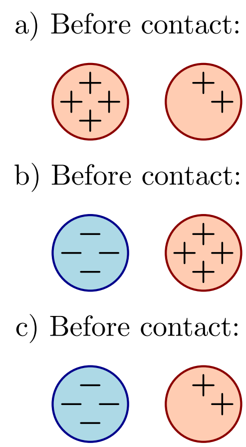
Phys 132: Exercises

Electrostatics

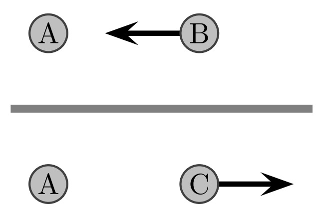
1 Charge distribution

Various identical metal spheres are separated and charged. The excess charged particles on each sphere are illustrated. Individual excess charged particles have the same magnitude of charge. The spheres are supported by insulating stands and are brought into contact and then later are separated. Determine the charge on each sphere after they have been in contact. Briefly explain your answers. *(132S22)*



2 Interacting charges

Pairs of objects that may or may not be charged are placed near each other and the observed interactions are as illustrated. What interaction would occur if C were placed near to B? Explain your answer. *(132S22)*



3 Interacting objects

Various objects, A, B, C and D, that may or may not be charged are placed near each other and the interactions are observed. It is observed that object A attracts object B, object A attracts object C and objects B and C do not interact. Object A repels object D. *(132S22)*

a) Can one determine the types of charge of B and C with certainty? If so what are they?

b) Can one determine the types of charge of A and D with certainty? If so what are they?

c) Suppose that D is placed near to B. Explain how it will interact with B.

4 Sphere, rod and wire

A metal sphere is initially uncharged. A positively charged rod is held near to but not touching the sphere. At this time a wire is briefly connected from the ground to the side of the sphere opposite to the rod and is then removed, while the rod is in place. Subsequently the rod is then removed. After all of this a small negatively charged ball is held near to the sphere. Will the sphere exert a force on the ball? If so is it repulsive or attractive? Explain your answer. *Hint: ground can supply and absorb charged particles. (132S22)*

5 Charge redistribution

Two identical metal balls are initially separated and have charges q_1 and q_2. The balls are brought into contact and the charge redistributes. *(132S22)*

a) Determine an expression for the charge on each ball after it has redistributed.

b) Assuming that $q_1 > q_2$, determine an expression for the number of electrons that flow from one ball to the other during the redistribution.

6 Electrons in copper

Copper consists of atoms that each have 29 electrons and mass 1.06×10^{-25} kg. Copper has a density of $8.69\,\mathrm{kg/m^3}$. *(132S22)*

a) Determine the number of electrons in a cube of pure copper with sides of length 2.0 cm. Assume that the copper is electrically neutral.

b) Suppose that electrons are removed from the copper so that it has a net charge of 5.0×10^{-9} C (this is a typical number for capacitors in electronic circuits). Determine the number of electrons that have been removed.

c) What fraction of the total number of electrons in the copper were removed to give it charge 5.0×10^{-9} C?

7 LHC accelerator beam dump

The Large Hadron Collider (LHC) is a high-energy physics particle accelerator at the CERN laboratory. The accelerator works by bombarding a target with high-velocity protons. Only a few of these hit the target and most of the protons need to be removed in a "beam dump." Beam dumps consist of a set of "bunches" with each bunch containing roughly the same number of protons. *(132S22)*

a) Determine an expression for the total charge deposited in a beam dump in terms of the number of bunches and the number of protons in each bunch.

b) According to CERN, a typical beam dump in the LHC consists of about 2500 bunches, each with about a billion protons. Determine the charge delivered in this beam dump.

c) Each beam has a total energy of about 560 MJ and the dump lasts $86\,\mu$s. Determine the power delivered in a single beam dump. Compare this to the power delivered by a typical electrical power plant.

8 Electric forces and charge magnitude

Two charged particles, A on the left and B on the right, are held at fixed locations. The charges on the particle can be adjusted while their locations stay fixed. *(132S22)*

a) Suppose that the charge of B is tripled. By how many times does this increase the force that A exerts on B? Explain your answer.

