Weds: ~~Mon:~~ Warm Up 4 (D2L)

Thurs: ~~Weds:~~ HW by 5pm

Ex 72, 82, 87, 89, 93, 94, 99, 100

Previously: Covered adding vectors with components.

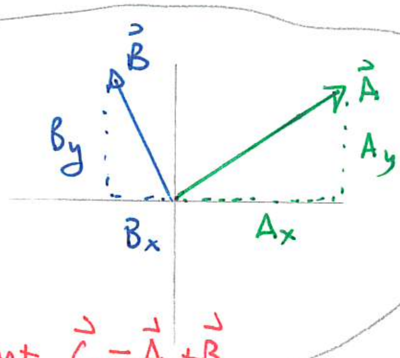
Today: * Getting vector components.

* Vectors and two-dimensional motion

Vector algebra with components

We can do basic vector manipulations using components

Given two vectors $\vec{A}, \vec{B}$



Want $\vec{C} = \vec{A} + \vec{B}$

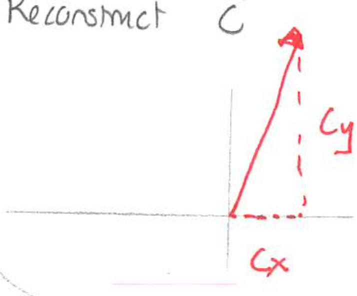
→ Get components A_x, A_y B_x, B_y

→ Add components

$C_x = A_x + B_x$ ← only horiz

$C_y = A_y + B_y$ ← only vert.

Reconstruct $\vec{C}$

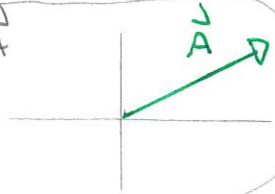


magnitude

$C = \sqrt{C_x^2 + C_y^2}$

Similarly for multiplication

Given $\vec{A}$



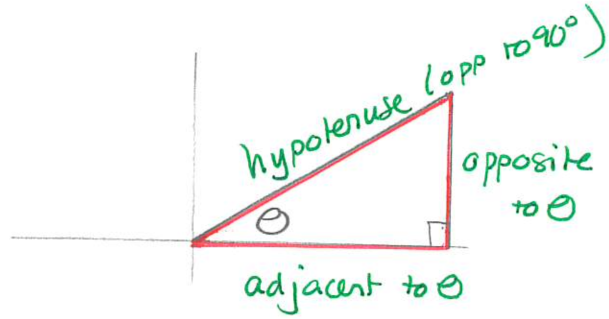
Want $b\vec{A}$

→ $C_x = b A_x$

$C_y = b A_y$

This requires that we are able to get the components. We can get these via trigonometry that deals with the sides of a right-angled triangle. Then the trigonometric functions each

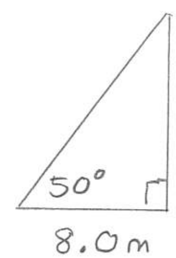
* take an angle θ as input
* produce a number



They satisfy

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

<u>Example</u> Determine the length of the hypotenuse in the illustrated triangle



<u>Answer:</u> We have

adjacent = 8.0m.

Then

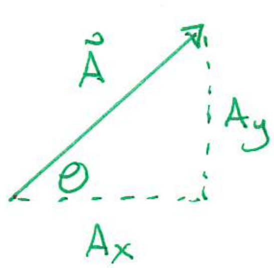
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \Rightarrow \underbrace{\cos 50^\circ}_{\text{calculator}} = \frac{8.0\text{m}}{\text{hyp}}$$

$$0.643 = \frac{8.0\text{m}}{\text{hyp}}$$

$$\Rightarrow \text{hyp}\ 0.643 = 8.0\text{m}$$

$$\Rightarrow \text{hyp} = \frac{8.0\text{m}}{0.643} = 12.4\text{m} \quad \blacksquare$$

