

Thurs: Discussion Quiz

EX 234, 235, 236, ~~237~~, 238, 239, 240, 245

Energy conservation

Newton's Laws eventually give an example of the conservation of energy

If the only force that does non-zero work on an object is gravity, then the total energy

$$E = K + U_{\text{grav}}$$

is constant. Here, the kinetic energy is

$$K = \frac{1}{2} m v^2$$

and the gravitational potential energy is

$$U_{\text{grav}} = mgy$$

Usually we consider two moments in the object's motion and then energy conservation implies

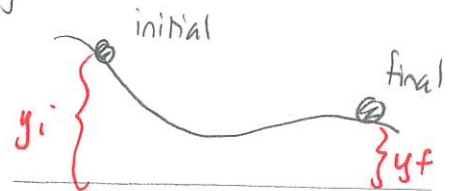
$$E_f = E_i$$

$$K_f + U_{\text{grav}f} = K_i + U_{\text{grav}i}$$

$$\frac{1}{2} m v_f^2 + mgy_f = \frac{1}{2} m v_i^2 + mgy_i$$

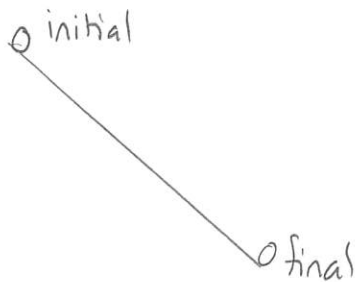
An alternative to this is

$$\Delta E = 0 \Rightarrow \Delta K + \Delta U_{\text{grav}} = 0$$



# Warm Up 1

Explanation:



$$E_f = E_i$$

$$K_f + U_{grav f} = K_i + U_{grav i}$$

$\begin{matrix} 0 & 0 \\ (y_f = 0) & v_i = 0 \end{matrix}$

$$\frac{1}{2} m v_f^2 = m g y_i$$

$$v_f^2 = 2 g y_i$$

does not depend on slope,

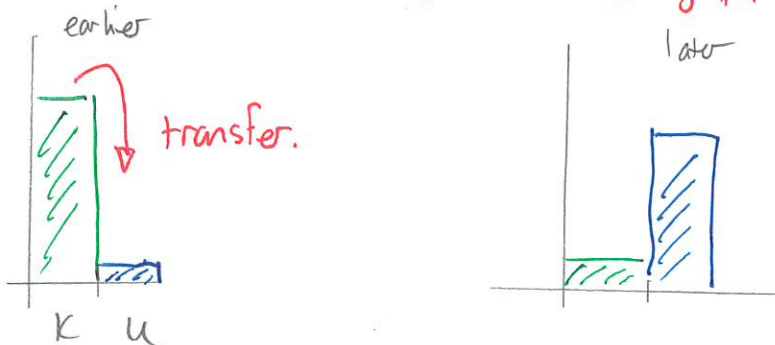
## Quiz 1 80%

Again we can contemplate energy. Both start with the same initial energy. At the maximum height the kinetic energy is zero. For example.

	start	ascends	max height	
K	200J	160J	0J	
$U_{grav}$	0J	40J	200J	$\rightarrow mgy_f$ lets one get $y_f$ .
E	200J	200J	200J	$\leftarrow$ All same.

There is a constant interchange in energy. We can view this via a bar graph

DEMO: PHET ESP (Basics) - show bar graph

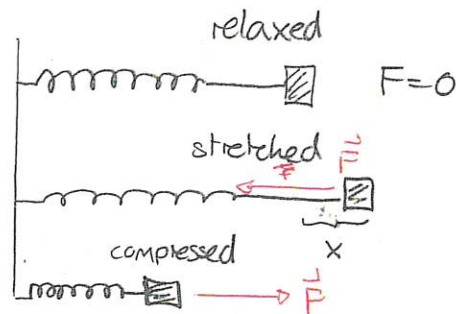


## Spring forces

Springs exert forces that vary depending on the amount by which the spring is stretched / compressed. To a good approximation the force exerted by the spring has magnitude

$$F_{sp} = kx$$

where  $x$  is the amount by which the spring is stretched or compressed. Here  $k$  is a constant which depends on the particular spring and is called the spring constant. It has units of N/m.