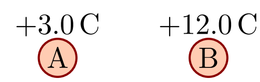
b) Suppose that the charge of B is tripled. By how many times does this increase the force that B exerts on A? Explain your answer.

c) Suppose that the charge of A is tripled and the charge of B is also tripled. By how many times does this increase the force that A exerts on B? Explain your answer.

d) Suppose that the charge of A is tripled and the charge of B is also tripled. By how many times does this increase the force that B exerts on A? Explain your answer.

9 Electric forces and pairs of charges

Two charged particles, A and B, are held fixed. Let $F_{\text{A on B}}$ be the magnitude of the force exerted by A on B and $F_{\text{B on A}}$ be the magnitude of the force exerted by B on A. Which of the following is true? Explain your answer. *(132S22)*



i) $F_{\text{B on A}} = \frac{1}{4} F_{\text{A on B}}$

ii) $F_{\text{B on A}} = F_{\text{A on B}}$

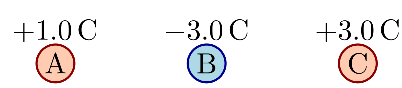
iii) $F_{\text{B on A}} = 3F_{\text{A on B}}$

iv) $F_{\text{B on A}} = 4F_{\text{A on B}}$

v) $F_{\text{B on A}} = 12F_{\text{A on B}}$

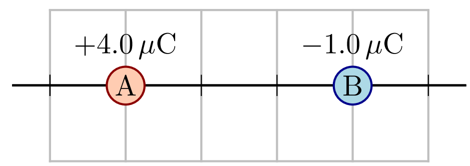
10 Electric forces and charges in a line

Three charged particles, A on the left, B in the center and C on the right, are held at fixed locations in a line. The distance between A and B is the same as between B and C. Rank the magnitudes of the net forces on each charge from smallest to largest. Explain your answer. *(132S22)*



11 Linear charge arrangements and zero force

Two charged particles are held fixed as illustrated; the grid units are each 0.010 m. A third charge is placed along the axis that connects the two charges (either between the charges or beyond one of them). Determine the location at which the net force on this third charge are zero. *Hint: Set up the problem using axes with the origin at one of the charges and using algebraic coordinates for the location of the third charge. (132S22)*



12 Electric and gravitational forces

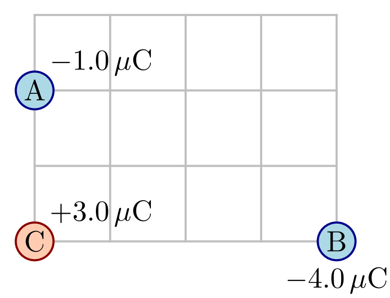
Two identical particles each have mass m and charge q and are held stationary. *(132S22)*

a) Determine an expression for the ratio of the electric force exert by one on the other to the gravitational force exerted by one on the other. Does this depend on the distance between the particles?

b) Determine a numerical value for the ratio of the electric to the gravitational force for two protons. What does you result say about the importance of electrical versus gravitational forces for the constituents of ordinary matter?

13 Two dimensional charge arrangements: force, 1

Three charged particles are held fixed as illustrated; the grid units are each 0.010 m. Determine the magnitude and direction of the net force on charge C. *(132S22)*



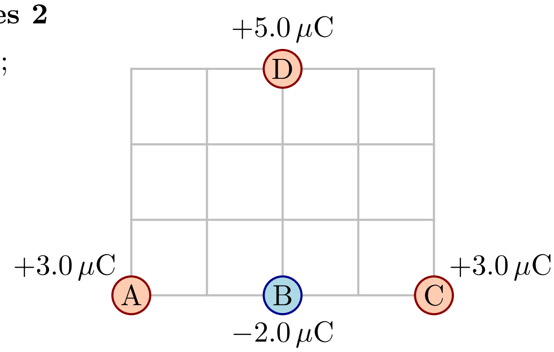
14 Two dimensional charge arrangements: forces 2

Four charged particles are held fixed as illustrated; the grid units are each 0.010 m. *(132S22)*

a) Determine the net force on charge B.

