Phys 132 2022

Lecture 21

Mon: HW

Tues: Warm Up 8

Weds: Review

Thurs: Ex II Ch 26.5-26.6

Ch 27, 28

Magnetic fields produced by straight currents

The magnetic fields produced by straight sections of current are amongst the simplest to analyze. The direction is obtained via

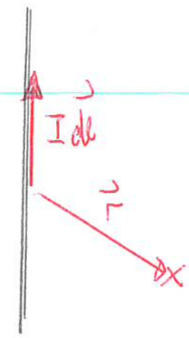
> The magnetic field produced by a straight section of current lies in a plane perpendicular to the current. The field vectors circle the current in a sense/direction given by the right hand rule

The magnitude is obtained via

> For an infinitely long wire carrying a uniform constant current I, the magnetic field at any location is
>
> $$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$
>
> where r is the distance from the wire (perpendicular) to the field point.

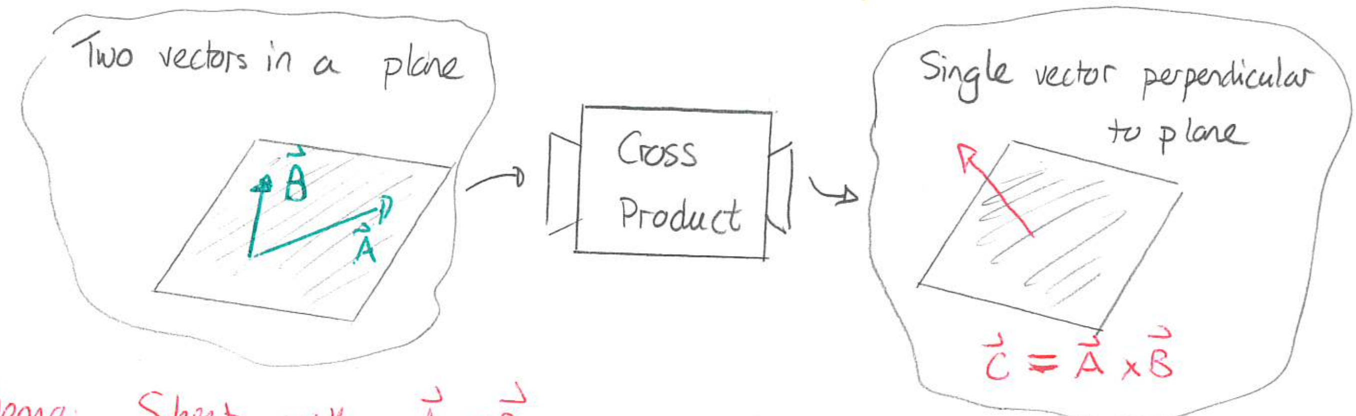
Here $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Tm/A}$ is the permeability of free space.

This result can be determined by applying a general rule for fields produced by currents, called the Biot-Savart law. The Biot-Savart law involves the cross product of vectors pertaining to the current and also the field point location.



Vector Cross Product.

The vector cross product is a piece of algebraic machinary that multiplies two vectors to obtain a third vector.



Demo: Sheet with $\vec{A} \times \vec{B}$

Two ways to define the cross product are:

Geometric rule

1) Magnitude of $\vec{C} = \vec{A} \times \vec{B}$ is

$$C = AB \sin\theta$$

where θ is angle between $\vec{A}, \vec{B}$

2) Direction is perpendicular to plane given by r.h.rule.

Quiz 1 60%

Quiz 2

Algebraic rule

Cross product satisfies

1) $\vec{A} \times (\alpha \vec{B}_1 + \beta \vec{B}_2) = \alpha(\vec{A} \times \vec{B}_1) + \beta(\vec{A} \times \vec{B}_2)$

2) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

and

$\hat{\imath} \times \hat{\jmath} = \hat{k}$ $\quad$ $\hat{\imath} \times \hat{\imath} = 0$

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$\hat{k} \times \hat{\imath} = \hat{\jmath}$

Biot-Savart Law

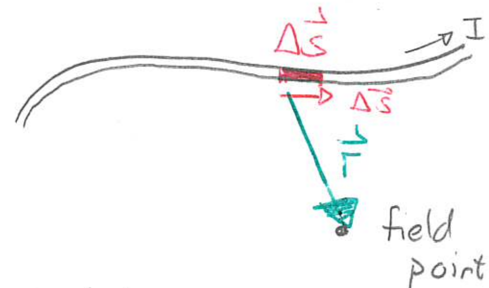
The fundamental rule for the magnetic field produced by stationary currents (non-time varying) is the Biot-Savart Law. It offers a procedure for determining magnetic fields by adding contributions from infinitesimal sections of current. The scheme is