For a vector this gives components via



$$\frac{A_x}{A} = \cos\theta \qquad \frac{A_y}{A} = \sin\theta \quad \Rightarrow \quad \boxed{\begin{aligned} A_x &= A\cos\theta \\ A_y &= A\sin\theta \end{aligned}}$$

magnitude A, always > 0

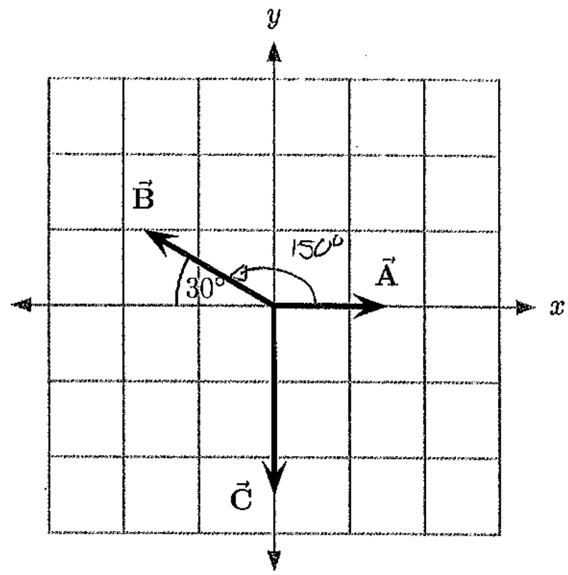
angle measured c.c.w from positive x-axis

68 Vector components: algebraic, 1

Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are illustrated. Their magnitudes are $A = 15\,\text{m}, B = 20\,\text{m}$ and $C = 25\,\text{m}$. *(111F2023)*

a) Determine the x and y components of each vector.

b) Determine the components of $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$.



Answer:

a) $A_x = 15\text{m}$ $\quad A_y = 0\text{m}$

$B_x = B\cos 150°$ $\quad B_y = B\sin 30° = 15\text{m}\sin 30°$

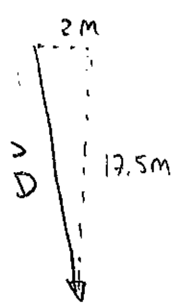
$= -\cancel{13}\text{m}$ $\quad = \overset{10}{\cancel{7.5}}\text{m}$

-17.3

$C_x = 0\text{m}$ $\quad C_y = -25\text{m}$

b) $D_x = A_x + B_x + C_x = 15\text{m} - \overset{17.3}{\cancel{13}}\text{m} + 0\text{m} = 2\text{m}$ $\quad 2.3$

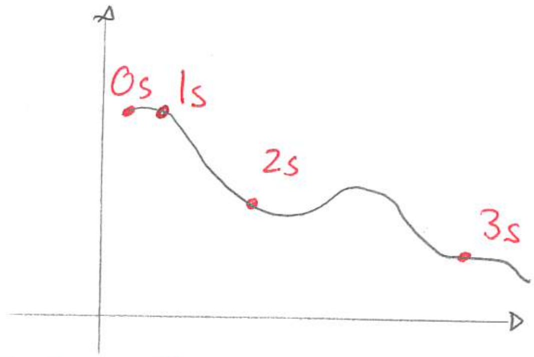
$D_y = A_y + B_y + C_y = 0\text{m} + \overset{10}{\cancel{7.5}}\text{m} - 25\text{m} = -\cancel{17.5}\text{m}$ $\quad -15$



Kinematics in Two Dimensions

We now adapt the language of one-dimensional kinematics to two dimensions. Consider an object moving along a two-dimensional surface. We can map the path (trajectory) as it moves. We want to

- describe the trajectory
- describe velocity using the trajectory.



Demo: Hawk Eye Video Tennis (about 1:20)
Hawk Eye cricket predictions (about 1:40)

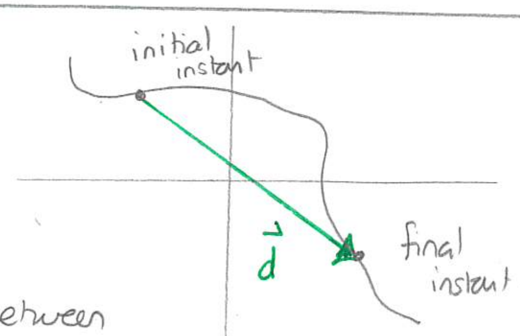
Again the ideas are

average velocity over an interval ≈ rate of change of position

The average velocity from time t_i to t_f is

$$\vec{v}_{avg} = \frac{\vec{d}}{\Delta t} = \frac{1}{\Delta t}\vec{d}$$

where $\vec{d}$ is the displacement vector between instants



Note:

1) average velocity is a <u>vector</u>
2) the direction of average velocity is the same as the direction of the displacement.

Quiz 1 40% – 90%

Quiz 2