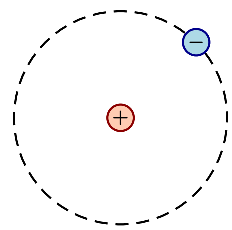
b) Determine the net force on charge D.

c) Determine the net force on charge C.



15 Orbiting charge

Two particles have charges that are equal in magnitude but opposite in sign. The positive charge is held fixed and the negative charge is launched in such a way that it orbits the positive charge. Let Q be the magnitude of either charge, m the mass of the orbiting charge and r the radius of orbit. *(132S22)*



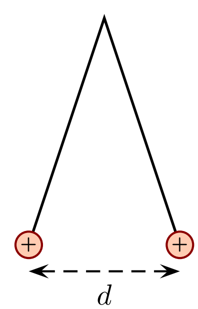
a) Starting with Newton's second law and using kinematics for uniform circular motion, determine an expression the speed of orbit v in terms of Q, m, r and constants.

b) Determine an expression for the time taken to complete one orbit, T in terms of Q, m, r and constants.

c) In a hydrogen atom the electron's orbital radius is approximately 10^{-10} m. Determine the time taken to complete one orbit.

16 Suspended charges

Two identical small metal spheres are each suspended from a string with length L. Each sphere is given the same charge q and they repel and reach an equilibrium state where they are a distance d apart. The mass of each sphere is m. *(132S22)*

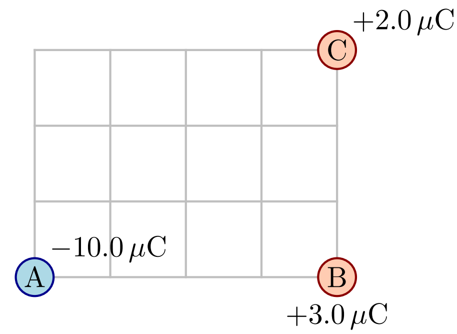


a) Determine an expression for q in terms of m, d, L and constants.

b) In a real case the mass of each ball is 8.0 g, each string is 25 cm long and the separation is 5.0 cm. Determine the charge on each ball.

17 Two dimensional charge arrangements: force, 3

Three charged particles are held fixed as illustrated; the grid units are each 0.010 m. Determine the magnitude and direction of the net force on charge C. *(132S22 Class)*



18 Levitating polystyrene ball

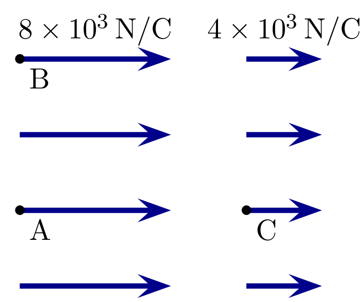
A small 2.5 g polystyrene ball is placed in a device which produces a 3000 N/C uniform electric field pointing vertically upwards; this field could easily be produced with standard laboratory equipment. *(132S22)*

a) What charge would the ball need in order to levitate at rest?

b) Suppose that the ball were levitating at rest and it suddenly lost half of its charge. How long would it take for the ball to drop a distance of 1.0 cm?

19 Electric field and forces at various locations

Hidden source charges produce the illustrated electric field. *(132S22)*



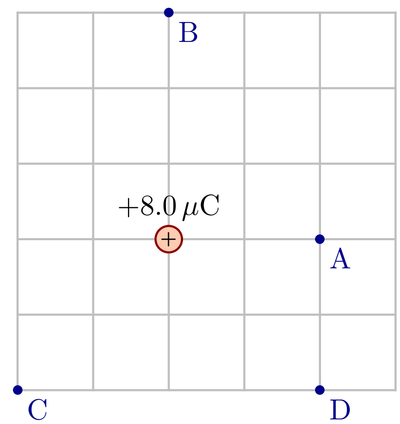
a) Determine the force exerted on a proton that is placed at A. Determine the acceleration of a proton at this point.

b) Determine the force exerted on an electron that is placed at B. Determine the acceleration of an electron at this point.

c) Determine the force exerted on an electron that is placed at C. Determine the acceleration of an electron at this point.

