

Thurs: Seminar, 12:30 WS 115

Fri: HW by 5pm Ex 166, 167, 168, ~~169~~ 170, 171, 173, 175, 176

Induced EMF

Whenever the magnetic flux through a loop changes with time, there will be an induced current and an induced EMF in the loop. The laws that describe this are:

Lenz's Law

The direction of the induced current is such that the induced magnetic field (produced by the induced current) opposes the change in the magnetic flux.

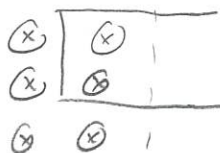
Faraday's Law:

Let Φ be the magnetic flux through a loop. Then the induced EMF has magnitude

$$\mathcal{E} = \left| \frac{d\Phi}{dt} \right|$$

Quiz 1 90%

Warm Up 1



← kick

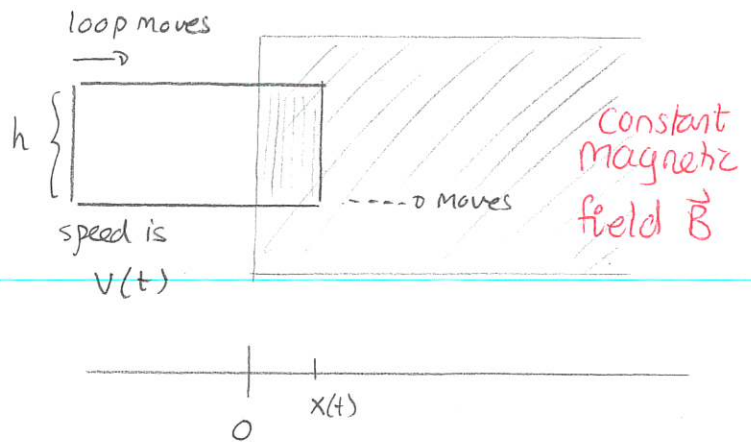
Demo: PSU-S Lenz's Law

We can assess the motional EMF as in the following example.

Consider a rectangular loop moving as illustrated

Then

$$\mathcal{E} = \left| \frac{d\Phi}{dt} \right|$$



and

$$\Phi = AB \cos \theta$$

Then we see that the area of the loop inside the field changes with time. So $A = A(t)$. Then:

$$\mathcal{E} = \left| \frac{d}{dt} [A B \cos \theta] \right| = \left| \frac{dA}{dt} \underbrace{B \cos \theta}_{\text{constant}} \right| = \left| \frac{dA}{dt} \right| B \cos \theta$$

We need to express area as a function of time. So with the illustrated co-ordinates:

$$A(t) = h x(t) \quad \Rightarrow \quad \frac{dA}{dt} = h \frac{dx}{dt} = h v(t)$$

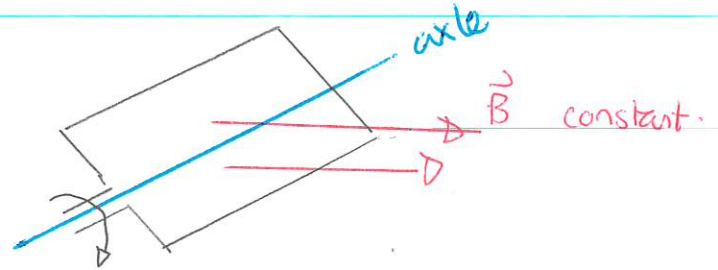
Thus, in this case,

$$\mathcal{E} = h |v(t)| B \cos \theta$$

Generators

We can see that motion of a loop relative to a field can generate a current. In order to do this constantly we need constant motion through a field. A rotating loop can accomplish this. We consider a

loop rotating through
a constant uniform field.

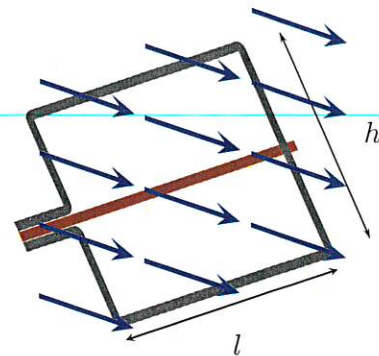


loop
forced to rotate

Warm Up 2

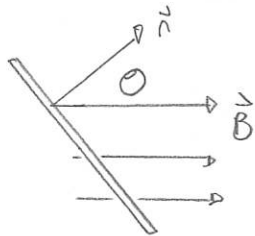
177 Generator

A generator consists of a loop which is forced to rotate about a fixed axle while it is in a uniform magnetic field as illustrated. Suppose that initially the loop is oriented vertically and then is forced (by a hand or something else external) to rotate with constant angular velocity, ω , about the illustrated axle. The aim of this exercise is to determine an expression for the EMF produced around the loop.



- Sketch a side view of the loop, indicate the normal and the angle between the normal and the field, θ . Express θ in terms of ω and time.
- Determine an expression for the flux through the loop as a function of time.
- Determine an expression for the EMF around the loop as a function of time.
- Does the EMF remain constant?
- Suppose that the loop is connected to a resistor with resistance R . Determine an expression for the power delivered to the resistor.

Answer a)



$$\theta = \omega t$$

$$b) \quad \Phi(t) = AB \cos \theta = h l B \cos(\omega t)$$

$$c) \quad \mathcal{E} = \frac{d\Phi}{dt} = -h l \omega B \sin(\omega t)$$

d) It varies with time.

$$e) \quad \mathcal{E} = IR \quad \text{and} \quad P = \mathcal{E}I \quad \Rightarrow \quad P = \mathcal{E}^2/R = \frac{\mathcal{E}^2}{R}$$

$$P = \frac{h^2 l^2 \omega^2}{R} \sin^2(\omega t)$$

In such a generator we see that the power

* increases as the angular velocity increases

* increases as the loop area increases

Demo: PhET generator

Quiz 2

Quiz 3

One way to effectively increase the loop area is to add many loops. Each loop will contribute the same EMF and they add together. So

$$\mathcal{E}_{N \text{ loops}} = N \mathcal{E}_{\text{single loop}}$$

Then with $\mathcal{E}_{\text{single loop}} = -\omega AB \sin(\omega t)$ we get

$$\mathcal{E}_{N \text{ loops}} = -N\omega AB \sin(\omega t)$$

The maximum value of the EMF is

$$\mathcal{E}_{\text{max}} = N\omega AB$$

Exercise Consider a square loop with sides 10cm long. Suppose the loop has 300 turns. It rotates through a uniform magnetic field with strength $5.0 \times 10^{-3} \text{ T}$. Determine the angular velocity s.t. $\mathcal{E}_{\text{max}} = 12 \text{ V}$.

Answer: $\mathcal{E}_{\text{max}} = N\omega AB \Rightarrow \omega = \frac{\mathcal{E}_{\text{max}}}{NAB}$ and $A = L^2$

$$\Rightarrow \omega = \frac{\mathcal{E}_{\text{max}}}{NL^2B} = \frac{12 \text{ V}}{300 \times (0.10 \text{ m})^2 \times 5.0 \times 10^{-3} \text{ T}} = 800 \text{ rad/s}$$

To convert $\omega = 800 \frac{\text{rad}}{\text{s}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 7600 \text{ rpm}$.