Electromagnetism and Optics: Class Exam II

17 March 2022

Name: Solution **Total:** /70

Instructions

- There are 9 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$e = 1.61 \times 10^{-19}\,\text{C} \qquad q_{\text{electron}} = -e \qquad q_{\text{proton}} = +e$$
$$m_{\text{electron}} = 9.11 \times 10^{-31}\,\text{kg} \qquad m_{\text{proton}} = 1.67 \times 10^{-27}\,\text{kg}$$
$$k = 9.0 \times 10^{9}\,\text{Nm}^2/\text{C}^2 \qquad \epsilon_0 = 8.85 \times 10^{-12}\,\text{C}^2/\text{Nm}^2$$

Question 1

In a procedure to coat an object with copper, copper ions flow through a solution and accumulate on the object. Each copper ion has the same charge as two protons. The current (consisting of copper ions) is 0.025 A. Determine the total number of copper ions that accumulate on the object in 30 minutes.

$I = \frac{\Delta Q}{\Delta t} \Rightarrow \Delta Q = I\Delta t$] +1

Let N be number of ions and q charge on each. Then

$\Delta Q = Nq \Rightarrow Nq = I\Delta t \Rightarrow N = \frac{I\Delta t}{q}$.

+1

Here $q = 2e$ gives

+1

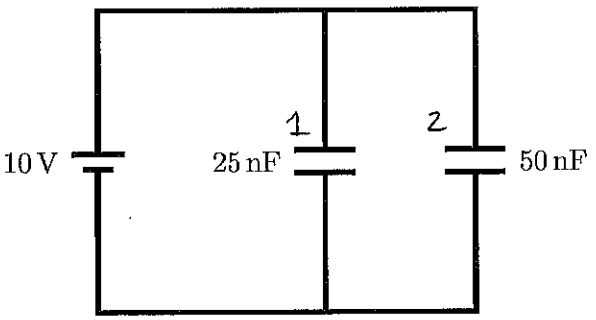
$$N = \frac{I\Delta t}{2e} = \frac{0.025\,\text{A} \times 30\,\text{min} \times 60\,\text{s/min}}{(1.6\times10^{-19}\,\text{C}) \times 2}$$] +3

$$N = 1.4 \times 10^{20}$$

/6

Question 2

Two capacitors are connected to a battery as illustrated.



+5 a) Determine the charge across each capacitor.

The potential difference is the same for each.

For 1

$$\Delta Q_1 = C_1 \Delta V$$

$$= 25 \times 10^{-9} F \times 10 V = 2.5 \times 10^{-7} C.$$

For 2

$$Q_2 = C_2 \Delta V$$

$$= 50 \times 10^{-9} F \times 10 V = 5.0 \times 10^{-7} C.$$

+5 b) Determine the total energy stored in the pair of capacitors.

For 1

$$U_1 = \frac{1}{2} \frac{Q_1^2}{C_1} = \frac{1}{2} C_1 (\Delta V_1)^2 \Rightarrow U_1 = \frac{1}{2} 25 \times 10^{-9} F \times (10V)^2$$

$$= 1.25 \times 10^{-6} J$$

For 2

$$U_2 = \frac{1}{2} C_2 (\Delta V_2)^2 = \frac{1}{2} 50 \times 10^{-9} F \times (10V)^2 = 2.5 \times 10^{-6} J$$

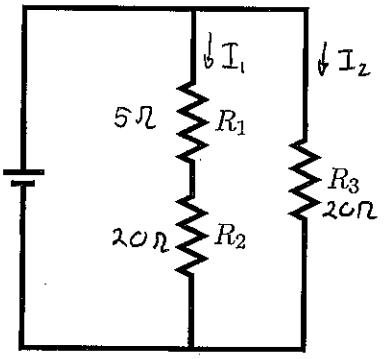
The total energy is $U = U_1 + U_2$

$$U = 3.75 \times 10^{-6} J$$

/10

Question 3

Resistors, labeled 1, 2, and 3, are connected to an ideal battery as illustrated. The resistances are $R_1 = 5\,\Omega$ and $R_2 = R_3 = 20\,\Omega$. Let P_1 be the power produced by resistor 1, P_2 the power produced by resistor 2 and P_3 the power produced by resistor 3. Which of the following (choose one) is true?



i) $P_1 = P_2 = P_3$
ii) $P_2 = P_3 < P_1$
iii) $P_2 = P_3 > P_1$
iv) $P_1 = P_2 < P_3$
v) $P_1 < P_2 < P_3$
vi) $P_2 < P_1 < P_3$

For each resistor $P = I\Delta V$
$= I\,IR$
$= I^2R$.

