Thurs: Discussion quiz

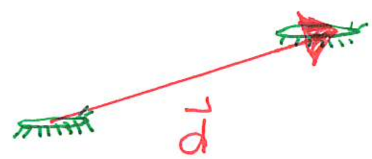
111 Ex: ~~52, 59, 61, 62,~~ 52, 56, 59, 61, 62, 64, 66

Fri:

* This class: - adding vectors
  - vector components

Vector algebra

Recall that a displacement vector is an arrow from the initial to final location of an object. The



displacement vector is denoted with an arrow above the label. A crucial feature of such vectors is

A displacement vector $\vec{d}$ has
* a magnitude (length), denoted d (never negative) AND
* a direction.

We need to define mathematical operations like addition, subtraction and multiplication for vectors.

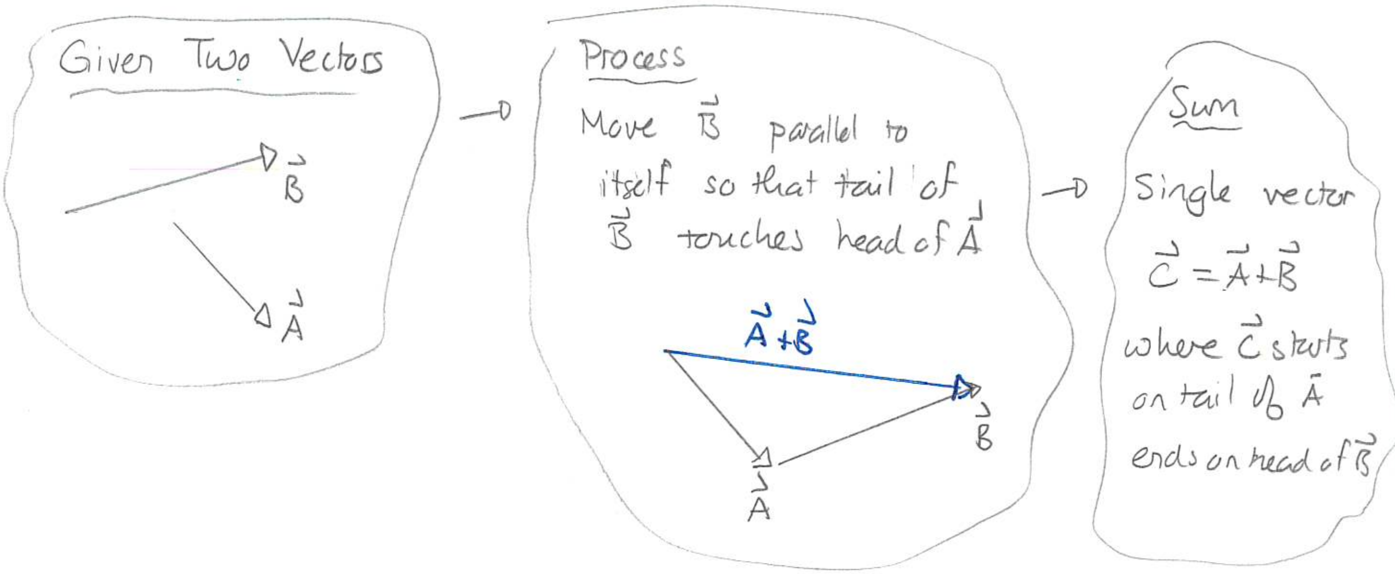
Vector addition

Consider two displacement vectors $\vec{A}, \vec{B}$. Then the idea behind the sum of $\vec{A}$ and $\vec{B}$ is

Sum of $\vec{A}$ and $\vec{B}$ $\simeq$ single displacement same as successive displacements of $\vec{A}$ and $\vec{B}$

DEMO: PhET Vector Addition – Explore 2D
* Produce Two Vectors – Show how to add.

The procedure for adding is:



Quiz 1 40% – 90%

Quiz 2 60% –

Note that vector addition $\vec{C}=\vec{A}+\vec{B}$ is different to addition of the magnitudes. In the previous example $C=5$, $A=4$, $B=3$ and $C \neq A+B$.

Vector Subtraction

Vector subtraction involves reversal of a vector. Here $-\vec{B}$ has the same magnitude as $\vec{B}$ but exactly the opposite direction.