### Demo: PhET Springs / Masses

- ~~into~~ Energy → Natural length
- lots of damping
- vary mass / constant

Because the force varies as the object moves it can be difficult to use Newton's laws to assess the motion of an object attached to the spring. But we can do so using work and energy.

### Warm Up 2

We can determine the work done by the spring via a potential energy associated with the spring. The elastic potential energy is

$$U_s = \frac{1}{2} kx^2$$

Here  $x$  is the amount by which the spring is stretched/compressed.

This enters into a revised conservation of energy law.

If the only forces that do non-zero work are gravity and spring forces then the total energy

$$E = K + U_g + U_s$$

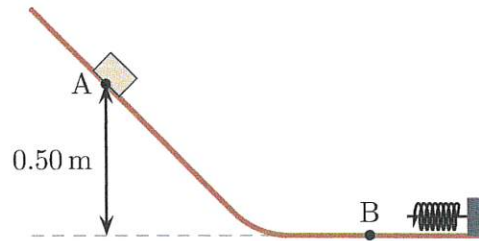
remains constant.

Demo: PhET Springs

- Show energy bar graph

### 252 Box, ramp and spring

A 6.0 kg box is released from rest at on the illustrated frictionless track. The box slides down the ramp, passes B and collides with a spring (constant 6000 N/m). While it moves friction and air resistance can be ignored. (111F2023)



- Determine the speed of the box at point B.
- ~~Determine the kinetic energy of the sled when it is at point B.~~
- Determine the maximum distance by which the spring is compressed.

Answer: a) Energy is conserved

$$E_B = E_A$$

$$K_B + U_{\text{grav}B} = K_A + U_{\text{grav}A}$$

$$\frac{1}{2} m v_B^2 + m g y_B = \frac{1}{2} m v_A^2 + m g y_A$$

$$\frac{1}{2} 6.0 \text{ kg } v_B^2 = 6.0 \text{ kg} \times 9.8 \text{ m/s}^2 \times 0.50 \text{ m}$$

$$3.0 \text{ kg } v_B^2 = 29.4 \text{ J}$$

$$\Rightarrow v_B^2 = \frac{29.4 \text{ J}}{3.0 \text{ kg}} = 9.8 \text{ m}^2/\text{s}^2$$

$$\Rightarrow v_B = \sqrt{9.8 \text{ m}^2/\text{s}^2} = 3.1 \text{ m/s}$$

$$v_A = 0$$

$$y_A = 0.50 \text{ m}$$

$$v_B = ?$$

$$y_B = 0 \text{ m}$$

(Note: m could cancel)

- b) Energy is conserved, let f be the point where the spring has max compression

$$E_f = E_B = E_A = 29.4 \text{ J}$$

~~$$K_f + U_{\text{grav}f} + U_{\text{spring}f} = 29.4 \text{ J}$$~~

$$\frac{1}{2} k x_f^2 = 29.4 \text{ J}$$

$$\frac{1}{2} 6000 \text{ N/m } x_f^2 = 29.4 \text{ J}$$

$$3000 \text{ N/m } x_f^2 = 29.4 \text{ J} \Rightarrow$$

$$v_f = 0 \Rightarrow K_f = 0$$

$$y_f = 0 \Rightarrow U_{\text{grav}f} = 0$$

$$x_f^2 = \frac{29.4 \text{ J}}{3000 \text{ N/m}} = 0.0098 \text{ m}^2$$

$$\Rightarrow x_f = \sqrt{0.0098 \text{ m}^2} \Rightarrow x_f = 0.099 \text{ m}$$