall that the state of a gas can be described by



n = number of moles of gas ($= N/N_A$)

V = volume of gas in m^3

P = pressure exerted by gas in Pa

T = temperature of gas in Kelvin (K) $\qquad T_K = T_C + 273°K$

These are not all independent and, for a sufficiently dilute gas, are related by the ideal gas law:

$$PV = nRT$$

where $R = 8.314\ J/mol \cdot K$ is the same for all gases. The ideal gas law applies when the gas is in thermal equilibrium. Gases can be made to change their states from one equilibrium state to another. At the beginning and end of such a process for an ideal gas, the ideal gas law applies.

Quiz 1 5% - 60%

To analyze this

$$PV = nRT$$

$$\frac{V}{T} = \frac{nR}{P} = \text{constant} \quad \Rightarrow \quad \frac{V_{final}}{T_{final}} = \frac{V_{initial}}{T_{initial}}$$

$$\Rightarrow V_{final} = V_{initial} \frac{T_{final}}{T_{initial}}$$

$$= V_{initial} \frac{373}{283}$$

$$= 1.37\ V_{initial}.$$

Quiz 2 20% -

Demo: Bottle / steam / balloon

Gases and energy

What type of energy does a gas contain? Can the energy be extracted from the gas? These were key questions from the early development of thermodynamics and the associated steam engines.

Demo: Rotative Steam Engine Watt Science Museum

A modern theory of gases as consisting of atoms can help answer these questions. The gas consists of individual atoms that can possess energy:

1) kinetic energy associated with the motion of each atom
2) potential energy associated with interactions between the atoms.

In an ideal gas there are no potential energies associated with interactions and so the gas has:

Thermal / internal energy, E_{th}, arising from the kinetic energies of the individual molecules or atoms	IDEAL GAS.

DEMO: PhET GAS PROPERTIES

- Energy Tab
- Add 100 heavy molecules
- Observe speeds

We observe that

1) at a given temperature there will be a variety of speeds
2) as temperature increases the typical speeds increase.

Given this range how could we calculate the kinetic energies? There are two possibilities

THERMODYNAMICS

* uses simple models or experimental evidence to determine rules for thermal energy
* does not consider individual molecules

STATISTICAL PHYSICS

* gives rules about probabilities of various speeds
* gives techniques for averaging to get the average kinetic energy per molecule

For an ideal gas that is monoatomic (only one atom per molecule) the thermal energy is

$$E_{th} = \frac{3}{2} nRT = \frac{3}{2} PV$$

Before describing the way in which the gas can supply or absorb energy in gas processes, we consider how the energy relates temperatures to speeds.

Temperature and motion of monoatomic gases

We consider a gas consisting of one type of monoatomic molecule (e.g. Helium gas). The only motional energy associated with these is translational kinetic energy. Let

K_{tot} = total kinetic energy of all molecules.

Then with N molecules the average kinetic energy per molecule is

$$K_{avg} = \frac{K_{tot}}{N}.$$

But $K_{tot} = E_{th} = \frac{3}{2} nRT$ gives

$$K_{avg} = \frac{3}{2}\frac{nRT}{N}$$

$$= \frac{3}{2}\frac{N/N_A \, RT}{N} \Rightarrow K_{avg} = \frac{3}{2}\frac{R}{N_A}T.$$

The term R/N_A is called Boltzmann's constant

$$k_B = R/N_A = 1.38 \times 10^{-23}\ J/K.$$

Thus

> For a monoatomic ideal gas the average kinetic energy per molecule is:
>
> $$K_{avg} = \frac{3}{2} k_B T$$

So, for these gases, temperature serves as a measure of average kinetic energy.

Quiz 3 80% – 95%

Quiz 4 70% – 80%

We can relate this to a typical speed measure, called the root-mean-square speed

$$V_{rms} = \sqrt{\text{average}(v^2)}$$

and can show

$$V_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

where m is the mass of one molecule

Proof

$$K_{avg} = \frac{1}{2} m \,\text{average}(v^2) = \frac{3}{2} k_B T$$

$$\Rightarrow \text{average}(v^2) = \frac{3 k_B T}{m}$$

$$\Rightarrow \sqrt{\text{average}(v^2)} = \sqrt{\frac{3 k_B T}{m}} \Rightarrow V_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

331 Helium atom speed

Helium is a monoatomic gas. Each atom has mass 6.65×10^{-27} kg. Determine the typical speed of a helium atom at room temperature (20° C).

Answer:

$$V_{rms} = \sqrt{\frac{3 k_B T}{m}}$$

$$T = 20 + 273 = 293\,K$$

$$V_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23}\ J/K \times 293\,K}{6.65 \times 10^{-27}\,kg}}$$

$$= 1350\,m/s$$