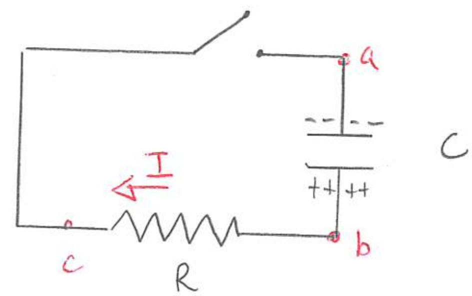
Weds: Discussion / quiz

Ex 106, 109, 111, 112, 114, 115

Thurs: Warm Up 7

## RC Circuits

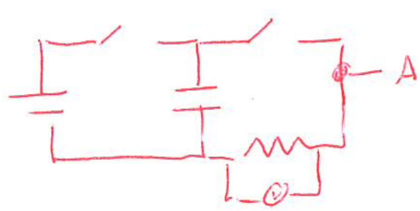



An RC circuit consists of a capacitor and a resistor in series. The capacitor is initially charged and the switch is then closed. The capacitor will discharge through the resistor. The questions are:

1) How does the charge behave as time passes after the switch is closed?

2) Is there some characteristic rate at which the capacitor discharges?

Demo: PhET AC Circuit Kit




Show discharging current, voltage.

As the capacitor discharges, the resistor impedes the flow of charge. So the rate of discharge will partly be determined by the resistance.

The voltage will also play a role and this is connected to the charge via the capacitance. We expect this to play a role as well.

The analysis involves

1) $Q$ = charge on positive capacitor plate.
2) $I$ = magnitude of current through resistor.
3) $V_{cap}$ = voltage across capacitor.

With the illustrated set-up

As time passes $Q$ decreases

$|I| = \left|\frac{dQ}{dt}\right|$

$\boxed{I = -\frac{dQ}{dt}}$ makes $I > 0$.

Around the loop $\Delta V_{a \to b} + \Delta V_{b \to c} + \cancel{\Delta V_{c \to a}}^{0} = 0$

and $\Delta V_{a \to b} = V_{cap}$

$\Delta V_{b \to c} = -IR$

Thus $V_{cap} - IR = 0$.

Then $C V_{cap} = Q \Rightarrow V_{cap} = Q/C$. Thus

$$Q/C + R\frac{dQ}{dt} = 0 \Rightarrow \boxed{\frac{dQ}{dt} = -\frac{1}{RC} Q}$$

Quiz 1 80%

Quiz 2

## 123 Discharging capacitor

A capacitor with capacitance, $C$ is connected to a resistor with resistance $R$. At time $t = 0$ the charge on the capacitor is $Q_0$. Subsequently the charge satisfies

$$\frac{dQ}{dt} = -\frac{1}{RC}Q. \tag{1}$$

The aim of this exercise is to solve this for $Q$ at all later times.

a) A solution to Eq. (1) is a function $Q(t)$ such that when it is substituted into both sides of equation Eq. (1) the resulting statement is *true for all times* $t$. To test this consider the possibility $Q(t) = t^3$. Substitute into both sides of Eq. (1) and carry out the relevant mathematics. Is the resulting equality *true for all times* $t$?

b) Consider the possible solution $Q = Ae^{\alpha t}$ where $A$ and $\alpha$ are constants. Substitute into both sides of Eq. (1) and carry out the relevant mathematics. Is the resulting equality *true for all times* $t$? What condition on $\alpha$ would guarantee this?

c) Determine a value for $A$ such that the charge at $t = 0$ is $Q_0$.

d) The resulting expression is written as

$$Q = Q_0 e^{-t/\tau} \tag{2}$$

where $\tau$ is a constant called the capacitive time constant. Determine an expression for $\tau$ in terms of $R$ and $C$.

The capacitive time constant quantifies the rate at which the capacitor charge decays. In the following, suppose that $R = 2.0 \times 10^5\,\Omega$ and $C = 5.0 \times 10^{-7}\,\mathrm{F}$.

e) Determine the capacitive time constant. What fraction of the initial charge remains at $t = 0.1\,\mathrm{s}$? What fraction of the initial charge remains at $t = 0.2\,\mathrm{s}$?

f) In experiments data for $Q$ versus $t$ is conveniently represented by plotting $\ln(Q)$ versus $t$. Determine an expression for $\ln(Q)$ in terms of $t$ and constants. What sort of plot would this yield? What is the slope of this plot?

Answer: a) If $Q = t^3$ then

$$\frac{dQ}{dt} = 3t^2$$

$$-\frac{1}{RC} Q = -\frac{1}{RC} t^3$$

Setting these equal: $3t^2 = -\frac{1}{RC} t^3$

$$-3RC = t$$

not true at all times.

b) $$\frac{dQ}{dt} = \alpha A e^{\alpha t}$$

$$\frac{dQ}{dt} = -\frac{1}{RC} Q \iff \alpha \cancel{A e^{\alpha t}} = -\frac{1}{RC} \cancel{A e^{\alpha t}}$$

$$\iff \alpha = -\frac{1}{RC}$$

works for all $t \iff \alpha = -1/RC$

c) $$Q = Ae^{-t/RC}$$

At $t = 0$ $\quad Q_0 = A \underbrace{e^{-0/RC}}_{1} \Rightarrow A = Q_0$

d) Based on the previous $\tau = RC$

e) $\tau = RC = 2.0 \times 10^5 \Omega \times 5.0 \times 10^{-7} F = 10 \times 10^{-2} s = 0.10 s$

At $t = 0.10s$ $\quad Q = Q_0 e^{-0.10s/0.10s}$

$$= Q_0 e^{-1}$$

$$Q/Q_0 = e^{-1} = 0.37$$

At $\underbrace{t = 0.20\,s}$ $\quad Q = Q_o e^{-0.20s/0.10s}$

$$= Q_o e^{-2}$$

$$\Rightarrow Q/Q_o = e^{-2} = 0.14$$

f) $\ln Q = \ln\left[Q_o e^{-t/\tau}\right]$

$$= \ln Q_o + \ln e^{-t/\tau}$$

$$= \ln Q_o - t/\tau$$

$$\underbrace{\ln Q}_{y} = \underbrace{-\tfrac{1}{\tau}}_{\text{slope}} \underbrace{t}_{x} + \underbrace{\ln Q_o}_{\text{intercept}} \quad \Rightarrow \quad \text{slope} = -\tfrac{1}{\tau}.$$

Note that the time constant quantifies the decay in the sense.

At time $t_o$ charge is $Q_i$

At time $t_o + \tau$ charge is $Q_f$

$\Rightarrow Q_f = Q_o e^{-(t_o+\tau)/\tau}$

$= Q_o e^{-t_o/\tau} e^{-1}$

$Q_i = Q_o e^{-t_o/\tau}$.

$\Rightarrow Q_f = Q_i e^{-1}$

$$Q_f = Q_i e^{-1}$$

$$Q_f = Q_i \, 0.37$$

This is exponential decay.