Weds: Warm Up 10

Thurs: Discussion /quiz.

Work done by gravity /gravitational potential energy

Recall the system for work and energy

Work done by a constant
force for a straight line displacement

$$W = Fd\cos\theta$$

Net work

$$W_{net} = \sum W_i$$
all forces i

Can be adapted to
non-constant forces

Work-kinetic energy theorem

$$W_{net} = \Delta K = K_f - K_i$$

$$= \frac{1}{2}MU_f^2 - \frac{1}{2}MU_i^2$$
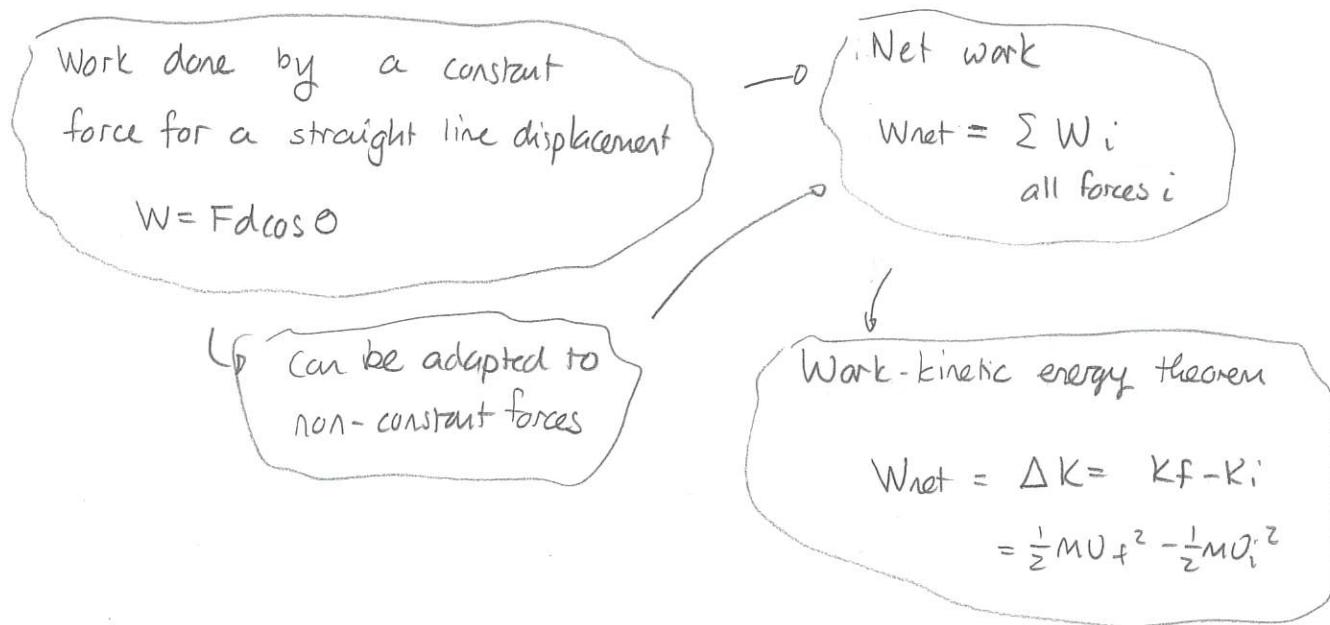
We would like more convenient ways to calculate work.

Consider an object moving under Earth's gravitational force. If the object moves vertically down then

$$F_{grav} = mg$$

$$W_{grav} = F_{grav}\, d\cos\theta \quad \theta = 0°$$

$$= mg\, d$$

$$= mg(y_i - y_f)$$

$$= mgy_i - mgy_f$$
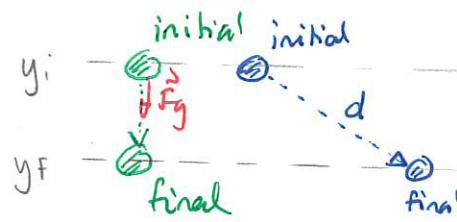
We can show that this is the same if the object follows a slanted trajectory

Proof:
$$W_{grav} = F_{grav} \, d \cos\theta$$
$$= mg \, d \cos\theta$$
$$= mg \, (y_i - y_f)$$

Remarkably, this is true regardless of the trajectory that the object follows: We get

The work done by gravity is
$$W_{grav} = mg(y_i - y_f)$$
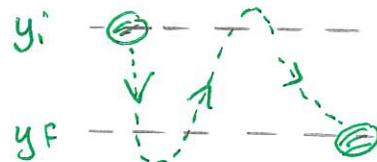
where y_i = initial vertical position
y_f = final vertical position.

This motivates the definition of gravitational potential energy

The gravitational potential energy of an object with mass m is
$$U_{grav} = mgy$$

where y is the vertical position above Earth.

Units: Joules   J

Then
$$W_{grav} = U_{grav\,i} - U_{grav\,f} = -[U_{grav\,f} - U_{grav\,i}]$$

$$\Rightarrow \boxed{W_{grav} = -\Delta U_{grav}}$$

Energy conservation

Now consider the special situation where gravity is the only force that does non-zero work. We can show:

If gravity is the only force that does non-zero work on an object then the total energy

$$E = K + U_{grav}$$

stay constant throughout the motion. Thus at any initial / final moment

$$K_f + U_{grav\,f} = K_i + U_{grav\,i}$$
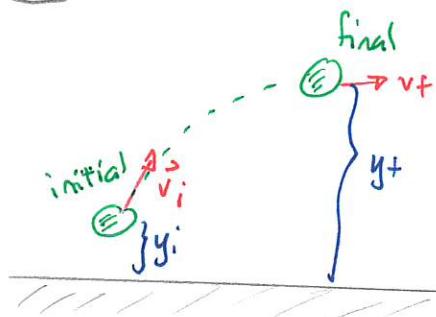
This is an example of the conservation of energy

Proof: $W_{net} = K_f - K_i$

   But $W_{net} = W_{grav}$ (no other force does non-zero work)

   $\Rightarrow W_{grav} = K_f - K_i$

   $- \left[U_{grav\,f} - U_{grav\,i} \right] = K_f - K_i$

   $U_{grav\,i} - U_{grav\,f} = K_f - K_i$

   $\Rightarrow K_f + U_{grav\,f} = K_i + U_{grav\,i}$ ▧

Quiz1 30% ~ 60%
Quiz2 70% — 90%