Phys111 2023

Lecture 31

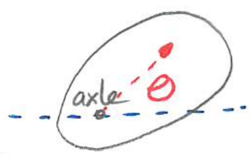
Weds: Warm Up 12 — Group Exercise (graded)

Thurs: HW 284, 286, 287ab, 292, 293, 299, 301, 302

Rotational kinematics

The scheme for rotational kinematics is.

Describe configuration via angular position



→ Angular velocity ~ rate of change of angular position

$$\omega = \frac{\Delta \theta}{\Delta t}$$

→ Angular acceleration ~ rate of change of angular velocity

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

Note that angular velocity can be used to get linear speed

For a point a distance r from the axle; speed is

$$v = \omega r$$

285 Accelerating disk

A disk initially rotates with angular velocity 30 rad/s. It speeds up at a constant rate, reaching 100 rad/s at an instant 14 s later. Determine the angular acceleration of the wheel in rad/s^2. *(111F2023)*

<u>Answer:</u>

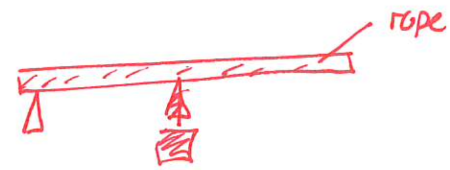
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{100\,\text{rad/s} - 30\,\text{rad/s}}{14\,\text{s}}$$

$$= 5.0\,\text{rad/s}^2$$

Torque

Angular acceleration will arise when forces act. We need to be able to connect forces to angular acceleration

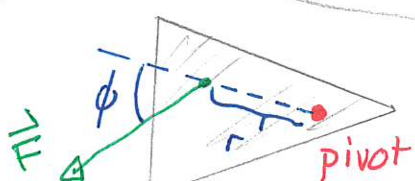
DEMO: Meter stick



The rotational effects of the force depend on:

1) magnitude of the force
2) point at which the force acts
3) angle at which the force acts.

These are combined in the rotational analog of force: torque.



1) Identify pivot/axle. Identify force and location where it acts

2) Draw a line from axle/pivot to force and extend.

3) Let ϕ be angle c.c.w from extension to force. Let r be distance from pivot to force

4) torque produced by force is

$$\tau = r F \sin\phi$$

"tau" — magnitude of $F \geq 0$, distance ≥ 0

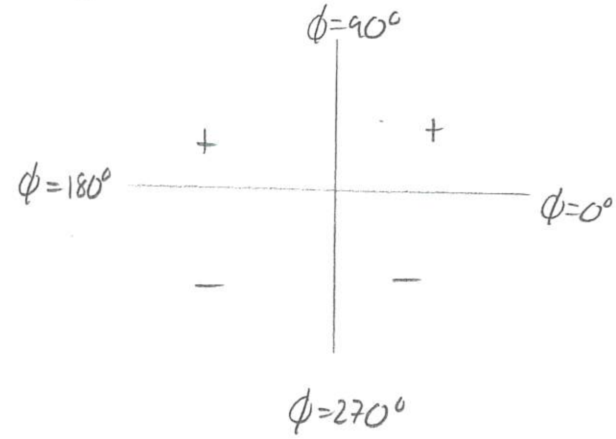
Units: $N \cdot m$

Torques determine angular acceleration

Force tends to change angular velocity => produces angular acceleration

Quiz 1 80% - 100%

Note that torques can be positive or negative. This depends on $\sin\phi$. Then, if multiple forces act on an object, each will produce a torque. In this case, the net torque on the object is



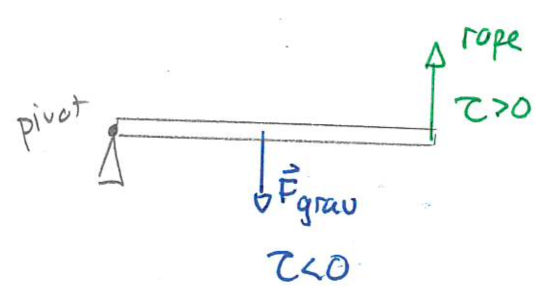
$$\boxed{\tau_{net} = \tau_1 + \tau_2 + \ldots = \sum_{\text{all forces } i} \tau_i}$$

Quiz 2 30%

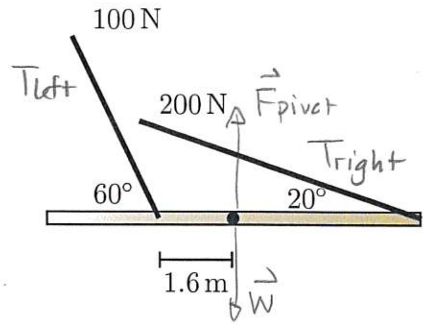
This illustrates aspects of the sign of torque.

τ is positive $\Rightarrow$ force tends to produce counterclockwise rotation
τ is negative $\Rightarrow$ force tends to produce clockwise rotation

Note that the Earth's gravitational force can also produce a torque. To compute this, place the gravitational force at the center-of-mass. For symmetrical objects, this will be the center.



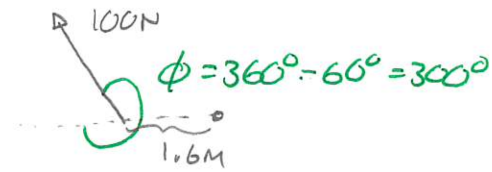
291 Multiple torques on a rod



A 3.0 kg rod with length 8.0 m can pivot about its midpoint. Two ropes pull as illustrated. The thickness of the rod is negligible. Determine the net torque on the rod. *(111F2023)*

Answer: There are four forces. The pivot and weight each produce zero torque because the distance $r = 0$.

Left tension



$$\tau_{left} = rF\sin\phi$$

$$= 100\,N \times 1.6\,m \times \sin 300° = -139\,N\cdot m$$

Right tension

$$\tau_{right} = rF\sin\phi$$

$$= 4.0\,m \times 200\,N \sin 20°$$

$$= 273\,N\cdot m$$

The net torque is

$$\tau_{net} = \tau_{left} + \tau_{right} + \cancel{\tau_{grav}}^{0} + \cancel{\tau_{pivot}}^{0}$$

$$= -139\,N\cdot m + 273\,N\cdot m$$

$$= 135\,N\cdot m$$

Rotational dynamics

One can apply Newton's Second Law to show that, for a rigid object, the net torque about a pivot is related to the angular acceleration via:

$$\tau_{net} = I\alpha$$

where I is the moment of inertia of the object. This depends on the mass present and it's distribution. For example:

solid disk mass M radius R	$I = \frac{1}{2}MR^2$
solid sphere mass M radius R	$I = \frac{2}{5}MR^2$