20 Electric field at various locations

A charged particle is held fixed as illustrated; the grid units are each 0.010 m. Determine the electric field vector components at each point A,B,C and D. Draw the vectors on the diagram (or on a copy of the diagram). *(132S22)*

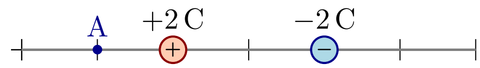


21 Air breakdown field

Air breaks down (produces sparks and lightning) when the electric field reaches about 3.0×10^6 N/C. A metal sphere with radius 0.20 m carries a charge uniformly distributed on its surface and in this case, outside the sphere, the electric field is exactly the same as that produced if all the charge were concentrated at the center. Determine the charge that the sphere must carry so that the air at its surface breaks down. *(132S22)*

22 Electric fields and forces

Two charged particles (sources) are held at fixed locations. A probe charge is placed at location A. Which of the following is true? Explain your answer. *(132S22)*



i) The electric field at A is zero because the total charge is zero.

ii) The electric field at A points in one direction for a positive probe and the opposite for a negative probe; the force on the probe points in the same direction regardless of the probe.

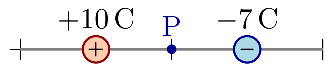
iii) The electric field at A points in one direction for a positive probe and the opposite for a negative probe; the force on the probe points one direction for a positive probe and the opposite for a negative probe.

iv) The electric field at A points in the same direction regardless of the probe charge; the force on the probe points in the same direction regardless of the probe charge.

v) The electric field at A points in the same direction regardless of the probe charge; the force on the probe points in different directions depending the probe charge.

23 Electric fields and forces produced by two charged particles.

Two charged particles are held at fixed locations. Various probe charges are placed at the midpoint. *(132S22)*



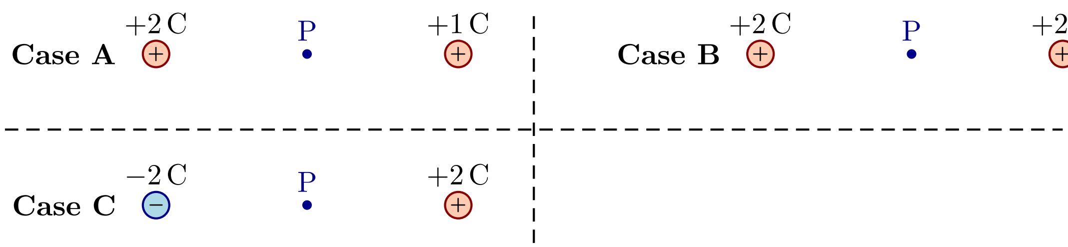
a) Does the direction and magnitude of the electric field produced by the two illustrated particles change when a positive probe at P is replaced by a negative probe at P?

b) Do the direction and magnitude of the force exerted on a probe placed at P change when a positive probe is replaced by a negative probe?

Explain your answers.

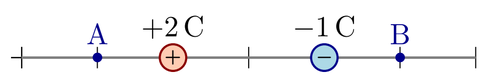
24 Electric field ranking

Three isolated pairs of charges are separated by the same distance in the three situations illustrated below. Rank the magnitudes of the electric fields at the midpoints (labeled P) from smallest to largest. Explain your answers. *(132S22)*



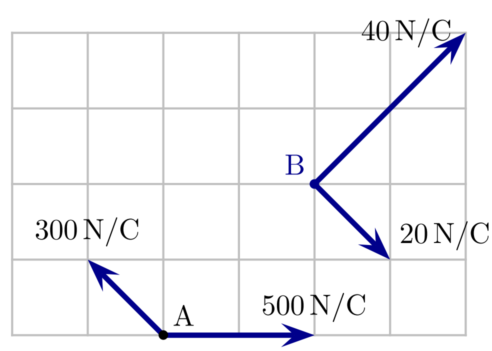
25 Electric fields at different locations

Two charged particles are held at fixed locations. Is the magnitude of the electric field produced by these charges at point A larger than, smaller than or the same as at point B? Explain your answer. *(132S22)*