1) Identify location of field point
2) Consider infinitesimal shaded segment
3) Identify vector $\Delta\vec{s}$ along this in the direction of current
4) Construct $\vec{r}$ from this segment to field point. Let $\hat{r}$ be the unit vector along this.
5) Then contribution to field from this segment is

$$\vec{B}_{segment} = \frac{\mu_0}{4\pi} I \frac{\Delta\vec{s} \times \hat{r}}{r^2}$$

6) Add contributions from all segments.

$$\vec{B} = \sum_{\text{all segments.}} \vec{B}_{segment}$$



Quiz 3 80% - 100%

Quiz 4 100%

Note that $\hat{r} = \vec{r}/r$ and so

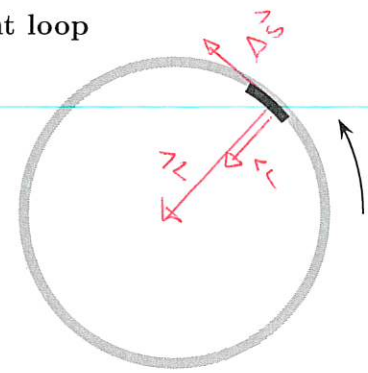
$$\vec{B}_{segment} = \frac{\mu_0}{4\pi} I \frac{\Delta\vec{s} \times \vec{r}}{r^3}$$

Currents and Magnetic Fields

125 Magnetic field at the center of a circular current loop

A circular loop with radius R carries current I as illustrated. The aim of this exercise is to use the Biot-Savart law to determine the magnetic field at the center of the loop. First consider the contribution to the field from the shaded section. This is

$$\vec{\mathbf{B}}_{\text{segment}} = \frac{\mu_0}{4\pi} I \frac{\Delta\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}. \qquad (3)$$



a) Use the diagram to indicate the vectors $\vec{\mathbf{r}}, \hat{\mathbf{r}}$ and $\Delta\vec{\mathbf{s}}$.

b) Determine the direction of $\Delta\vec{\mathbf{s}} \times \hat{\mathbf{r}}$. Would the direction be different for a different segment of the current in the loop?

c) Let the magnitude of $\Delta\vec{\mathbf{s}}$ be Δs. Determine an expression for the magnitude of $\Delta\vec{\mathbf{s}} \times \hat{\mathbf{r}}$ and use this and Eq. (3) to determine an expression for the magnitude of $\vec{\mathbf{B}}_{\text{segment}}$.

d) What is the value of r in this situation? Use your answer to determine an expression for the magnitude of $\vec{\mathbf{B}}_{\text{segment}}$.

e) Add the contributions from all segments to obtain an expression for the field at the center of the loop.

f) Suppose that you aimed to use this to determine the magnetic field at a point that is not at the center of the loop. What mathematical difficulties would complicate the calculation compared to that for the center of the loop?

Answer: a) See diagram

b) Out of page - all are same

c) $\vec{B}_{segment} = \frac{\mu_0}{4\pi} \frac{I}{r^3} \Delta\vec{s} \times \vec{r}$

$|\Delta\vec{s} \times \vec{r}| = (\Delta s)\, r \sin 90^\circ$
$= r\Delta s$

$B_{segment} = \frac{\mu_0}{4\pi} \frac{I}{r^3} \Delta s\, r$

$= \frac{\mu_0}{4\pi} \frac{I}{r^2} \Delta s$

d) The loop has radius R; thus $r = R$

$$\Rightarrow \quad B_{segment} = \frac{\mu_0}{4\pi}\frac{I}{R^2}\Delta s$$

e) $$B = \sum B_{segment} = \frac{\mu_0}{4\pi}\frac{I}{R^2}\underbrace{\sum_{\text{all segments}} \Delta s}_{\text{circumference of loop } = 2\pi R}$$

Thus

$$B = \frac{\mu_0}{4\pi}\frac{I}{R^2}2\pi R \quad \Rightarrow \quad B = \frac{\mu_0 I}{2R}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{2R} \text{ out of page.}$$

f) The directions would not change. However r would vary for different segments. Thus the sum would involve

$$\sum_{\text{all segments}} \frac{\Delta s}{r^2}$$

and this would be difficult to compute.