The current from the battery splits. I_2 is larger than I_1 since resistance on right is less than on left.

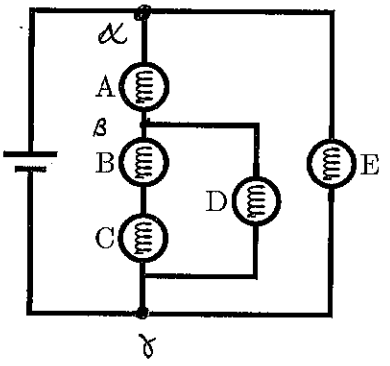
So $P_3 > P_2$

Then $P_2 > P_1$ since currents are same and $R_2 > R_1$

/5

Question 4

A battery and identical bulbs are connected in the illustrated circuit. Rank the bulbs in order of increasing brightness. Explain your answer.



Power only depends on current since $P = IR^2$ and resistances are all equal.

The resistance of the A,B_C,D combination is larger than E. So at α more current flows right. This means that E is brighter than any others.

The current that flows left all passes through A but splits at β. So A is brighter than B, C, D.

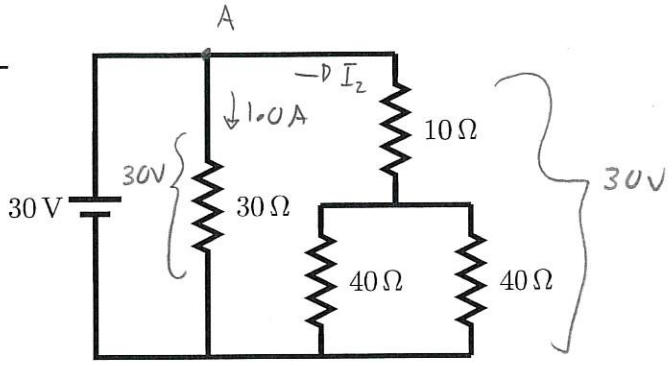
At β more current flows right since R is lower, So D is brighter than B, C. The last two are equal since they are in series.

B = C < D < A < E

/6

Question 5

Determine the current that flows through **each resistor** and **the battery** in the illustrated circuit.

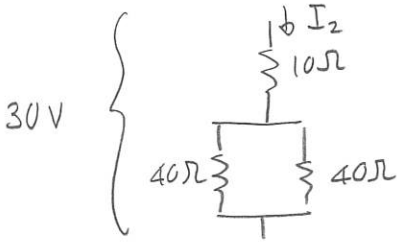


There are two parallel arms whose voltages are each 30V.

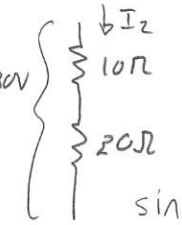
Across the 30Ω resistor

$\Delta V = IR \Rightarrow 30V = I30\Omega \Rightarrow I = 1.0A$

Now consider the remaining combination:



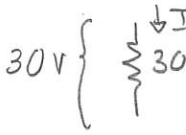
$\approx$



since $\frac{1}{R_{eq}} = \frac{1}{40} + \frac{1}{40}$

$= \frac{1}{20}$ +3

This is equivalent to



+1

resistor	current	
30Ω	1.0A	+3
10Ω	1.0A	+3
40Ω	0.5A	+3
40Ω	0.5A	
battery	2.0A	

So here $I_2 = 1.0A$ again. This flows through the 10Ω resistor

This splits evenly at the two 40Ω resistors $\rightsquigarrow$ 0.5A in each

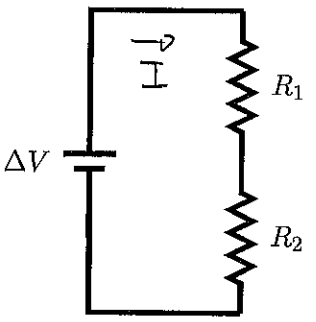
At the junction A, $I_{in} = 1.0A + 1.0A = 2.0A$ +2