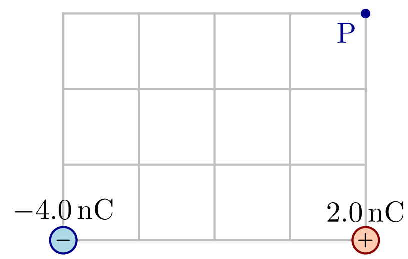
26 Net electric field

Two hidden arrangements of charged particles each produce an electric field, i.e. one electric field vector at each point. These are illustrated at two points. The net electric field at any point is the sum of the two field vectors at each point. Use vector components to determine the net electric field at point A and the net electric field at point B. *(132S22)*



27 Electric field produced by two point charges

Two charged particles are held fixed as illustrated; the grid units are each 0.010 m. The aim of this exercise will be to determine the field at point P. *(132S22 Class)*



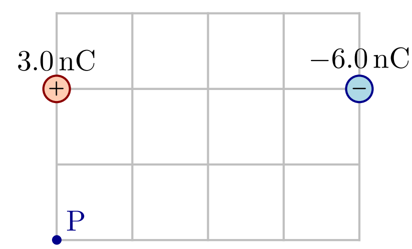
a) Indicate the directions of the electric fields produced by each source charge at point P.

b) Determine the magnitude of the electric field produced by each source charge at point P.

c) Using vector components add the two electric fields. Express the total electric field in terms of standard unit vectors.

28 Electric field produced by two point charges

Two charged particles are held fixed as illustrated; the grid units are each 0.010 m. The aim of this exercise will be to determine the field at point P. *(132S22)*



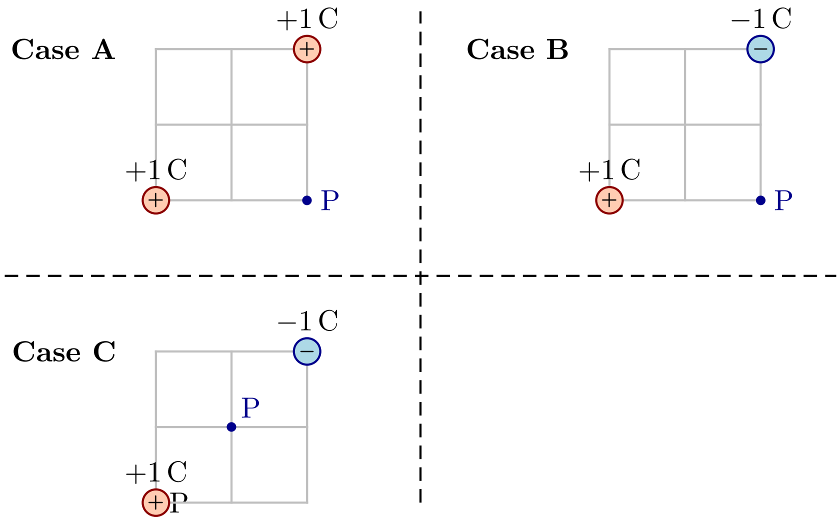
a) Indicate the directions of the electric fields produced by each source charge at point P.

b) Determine the magnitude of the electric field produced by each source charge at point P.

c) Using vector components add the two electric fields. Sketch the net electric field vector and determine its magnitude.

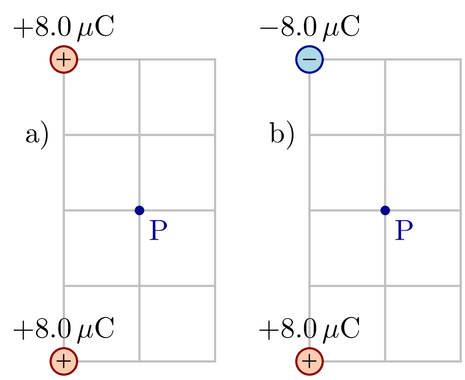
29 Ranking electric fields in two dimensions

Three arrangements of two charged particles are as illustrated. Rank the electric fields in order of increasing magnitude, indicating equal cases where applicable. Explain your answer. *(132S22)*



