Weds: Discussion/Quiz

Ex 36, 37, 38ab, 39a, 43, 44, 45

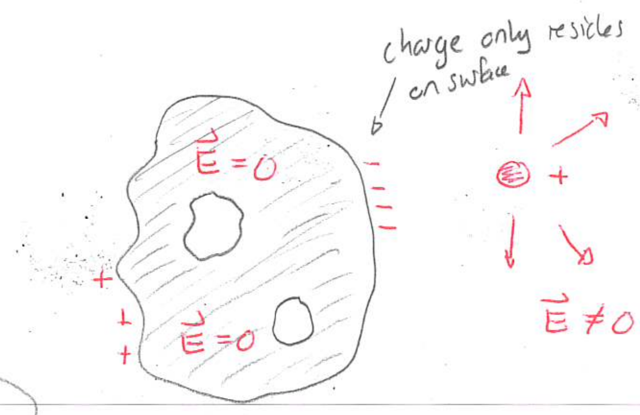
Thurs: Warm Up 3 D2L

Seminar

Fields in conductors

A conductor is a material in which any excess charge can flow freely without resistance. In an electrostatic situation there cannot be any net electric forces on the charges within a conductor otherwise they would move. Thus

> In electrostatic equilibrium the electric field is zero inside the conductor
>
> $\vec{E} = 0$



Then Gauss' law gives that

> The charge density anywhere inside a perfect conductor in equilibrium is zero

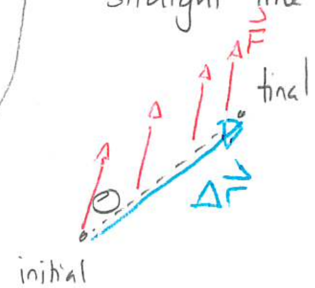
In these cases the field inside the hollows is also zero.

Demo Falstad - Conducting cylinder + add point charge

Work + energy overview

Work and energy offer a partial alternative to forces + accelerations for assessing classical physics situations. The general process is:

For constant force, straight line motion



Work done by force

$$W = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos\theta$$

For general forces / trajectories

$$W = \int \vec{F}\cdot d\vec{r}$$

along trajectory

Work - Kinetic Energy Theorem

$$W_{net} = \Delta K$$

where W_{net} = sum of all works by individual forces

Kinetic energy

$$K = \frac{1}{2}mv^2$$

Quiz 1 90% – 100%

Quiz 2 70% – 90%

Then a special case is that where the force is conservative. In this case there exists a potential energy U such that

$$W = -\Delta U$$

Then we can arrive at

If the mechanical energy of the system is

$$E_{mech} = K + U_{force\,1} + U_{force\,2} + \cdots$$

Then

$$\Delta E_{mech} = \Delta K + \Delta U_{force\,1} + \cdots = W_{non\text{-}cons}$$

work done by non-conservative forces

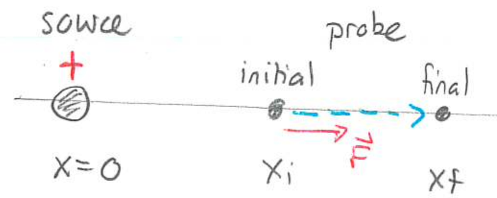
This gives the <u>conservation of energy</u>

If the non-conservative forces do zero work then $\Delta E_{mech} = 0 \iff \Delta K + \Delta U = 0 \iff$ mechanical energy is constant

Electric potential energy

We now consider the question of whether the electrostatic force is conservative. The easiest situation is to consider two point charges.

Then suppose that the probe moves a small displacement



$$\Delta \vec{r} = \Delta x \, \hat{i}$$

Then $\vec{F} = k \frac{q_1 q_2}{r^2} \hat{i} \Rightarrow \vec{F} = k \frac{q_1 q_2}{x^2} \hat{i}$ and the work done is

$$\Delta W = k \frac{q_1 q_2}{x^2} \Delta x$$

Adding over all small segments gives

$$W = \sum k \frac{q_1 q_2}{x^2} \Delta x \rightarrow \int_{x_i}^{x_f} k \frac{q_1 q_2}{x^2} \, dx$$

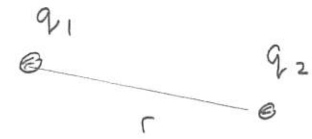
This gives

$$W = -k q_1 q_2 \frac{1}{x} \Big|_{x_i}^{x_f} \Rightarrow W = -k \frac{q_1 q_2}{x_f} + k \frac{q_1 q_2}{x_i}$$

$$= -\left[\frac{k q_1 q_2}{x_f} - \frac{k q_1 q_2}{x_i} \right]$$

Thus if we define the electrostatic potential energy for two point charges as

$$\boxed{U_{elec} = k\,\frac{q_1 q_2}{r}}$$



we get $W = -\Delta U_{elec}$ in the previous example. Some features of this are:

1) the electrostatic potential energy of two point charges infinitely far apart is zero.

2) the electrostatic potential energy of a collection of point charges is the work needed to assemble them from a configuration where they are infinitely far apart at rest.