Thus current in battery is 2.0A

/15

Question 6

Two resistors, with resistances R_1 and R_2, are connected to a battery as illustrated. The battery provides potential difference ΔV.



a) Derive an expression for the current (in terms of R_1, R_2 and ΔV) that flows through each resistor in the illustrated circuit.

$\Delta V = I R_{eq}$ $\quad R_{eq} = R_1 + R_2$

$\Rightarrow \Delta V = I(R_1 + R_2) \Rightarrow I = \dfrac{\Delta V}{R_1 + R_2}$

The current in each resistor is the same. So $I = \dfrac{\Delta V}{R_1 + R_2}$ +3

b) Derive an expression for the potential difference (in terms of R_1, R_2 and ΔV) across the resistor with resistance R_1 in the illustrated circuit.

$\Delta V_1 = I R_1$

$= \dfrac{\Delta V}{R_1 + R_2} R_1 \Rightarrow \Delta V_1 = \dfrac{R_1}{R_1 + R_2} \Delta V$

+3

c) Determine the ratio R_1/R_2 such that the potential difference across the resistor with resistance R_1 is one third of the potential difference provided by the battery.

We need $\Delta V_1 = \frac{1}{3} \Delta V$ +6

$\Rightarrow \dfrac{R_1}{R_1 + R_2} \Delta V = \frac{1}{3} \Delta V$

$R_1 = \frac{1}{3} R_1 + \frac{1}{3} R_2$

$\frac{2}{3} R_1 = \frac{1}{3} R_2 \Rightarrow R_1/R_2 = 1/2$ /12

Question 7

A heater is designed to produce 1200 W of power when connected to a 120 V outlet. Assuming that the heater obeys Ohm's law, determine the resistance of the heater.

$P = I\Delta V$ and $I = \Delta V/R$

$\Rightarrow P = (\Delta V)^2/R \Rightarrow R = \frac{(\Delta V)^2}{P} = \frac{(120V)^2}{1200W} = 12\Omega$

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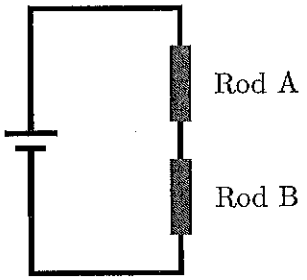
Question 8

Two metal rods are connected to a battery as illustrated. The cylinders have the same lengths and also the same radius. The resistivity of rod A is five times that of rod B. Which of the following (choose one) is true regarding the currents through the rods?



i) $I_{\text{in A}} = I_{\text{in B}}$ In series currents are same everywhere.

ii) $I_{\text{in A}} = 25\, I_{\text{in B}}$

iii) $I_{\text{in A}} = 5\, I_{\text{in B}}$

iv) $I_{\text{in A}} = \frac{1}{5} I_{\text{in B}}$

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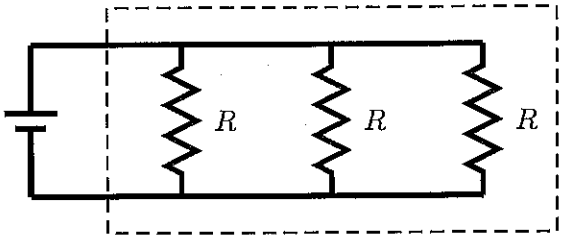
Question 9

Three identical resistors are connected to a battery as illustrated in the following circuit. Which of the following (choose one) is the equivalent resistance of the portion of the circuit within the box?



i) $R_{\text{eq}} = R/3$

ii) $R_{\text{eq}} = R/2$

iii) $R_{\text{eq}} = R$

iv) $R_{\text{eq}} = 2R$

v) $R_{\text{eq}} = 3R$

$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$

$\Rightarrow R_{eq} = R/3$

/5