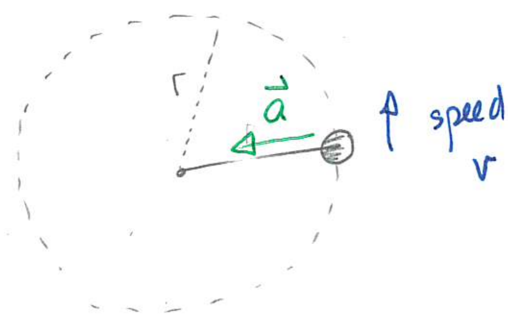
Mon: Warm Up 7

Weds: HW by 5pm   Ex 185, 188, 194, 195, ~~203~~ 198, 207, 210, 211

Dynamics of Uniform Circular Motion

Uniform circular motion is where an object moves with constant speed in a circle. Applying the usual kinematics eventually gives:

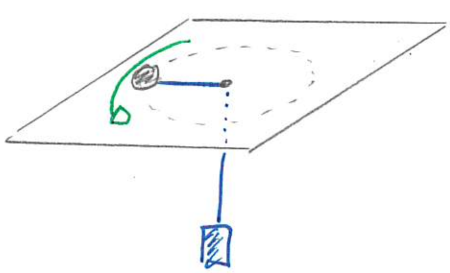


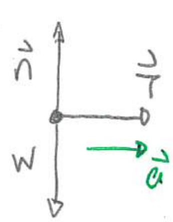
> For uniform circular motion the acceleration:
>
> * points radially inward
> * has magnitude
>
> $$a = \frac{v^2}{r}$$

This is called centripetal acceleration. Then Newton's 2nd Law implies:

> In uniform circular motion the net force is radially inward.

We can consider an object moving in a horizontal circle such as on a frictionless table. Then when the object is viewed while it is at the left edge:



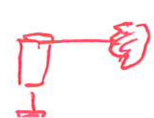


$$\sum F_{ix} = ma_x$$

$$\Rightarrow T = ma = m\frac{v^2}{r}$$

Quiz 1 80%

DEMO:



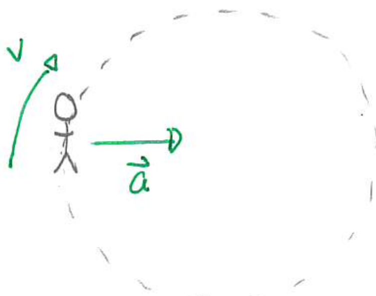
190 Merry-go-round dynamics

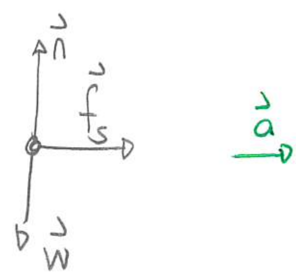
A 50 kg child sits at the edge of a merry-go-round with radius 2.5 m. The merry-go-round rotates with frequency 15 revolutions per minute. *(111F2023)*

a) Determine the direction of the friction force on the child.

b) Determine the magnitude of the friction force on the child.

Answer: Top view Side view





a) The net force points inward. This must be static friction.

b) We need

Time / frequency $\rightarrow$ Speed $\rightarrow$ Acceleration $\rightarrow$ friction

$$\sum F_x = ma_x$$

$$\Rightarrow f_s = ma \Rightarrow \boxed{f_s = m\frac{v^2}{r}}$$

Now $v = \frac{2\pi r}{T}$ where $T =$ period of motion $T = \frac{1}{f}$. Here

$$f = \frac{15\,\text{rev}}{\text{min}} = \frac{15\,\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}}$$

$$\Rightarrow f = 0.25\,\text{rev/s} \qquad \Rightarrow T = \frac{1}{f} = \frac{1}{0.25\,\text{rev/s}} = 4.0\,\text{s}$$

Then

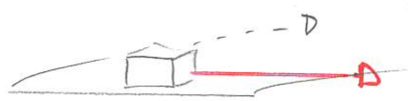
$$V = \frac{2\pi r}{T} = \frac{2\pi \times 2.5m}{4.0s} = 3.93 m/s$$

Thus $$f_s = m\frac{v^2}{r}$$

$$= 50kg \times \frac{(3.93m/s)^2}{2.5m}$$

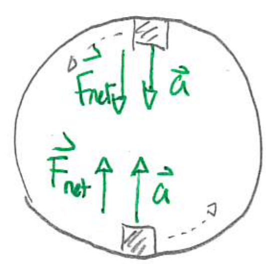
$$= 310 N$$

This explains how vehicles can turn on flat surfaces. There must be an inward friction force



(static) friction radially inward.

Vertical circular motion



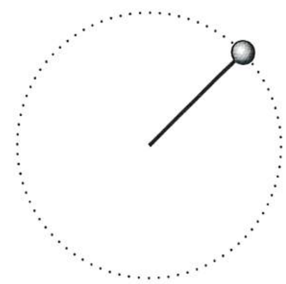
The same analysis can be applied to objects that move in vertical circles. In general the motion is not uniform - the speed varies. However, at the top and bottom the acceleration is often radially inward, and has the usual centripetal acceleration.

Quiz 2 80%

Quiz 3 60% - 80%

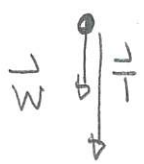
197 Ball swinging in a vertical circle, 1

A 0.80 kg ball swings with in a vertical circle at the end of a 0.50 m long string. The speed of the ball at the highest point in the circle is 3.0 m/s. Determine the tension in the string at this moment. *(111F2023)*



Answer: At top

(1)



(2) $\sum F_{iy} = ma_y$

$$a_y = -\frac{v^2}{r} = -\frac{(3.0\,m/s)^2}{0.50\,m} = -18\,m/s^2$$ Centripetal accel.

$$\sum F_{iy} = -m \times 18\,m/s^2$$

(3) $w = mg = 0.80\,kg \times 9.8\,m/s^2 = 7.8\,N$

(4) $\sum F_{iy} = ma_y$

$$\Rightarrow -T - 7.8\,N = -\underbrace{0.80\,kg \times 18\,m/s^2}_{14.4\,N}$$

$$14.4\,N - 7.8\,N = T$$

$$\Rightarrow T = 6.6\,N$$

Quiz 4