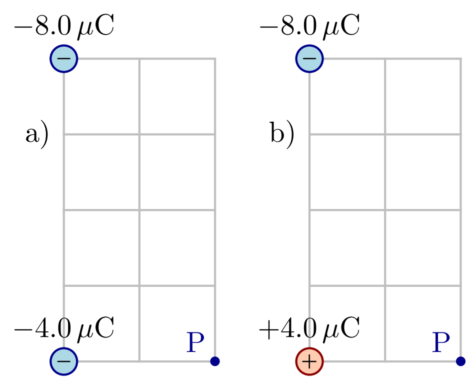
30 Electric field produced by two charges in two dimensions

In each of the following, fixed charges are held fixed as illustrated. The grid units are each 0.020 m. Determine the electric field at point P in each case. *(132S22)*



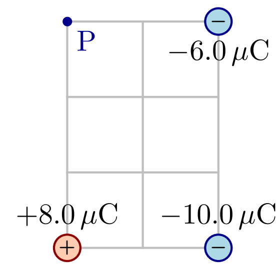
31 Electric field produced by two charges in two dimensions

In each of the following, fixed charges are held fixed as illustrated. The grid units are each 0.020 m. Determine the electric field at point P in each case. *(132S22)*



32 Electric field produced by multiple charges in two dimensions

Three charged particles are held fixed as illustrated; the grid units are each 0.020 m. *(132S22)*



a) Determine the electric field at point P.

b) A particle with charge $+3.0\,\mu$C is placed at P. Determine the force exerted on this particle by the other charged particles.

c) A particle with charge $-3.0\,\mu$C is placed at P. Determine the force exerted on this particle by the other charged particles.

33 Carbon monoxide dipole

Carbon monoxide is a molecule consisting of a single carbon and a single oxygen atom. The separation between these is roughly 1.0×10^{-10} m. The dipole moment of carbon monoxide is 4.07×10^{-31} C$\cdot$ m. *(132S22)*

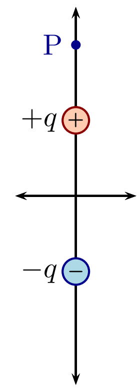
a) If the carbon monoxide molecule were to be modeled as a dipole consisting of a positive and a negative charge separated by 2.0×10^{-10} m, determine the magnitudes of these charges.

b) Determine the electric field along the dipole axis at a distance 20 nm from the center of the carbon monoxide molecule.

c) An electron is held at rest along the dipole axis at a distance 3.0×10^{-7} m from the center of the carbon monoxide molecule and is then released. Determine the acceleration of the electron at the moment it starts to move.

34 Electric field produced by a dipole

A point dipole consists of two opposite charges oriented along the y axis as illustrated. The distance between the point charges is d. With the origin midway between the two charges, the vertical component of the field at point P is



$$E_y = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(y-d/2)^2} - \frac{1}{(y+d/2)^2}\right]$$

where y is the vertical coordinate of P. The aim of this exercise is to show how a strategy of using approximations lets one arrive at the formula for the dipole field. *(132S22)*

a) Show that $(y \pm d/2)^2 = y^2(1 \pm d/2y)^2$.

b) Show that

$$E_y = \frac{1}{4\pi\epsilon_0}\frac{q}{y^2}\left[\frac{1}{(1-d/2y)^2} - \frac{1}{(1+d/2y)^2}\right].$$

Calculus gives a series (called a Taylor series) for any function. In this case, the relevant series is

$$\frac{1}{(1+x)^2} = 1 - 2x + 3x^3 - 4x^3 + \cdots .$$

If x is much less than 1 (written $x \ll 1$) then x^2, x^3, x^4 are all much smaller than x. Thus, to a good approximation,

$$\frac{1}{(1+x)^2} \approx 1 - 2x.$$

Similarly

$$\frac{1}{(1-x)^2} \approx 1 + 2x.$$

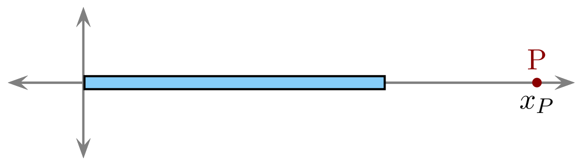
c) Assuming that $y \ll d$, then with $x = d/2y$, use the series approximation to show that

$$E_y \approx \frac{1}{4\pi\epsilon_0}\frac{q}{y^2}\frac{2d}{y} = \frac{1}{4\pi\epsilon_0}\frac{2p}{y^3}$$

where $p = qd$.

