Weds: HW 5pm

Ex 185, 188, 194, 195, 198, 207, 210, 211

Thurs: Review Exam 2

Prev exams Class Exam 2 2016, 2019 } All questions

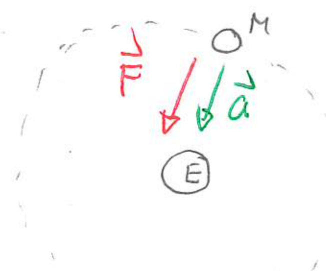
Mon: Exam 2

Gravitation

Earth's gravity is responsible for the motion of:

1) freely falling objects
2) projectiles.

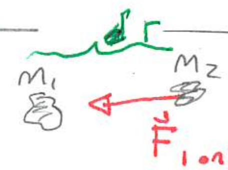
Is it also responsible for the motion of the Moon or satellites around Earth? What about the Sun and planets in the solar system. Consider the Moon in a circular orbit around Earth. If the motion is uniform then the acceleration is radially inward. There must be some force pointing radially inward. Can Earth's gravity provide this?



DEMO: Newton's Cannon

In general gravitational forces are widespread. Newton's universal law of gravitation says

> Every object with non-zero mass exerts a gravitational force on every other object with non-zero mass. The force
>
> 1) is attractive (along line connecting objects)
> 2) has magnitude
>
> $$F_{1\,on\,2} = G\frac{m_1 m_2}{r^2}$$
>
> r → distance between objects



The quantity G is called the universal gravitational constant and is the same for any pair of objects. It was eventually measured by Cavendish about 100 years after Newton proposed the law of gravitation. The best accepted value is

$$G = 6.67 \times 10^{-11} \ Nm^2/kg^2$$

Warm Up 1

~~DEMO: Cavendish experiment.~~

Quiz1 40% – 80%

204 Ordinary objects and gravitational forces

A 100 kg person stands near a 4000 kg elephant. Their centers of mass are 1.8 m apart. Determine the gravitational force exerted by the person on the elephant. *(111F2023)*

Answer:
$$F_{p\,on\,e} = G\frac{m_p m_e}{r^2}$$

$$= 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}\ \frac{100\,kg\times4000\,kg}{(1.8\,m)^2}$$

$$F_{p\,on\,e} = 8.2\times10^{-6}\,N$$

This is very small compared to Earth's gravitational force on the elephant

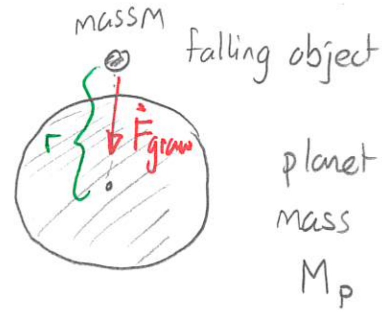
$$W = mg = 4000\,kg\times9.8\,m/s^2$$
$$= 39200\,N$$

DEMO: Cavendish expt here

Gravitational acceleration near to a planet

We can use Newton's Law of gravitation to understand and predict the acceleration due to gravity near a planet's surface.

Consider a falling object a distance r from the center of a planet. Newton's Second Law gives



$$\vec{F}_{net} = m\vec{a}$$

$$\Rightarrow F_{grav} = ma$$

$$\Rightarrow G\frac{\cancel{m}M_p}{r^2} = \cancel{m}a$$

$$\Rightarrow a = G\frac{M_p}{r^2}$$

This predicts:

1) the acceleration of a freely falling object is independent of the object's mass
2) the acceleration points radially inward.
3) near the surface of a planet the acceleration is approximately the same at all locations.

For example Earth's radius is 6.37×10^6 m. If we hold an object 100m above Earth's surface then we use $r = 6.37\times10^6\text{ m} + 100\text{m} = 6370100\text{m}$ which will produce a negligibly different acceleration than at Earth's surface. This can be used to determine Earth's mass. At Earths surface

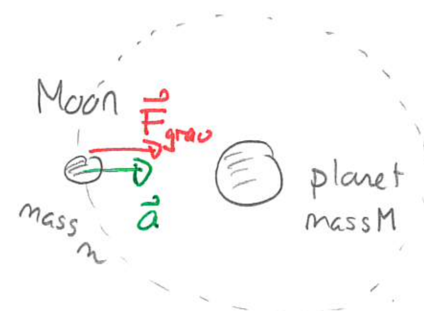
$$a = g = 9.8\text{m/s}^2$$

$$r = 6.37\times10^6\text{m}$$

$$\Rightarrow g = G\frac{M_e}{r_E^2} \Rightarrow M_E = \frac{g r_E^2}{G} = \frac{9.8\text{m/s}^2\times(6.37\times10^6\text{m})^2}{6.67\times10^{-11}\text{ Nm}^2/\text{kg}^2} = 6.0\times10^{24}\text{kg}$$

Objects in Orbit.

Newton's Law of Gravitation provides complete details about the orbits of objects (e.g. planets, moons,) under gravity. Consider a moon or satellite in a circular orbit around a planet with mass M. Let r be the radius of orbit. Then



$$\sum F_{ix} = ma_x$$

$$\Rightarrow \quad F_{grav} = ma$$

$$\Rightarrow \quad G\frac{mM}{r^2} = ma$$

Quizz

$$\Rightarrow \quad a = G\frac{M}{r^2}$$

Thus

> The acceleration of any orbiting object is independent of the object's mass.

Then

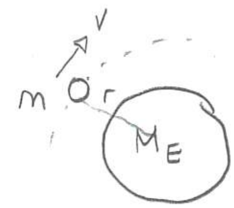
$$a = \frac{v^2}{r} \qquad \text{AND} \qquad v = \frac{2\pi r}{T}$$

give information about speed and period of orbit.

209 Newton's cannonball in a low orbit

Consider Newton's cannonball fired such that it orbits 3000 m (roughly the altitude of the Grand Mesa) above Earth's surface. Determine the speed of the cannonball. Ignore air resistance and any obstacles. *(111F2023)*

$$r = r_E + 3000\,m = 6.38\times10^6\,m + 3\times10^3\,m = 6.383\times10^6\,m$$



$$F_g = ma$$

$$\Rightarrow G\frac{\cancel{m}M_E}{r^2} = \cancel{m}a$$

$$\Rightarrow G\frac{M_E}{r^{\cancel{2}}} = \frac{v^2}{\cancel{r}}$$

$$\Rightarrow v^2 = G\frac{M_E}{r} \quad \Rightarrow v = \sqrt{\frac{GM_E}{r}}$$

$$\Rightarrow v = \sqrt{\frac{6.67\times10^{-11}\,Nm^2/kg^2 \times 6.0\times10^{24}\,kg}{6.383\times10^6\,m}}$$

$$v = 7.92\times10^3\,m/s$$

Warm up 2

Note that one can relate the orbital period to the orbital radius

$$v = \frac{2\pi r}{T}$$

mass of object being orbited

$$\Rightarrow \left(\frac{2\pi r}{T}\right)^2 = G\frac{M}{r}$$

$$\Rightarrow \frac{4\pi^2 r^2}{T^2} = G\frac{M}{r} \quad \Rightarrow \quad T^2 = \frac{4\pi^2}{GM} r^3$$

This is Kepler's Third Law