Phys 111 2023

Lecture 19

Thurs: Discussion / quiz

Ex: 160, 161, 162, 168, 172, 173

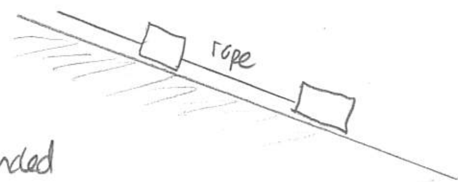
Next HW will be next Weds

Next Warm Up will be next Mon.

Connected objects

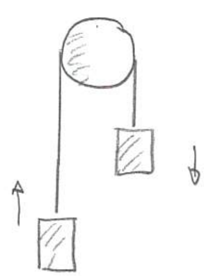
There are often situations where two objects move while being connected via a rope, string or cable. In this case we will assume that

* the rope does not stretch
* the rope is massless



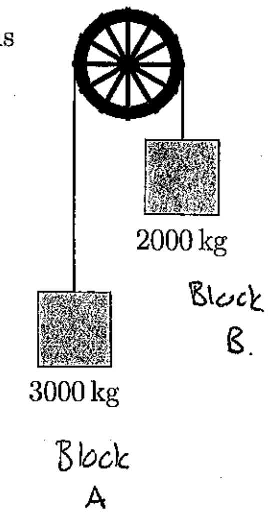
We consider the example of two objects suspended over a pulley

Warm Up 1



182 Counterbalanced elevator

A 3000 kg elevator is connected to a 2000 kg block by a rope that runs over a pulley. Determine the acceleration of the elevator. *(111F2023)*



Answer: Consider each block separately. We apply Newton's second law to each and combine the results. First there is a constraining piece of kinematics.

- the accelerations have the same size but are opposite.

Let a be the magnitude of acceleration. Then

For A : $a_y = a$

For B : $a_y = -a$.

Block A

$$\sum F_{iy} = m_A a_y$$

$$\Rightarrow T - W_A = m_A a$$

$$\Rightarrow T - m_A g = m_A a$$

$$\Rightarrow T - 3000\,\text{kg} \times 9.8\,\text{m/s}^2 = 3000\,\text{kg}\, a$$

$$\Rightarrow T - 29400\,\text{N} = 3000\,\text{kg}\, a$$

Block B

$$\sum F_{iy} = m_B a_y$$

$$T - W_B = m_B(-a)$$

$$T - m_B g = -m_B a$$

$$T - 2000\,\text{kg} \times 9.8\,\text{m/s}^2 = -2000\,\text{kg}\, a$$

$$T - 19600\,\text{N} = -2000\,\text{kg}\, a$$

We combine these. $T = 19600\,\text{N} + 2000\,\text{kg}\, a$

$$\Rightarrow 19600\,\text{N} + 2000\,\text{kg}\, a - 29400\,\text{N} = -3000\,\text{kg}\, a$$

$$\Rightarrow 2000\,\text{kg}\, a - 9800\,\text{N} = -3000\,\text{kg}\, a$$

$$\Rightarrow 5000\,\text{kg}\, a = 9800\,\text{N}$$

$$\Rightarrow a = \frac{9800\,\text{N}}{5000\,\text{kg}} \Rightarrow a = 1.96\,\text{m/s}^2$$

Circular motion

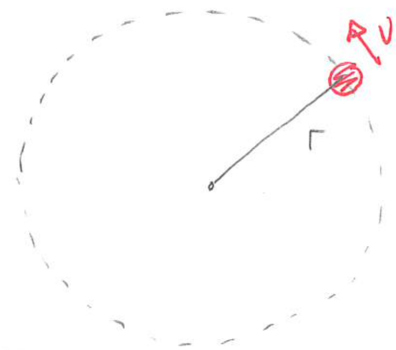
There are many situations where an object moves along a circular path. Examples include:

1) ball swinging on the end of a string
2) motion of planets/moon/satellites DEMO: The Sky Planets.
3) motion of charged particles in a uniform magnetic field.

The same basic framework of kinematics and Newton's Laws for dynamics can be applied to analyze circular motion. The language will be adapted for circular motion

The simplest type of circular motion is that where speed is constant. This is called uniform circular motion.

Consider an object moving in a circle of radius r with speed v.



Then

> The period of the motion is the time taken to complete one orbit. This is denoted T.
>
> units : s

If the period of orbit is 0.040s then this means that the object completes $1s/0.040s = 25$ orbits each second. This is properly described by the frequency

> The frequency of the motion, f, is the number of orbits completed each second and
>
> $$f = 1/T$$
>
> units: rev/s.

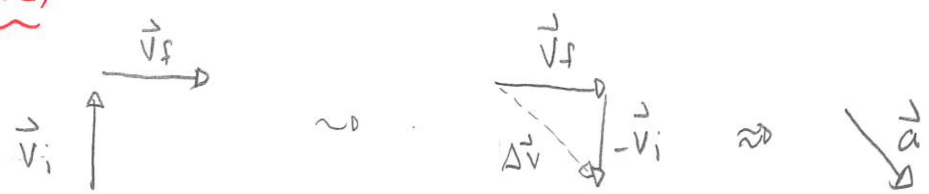
Then the speed of orbit is

$$V = \frac{\text{distance one orbit}}{\text{time one orbit}} \Rightarrow \boxed{V = \frac{2\pi r}{T}}$$

Now consider the acceleration. Again we use the average acceleration as a start:

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

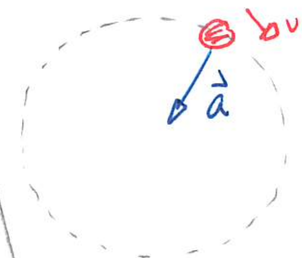
Quiz) 30%



This shows that:

In uniform circular motion, the acceleration points radially inward

We can arrived at the magnitude using calculus. Then



The magnitude of the acceleration for uniform circular motion is $a = v^2/r$ where v is the speed and r the radius of orbit

Centripetal acceleration

184 Merry-go-round

A merry-go-round is a large flat disk that spins around a vertical axis through its center. A child is at the edge of a merry-go-round with radius 3.0 m. The merry-go-round spins so that the child's acceleration is $1.5g$. Determine the period and frequency of orbit for this to occur. *(111F2023)*

Answer: We can get the period from the speed via:

$$v = \frac{2\pi r}{T} \Rightarrow vT = 2\pi r \Rightarrow \boxed{T = \frac{2\pi r}{v}}$$

We can get the speed from acceleration by

$$a = \frac{v^2}{r}$$

So we will

Get v from acceleration → Get period from v → Get frequency from period

Then $a = 1.5\,g = 1.5 \times 9.8\,m/s^2 = 14.7\,m/s^2$

$$14.7\,m/s^2 = \frac{v^2}{3.0\,m} \Rightarrow 3.0\,m \times 14.7\,m/s^2 = v^2$$

$$\Rightarrow 44.1\,m^2/s^2 = v^2 \Rightarrow v = \sqrt{44.1\,m^2/s^2}$$

$$\Rightarrow v = 6.6\,m/s$$

Then

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 3.0\,m}{6.6\,m/s} = \frac{18.8\,m}{6.6\,m/s} \Rightarrow \boxed{T = 2.8\,s}$$

Finally $f = \frac{1}{T} = \frac{1}{2.8\,s} \Rightarrow \boxed{f = 0.35\,rev/s}$

This means it does $60 \times 0.35\,rev/s = 21\,rev/min$