Phys 111 2023

Lecture 33

Fri: Review Exam 3

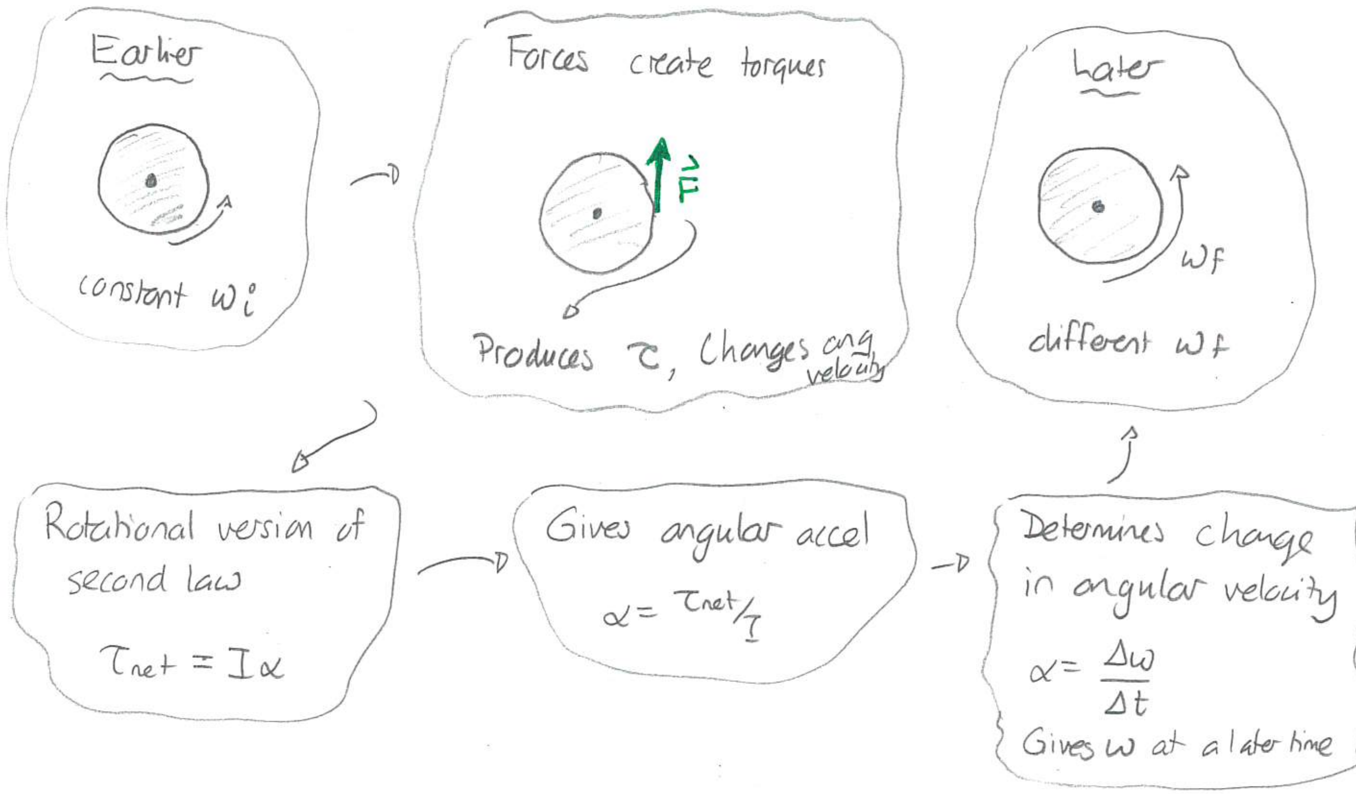
Mon Exam 3 covers energy, momentum, rotational motion

Prev exams: 2016 all except Q6b

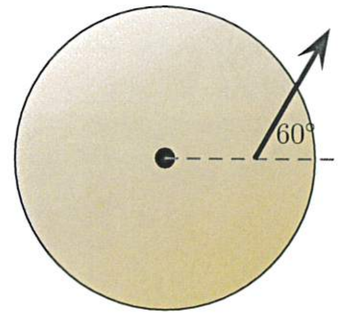
2019 all Q.

