Phys 111 2023

Lecture 30

Mon: No HW

Weds: Warm Up 12

Circular and rotational motion

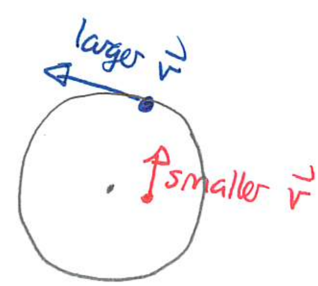
In many situations an object or system of objects will display circular or rotational motion. The same basic rules of Newton's system of mechanics apply by the language can be modified to provide a more convenient description.

DEMO: Ball rotating

Examples of rotational motion include:

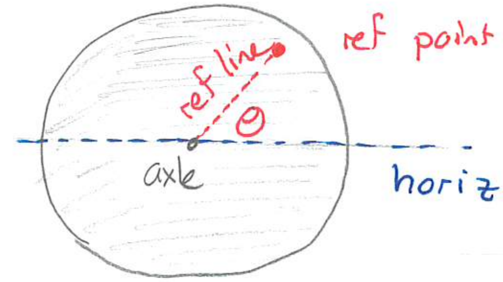
1) rotating wheels, disks, cylinders, balls
2) pivoting beams and rods in structures
3) biomechanical situations. DEMO: Front. Neurology article - show walking model.
4) rotating stars, planets, galaxies,... DEMO: Zurich animation video.
5) aspects of molecular and atomic motion

Rotational kinematics



The basic language of kinematics must be adapted to describe rotational motion conveniently. The primary complication with rotating objects is that at any given instant, different parts rotate at different velocities despite the fact that in some sense the entire object appears to rotate at one particular rate at that instant.

We can simplify the description by describing the state of the rotating object using an angle. We establish a reference point on the object. At any given instant we can draw a line from the reference point to the axle. We then describe



> The angular position of the object is
>
> θ = angle counterclockwise from x-axis to the reference line.

The angle must be measured in radians. These are angle units where

$$2\pi \text{ radians} = 360^\circ \Rightarrow 1\,\text{rad} = \frac{360^\circ}{2\pi}$$
$$\Rightarrow 1^\circ = \frac{2\pi\,\text{rad}}{360}$$

Quiz1 60%

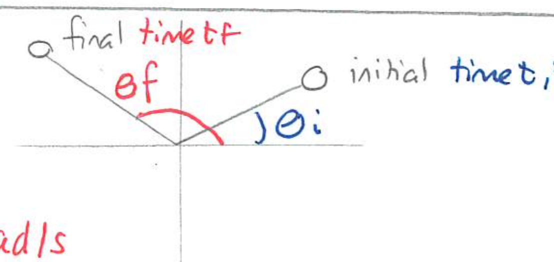
Note the following important angles:

degrees	0°	45°	90°	180°	270°	360°
radians	0	$\pi/4$	$\pi/2$	π	$3\pi/2$	2π

Then we would like to quantify the rate at which angular position changes. Here

angular velocity = rate of change of angular position

with definition

> The average angular velocity over an interval from t_i to t_f is
>
> $$\omega := \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$$
>
> omega
>
> 
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>
> Units: rad/s

282 Rotating wheel RPM

A wheel rotates at 300 rpm (revolutions per minute). Determine the angular velocity of the wheel in rad/s. *(111F2023)*

1 revolution = 2π rad.

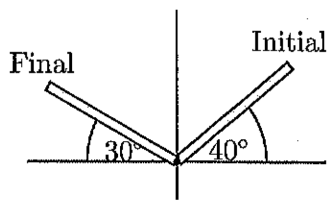
1 min = 60s

$$300 \text{ rpm} = \frac{\cancel{300}^{5} \text{ rev}}{1 \text{ min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{\cancel{60} \text{ s}} = 10\pi \text{ rad/s}$$

$$= 31.4 \text{ rad/s}$$

283 Rotating beam

A narrow metal beam rotates at a constant rate about the axle through one end. Its positions are indicated at two instants 0.60 s apart. *(111F2023)*



a) Determine the angular velocity of the beam in rad/s.

b) Determine the angular velocity of the beam in rpm.

Answer: a) $\theta_i = 40° = \frac{40°}{360°} \times 2\pi = \frac{2\pi}{9} \text{ rad.} = 0.698 \text{ rad}$

$$\theta_f = 180° - 30° = 150° = \frac{150° \times 2\pi}{360°} = \frac{5 \times 2\pi}{12} = \frac{10\pi}{12} \text{ rad.}$$

$$= 2.618 \text{ rad}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2.618 \text{ rad} - 0.698 \text{ rad}}{0.60 \text{ s}} = 3.2 \text{ rad/s}$$

b) $\omega = 3.2 \text{ rad/s} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}} \Rightarrow \omega = 31 \text{ rpm}.$

Note that there are signs to angular velocity. If an object rotates counterclockwise then θ increases with time and $\omega > 0$. So we find:

If an object rotates counterclockwise $\Rightarrow$ $\Delta\theta > 0$ $\Rightarrow$ ω positive
If an object rotates clockwise $\Rightarrow$ $\Delta\theta < 0$ $\Rightarrow$ ω negative.

Angular acceleration

The crucial feature about rotational motion that forces will describe is not the angular velocity but the rate at which angular velocity changes.

DEMO: Bicycle wheel - rotate
- apply braking force

We need

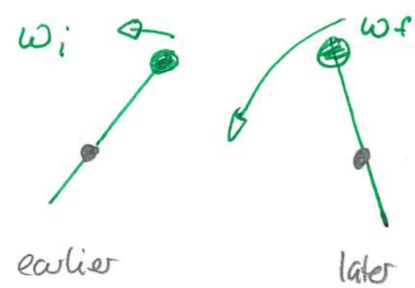
angular acceleration $\approx$ rate of change of angular velocity

and

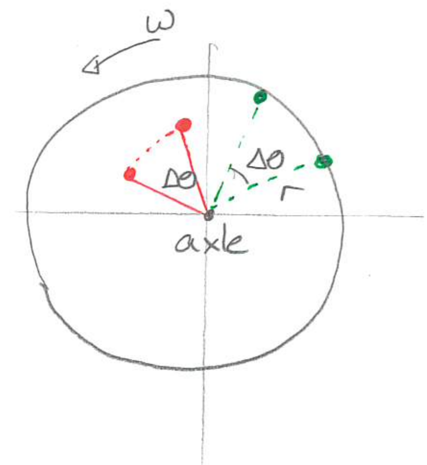
The average angular acceleration from time t_i to t_f is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i}$$

units: rad/s^2



Rotational velocity and linear velocity



Consider an object that rotates with constant angular velocity ω. Individual points have linear speeds and velocities that must somehow be related to the angular velocity. We can show that

> Consider an object rotating with angular velocity ω. Then consider a point at a distance r from the axle. This moves with linear speed
>
> $$v = \omega r$$

Quiz 2 $10\% \to 30\%$

Quiz 3 not done