35 Field of a uniformly charged rod

A uniformly charged rod is oriented as illustrated. The length of the rod is L and the total charge (positive) is Q. *(132S22)*



a) As an example, suppose that the length of the rod is 2.0 m and the total charge is 60 C. Determine the charge density and use it to determine the charge in any segment of the rod with length 0.010 m. Repeat this for a segment with length 0.0050 m.

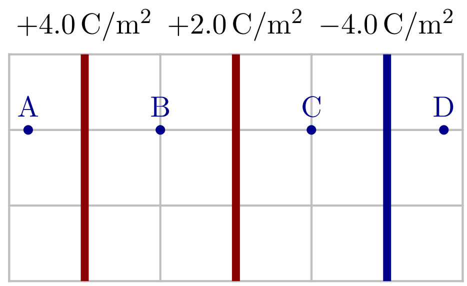
b) Consider a segment from $x \to x + \mathrm{d}x$ where x is some point in the rod. Write expressions for the length of the segment and the charge it contains (in terms of $\mathrm{d}x$ and λ).

c) Indicate the field produced by this segment and determine an expression for its components (in terms of $x_p, x, \mathrm{d}x$ and λ).

d) Write an expression (eventually in terms of calculus) for the components of the net electric field produced by all segments.

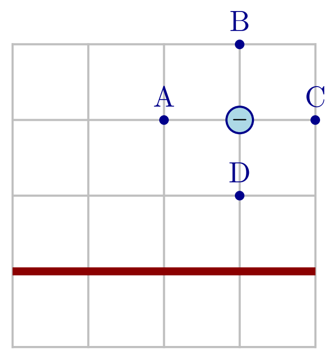
36 Electric fields produced by planes

Several parallel uniformly charged infinite planes, with indicated surface charge densities, are as illustrated. Rank the magnitudes of the electric fields at the indicated points, A, B, C and D. Explain your answer. *(132S22)*



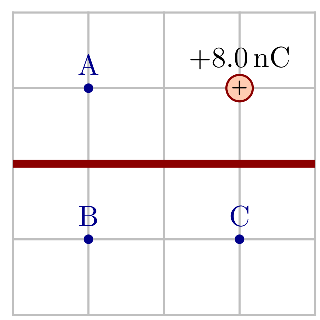
37 Electric field produced by point charge and plane

A point charge is held in the vicinity of a uniformly charged infinite plane as illustrated. The plane carries positive charge. Rank the magnitudes of the electric fields at the indicated points, A, B, C and D. Explain your answer. *(132S22)*



38 Electric field produced by point charge and plane

A point charge is held in the vicinity of a uniformly charged infinite plane as illustrated. The surface charge density on the sheet is $+1.5 \times 10^{-7}\,\mathrm{C/m^2}$. The grid units are each 0.040 m. *(132S22)*



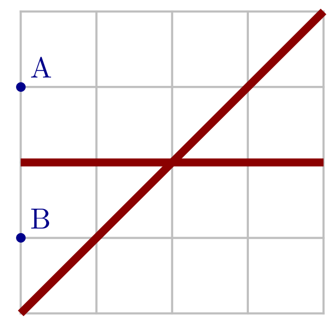
a) Determine the net electric field at point A.

b) Determine the net electric field at point B.

c) Determine the net electric field at point C.

39 Electric fields produced by planes

Two uniformly charged infinite planes intersect at an angle of 45°, as illustrated. Each carries charge density $5.0 \times 10^{-6}\,\mathrm{C/m^2}$. The grid units are $0.010\,\mathrm{m}$. *(132S22)*



a) Determine the net electric field at point A.

b) Determine the net electric field at point B.

40 Motion of a charge near a point source

A point source charge is held fixed as illustrated. Various probe charges are released from rest at point X. *(132S22)*

a) Suppose that a proton is released from rest at point P. Which of the following is subsequently true? Explain your choice.