Rotational Dynamics

The rotational state of motion of any object can change as a result of torques acting on the object



305 Jet-propelled disk



A 3.0 kg solid, uniform disk has radius 0.25 m and can rotate horizontally about a frictionless axle through its center. A small jet, which is attached 0.15 ~~c~~m from the center of the disk, exerts a 4.0 N force as illustrated. The aim of this exercise is to determine the angular acceleration of the disk. *(111F2023)*

a) Write the rotational version of Newton's second law.

b) Determine the moment of inertia of the disk.

c) Determine the net torque acting on the disk.

d) Determine the angular acceleration of the disk.

Answer:

a) $\tau_{net} = I\alpha$

b) $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 3.0\,kg \times (0.25\,m)^2 = 0.09375\,kgm^2$

c) $\tau_{net} = \tau_{grav} + \tau_{jet}$

gravity: $\tau_{grav} = \underset{0}{\cancel{r}}F\sin\phi = 0\,N\cdot m$

jet $\tau_{jet} = rF\sin\phi$

$= 0.15\,m \times 4.0\,N \times \sin 60°$

$= 0.52\,N\cdot m$

d) $\tau_{net} = I\alpha$

$0.52\,N\cdot m = 0.09375\,kgm^2\,\alpha$

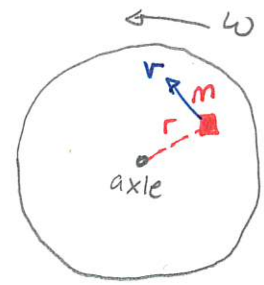
$\Rightarrow \alpha = \dfrac{0.52\,Nm}{0.09375\,kgm^2} = 5.5\,rad/s^2$

So if the disk was initially at rest then $\omega = \alpha t$ gives speed after t seconds

t	ω (rad/s)	rpm
1s	5.5 rad/s	53 rpm
5s	28 rad/s	264 rpm
10s	55 rad/s	530 rpm

Rotational kinetic energy

As with linear (translational) motion, the framework of Newton's laws allows for energy that describes rotational motion. Consider a disk that rotates about an axle with angular velocity ω. The various portions of the disk will be move. Since these portions have mass, they have kinetic energy. The shaded portion has kinetic energy

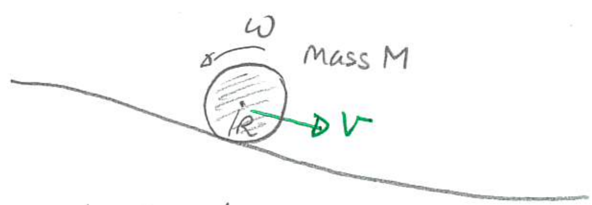


$$\frac{1}{2} m v^2 = \frac{1}{2} m (\omega r)^2 = \frac{1}{2} m r^2 \omega^2$$

We need to add all such portions and this entails adding all mr^2 contributions. This gives the moment of inertia and results in

> The rotational kinetic energy of an object rotating about some axis is
>
> $$K_{rot} = \frac{1}{2} I \omega^2$$
>
> where I = moment of inertia
>
> ω = angular velocity

We can apply this to an object that rolls along a surface without slipping. Assume that the object has a circular cross-section, with radius R



Then the entire mass moves with speed v and will have kinetic energy

$$\frac{1}{2} M v^2$$

The constituent parts have rotational kinetic energy $\frac{1}{2} I \omega^2$

The two combine to give total kinetic energy

$$K_{tot} = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$$

Quiz 1 30% – 70%

Quiz 2 10%

If the object rolls without slipping then $v = \omega R \Rightarrow \omega = v/R$ and

$$K_{tot} = \frac{1}{2} M v^2 + \frac{1}{2} I \frac{v^2}{R^2}$$

$$= \frac{1}{2}\left(M + \frac{I}{R^2}\right) v^2$$

Now consider energy conservation. At release the energy is entirely potential U_{grav} and at the bottom of the ramp it is entirely kinetic. Energy conservation gives:

$$K_{tot} = U_{grav}$$

at bottom

different.

$$\frac{1}{2}\left(M + \frac{I}{R^2}\right) v^2 = U_{grav}$$

same for both

same for both

DEMO: Rolling hoop/disk.

Angular momentum

One can develop a rotational version of momentum. For a rotating object

The angular momentum of the object is

$$L = I\omega$$

Units: $kg\,m^2/s$

where I = moment of inertia

ω = angular velocity

Then

If the net external torque on a system is zero the total angular momentum of the system stays constant

This is the conservation of angular momentum

→ sort out contrac'

Quiz 3 70%

DEMO: Hoberman sphere video

Quiz 4

DEMO: Train video.

Platform DEMO