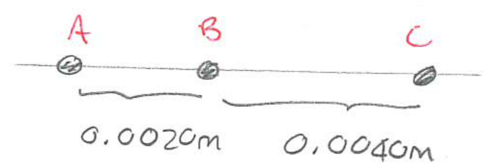
Quiz 3

Example: Determine the electrostatic potential energy of the illustrated arrangement. Here

$q_A = 5.0\mu C$

$q_B = -2.0\mu C$

$q_C = 5.0\mu C$



Answer: $U_{elec} = U_{elec\,AB} + U_{elec\,AC} + U_{elec\,BC}$

$$U_{elec\,AB} = k\,\frac{q_A q_B}{r_{AB}} = 8.99\times10^9\,\frac{Nm^2}{C^2}\;\frac{5.0\times10^{-6}C\times(-2.0\times10^{-6}C)}{0.0020m}$$

$$= -45\,J$$

$$U_{elec\,AC} = k\,\frac{q_A q_C}{r_{AC}} = 8.99\times10^9\,\frac{Nm^2}{C^2}\;\frac{5.0\times10^{-6}C\times 5.0\times10^{-6}C}{0.0060m} = 37.5\,J$$

$$U_{elec\,BC} = k\,\frac{q_B q_C}{r_{BC}} = 8.99\times10^9\,\frac{Nm^2}{C^2}\;\frac{(-2.0\times10^{-6}C)\times(5.0\times10^{-6}C)}{0.0040m} = -22.5\,J$$

$$U_{elec} = -45\,J + 37.5\,J - 22.5\,J = -30\,J.$$

46 Probes moving away from a fixed point charge

A +30 nC is held fixed. Various probe particles are placed near to this. First a proton is released from rest 0.0010 m from the fixed charge. The aim is to determine the speed of the proton when it is 0.0025 m from the fixed charge. *(132S22 Class)*

a) Suppose that you tried to use Coulomb's law to determine the speed. What mathematical difficulties would arise?

b) Use energy conservation to determine the velocity of the proton when it is 0.0025 m from the fixed charge.

Second an electron, initial 0.5cm ~~0.0010~~ m from the fixed charge, is launched directly away.

c) Determine the minimum launch speed of the electron such that it escapes the fixed charge.

Answer: a) The force $F = k\dfrac{q_1 q_2}{r^2}$ is not constant $\Rightarrow$ acceleration is not constant. So constant acceleration kinematics will not work.

b) $\Delta E_{mech} = 0$

+30nC   initial   final

$r_i = 0.0010\text{m}$ $r_f = 0.0025\text{m}$

$v_i = 0$ $v_f = ?$

$$\Rightarrow K_f + U_{elec\,f} = K_i + U_{elec\,i}$$

$$\Rightarrow \frac{1}{2}mv_f^2 + k\frac{q_1q_2}{r_f} = \cancel{\frac{1}{2}mv_i^2}^{\,0} + k\frac{q_1q_2}{r_i}$$

$$\Rightarrow \frac{1}{2}mv_f^2 = k\frac{q_1q_2}{r_i} - k\frac{q_1q_2}{r_f} = kq_1q_2\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$\Rightarrow v_f^2 = \frac{2kq_1q_2}{m}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{2 \times 8.99\times 10^9\,\text{Nm}^2/\text{C}^2 \times 30\times 10^{-9}\,\text{C} \times 1.6\times 10^{-19}\,\text{C}}{1.67\times 10^{-27}\,\text{kg}}\left(\frac{1}{0.0010\text{m}} - \frac{1}{0.0025\text{m}}\right)$$

$V_f^2 = 3.1 \times 10^{13}\ m^2/s^2 \quad \Rightarrow \quad V = 5.6 \times 10^6\ m/s$

c) The set up is similar but $r_f = \infty$ $\quad V_f = 0\ m/s$

$r_i = 0.0010\,m$ $\quad v_i = ??$

$$K_f + U_{elec\,f} = K_i + U_{elec\,i}$$

$$\cancel{\tfrac{1}{2} m v_f^2}^{\,0} + \cancel{k \frac{q_1 q_2}{r_f}}^{\,0} = \tfrac{1}{2} m v_i^2 + k \frac{q_1 q_2}{r_i}$$

$$0 = \tfrac{1}{2} m v_i^2 + k \frac{q_1 q_2}{r_i}$$

$$\Rightarrow \quad \tfrac{1}{2} m v_i^2 = -\frac{k q_1 q_2}{r_i}$$

$$\Rightarrow \quad v_i^2 = -\frac{2 k q_1 q_2}{m r_i} = \frac{-2 \times 8.99 \times 10^9 \frac{Nm^2}{C^2} \times (-1.6 \times 10^{-19} C) \times 30 \times 10^{-9} C}{9.11 \times 10^{-31} kg \times 0.50 m}$$

$$= 1.9 \times 10^{14}\ m^2/s^2$$

$$\Rightarrow \quad v_i = \sqrt{1.9 \times 10^{14}\ m^2/s^2} = 1.4 \times 10^7\ m/s \quad \boxed{}$$