Then vector subtraction is just adding a reversed vector. So
$\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$

Warm Up 1

Quiz 3

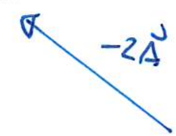
Vector multiplication by a scalar (number)

We can repeatedly add vectors, e.g. $\vec{A}+\vec{A}+\vec{A}=3\vec{A}$. This suggests we can multiply vectors by numbers. The rules are

Given a vector $\vec{A}$ and a number c

→ If c is positive then $c\vec{A}$ is a vector with length c times that of $\vec{A}$ and same direction

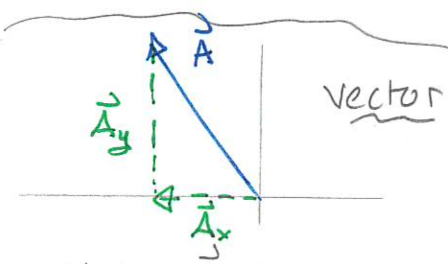
→ If c is negative then $c\vec{A}$ is a vector with length $|c|A$ and direction opposite to $\vec{A}$



Quiz 4

Vector components

We would like to do vector algebra (addition, subtraction, multiplication) without having to rely on diagrams. To do this we represent a vector in terms of components



Vector $\vec{A}$ decomposed into two component vectors

→ Two component numbers

A_x, A_y

→ horizontal

A_x = magnitude of $\vec{A}_x$ with + for right
− for left

A_y = magnitude of $\vec{A}_y$ " + for up
− for right

→ vertical

Demo: PhET Vector Addition

* Show component vectors
* Show components
* Show positive / negative.

Warm Up 2

Components are useful for vector algebra. For example, if

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$

then we can determine $\vec{D}$ by:

Get components

$\vec{A}$ and A_x, A_y

$\vec{B}$ and B_x, B_y

⋮

→ Components of $\vec{D}$ are

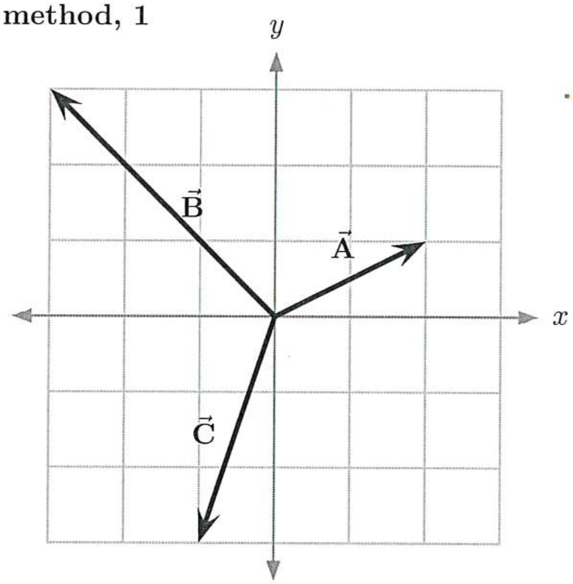
Horizontal $D_x = A_x + B_x + C_x$

Vertical $D_y = A_y + B_y + C_y$

↳ Reconstruct $\vec{D}$ from components

70 Vector addition: graphical and algebraic method, 1

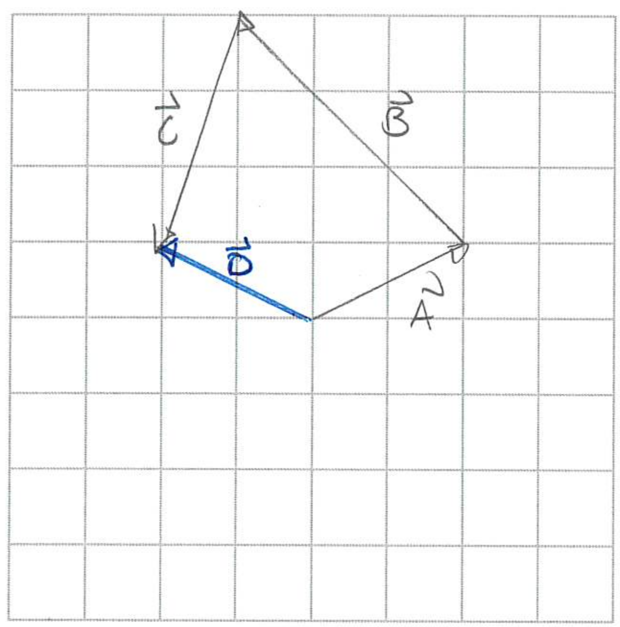
Displacement vectors, $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are illustrated. Let $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. *(111F2023)*



a) Using the graph sheet below, determine $\vec{\mathbf{D}}$ graphically via the head–to–tail method. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.

b) List the horizontal and vertical components of each of $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ and use these components to determine the components of $\vec{\mathbf{D}}$. Use the result to determine the magnitude of $\vec{\mathbf{D}}$.

Ans a)



b) $A_x = 2$ $\quad B_x = -3$ $\quad C_x = -1$ $\quad D_x = A_x + B_x + C_x = 2 - 3 - 1 = -2$

$A_y = 1$ $\quad B_y = 3$ $\quad C_y = -3$ $\quad D_y = A_y + B_y + C_y = 1 + 3 - 3 = 1$

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$= 2.2$$