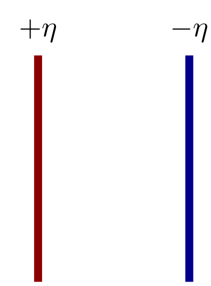
   i) The proton moves right with constant speed.
   ii) The proton moves right with constantly increasing speed.
   iii) The proton moves right with constantly decreasing speed.

b) Suppose that an electron is released from rest at point P. Which of the following is subsequently true? Explain your choice.

   i) The electron moves left with constant speed.
   ii) The electron moves left with constantly increasing speed.
   iii) The electron moves left with constantly decreasing speed.

41 Proton between parallel capacitor plates

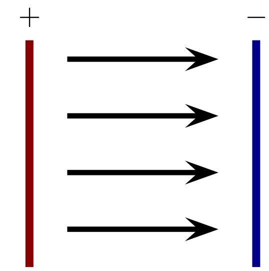
A parallel plate capacitor has plates separated by $0.0020\,\mathrm{m}$. The plates are uniformly charged as illustrated with magnitude $\eta = 6.0 \times 10^{-12}\,\mathrm{C/m^2}$. They are large enough that the field between the plates can be regarded as being produced by infinitely large plates. A proton is released from rest midway between the plates. Ignore gravity in this problem. *(132S22 Class)*



a) Describe the trajectory of the proton.

b) Determine how long it takes for the proton to hit a plate after release.

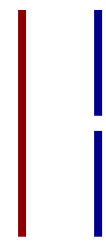
42 Particle moving in a uniform electric field



Two closely spaced plates are uniformly charged; that on the left is positive and that on the right is negative. The distance between the plates is 0.0080 m. The electric field between the plates points to the right. An electron is launched from the left plate with speed 3.0×10^6 m/s and travels to the right plate. It stops just before reaching the right plate. *(132S22)*

a) Determine the acceleration of the electron.
b) Determine the magnitude of the electric field.

43 Electron accelerator



Electrons can be accelerated from rest to an appreciable velocity by an arrangement of two charged plates. Suppose that two parallel flat plates are closely spaced as illustrated. The plates have uniform charge densities which are exactly opposite to each other. Electrons start at rest on the plate on the left, travel toward the plate on the right and some continue through the small hole in the right plate. Assume that the distance between the plates is small enough that the plates can be considered as infinite. Ignore gravity in this problem. *(132S22)*

a) Describe which plate should be positively charged.
b) Suppose that the electrons must arrive at the plate with velocity v. Determine an expression for the magnitude of the charge density on each plate (in terms of v and other quantities relevant to the situation) such that this occurs.
c) Do the electrons that pass through the hole continue onwards with constant velocity? Explain your answer.

44 Particle above a uniformly charged infinite sheet

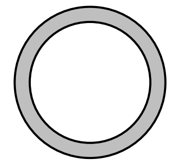
A horizontal infinite sheet is uniformly charged. A launcher fires an electron with speed 3.0×10^6 m/s at an angle of $60°$ above the horizontal. The electron reaches a maximum height of 0.025 m above the plate. *(132S22)*

a) Ignoring gravity, determine the electric field produced by the plate.
b) Determine the surface charge density of the plate.
c) Is it reasonable to ignore gravity? To answer this compare the magnitudes of the gravitational force on the electron to the electrostatic force on the electron.

Conductors

45 Conducting spherical shell

A thin spherical shell is as illustrated. The shell is hollow and the walls have non-zero thickness and are constructed from a perfect conductor. Charge Q is placed on the conductor. *(132S22)*



a) Describe whether the charge will distribute uniformly or not. Explain your answer.

b) Someone claims that a fraction of the charge will reside on the inner surface of the shell and the rest will reside on the outer surface. If this is true would the electric field within the shell walls be zero or not? Explain your answer.

c) Is it possible for the inner surface to have a non-zero charge density? Explain your answer.

Electrostatic Potential

46 Probes moving away from a fixed point charge

A +30 nC is held fixed. Various probe particles are placed near to this. First a proton is released from rest 0.0010 m from the fixed charge. The aim is to determine the speed of the proton when it is 0.0025 m from the fixed charge. *(132S22 Class)*

a) Suppose that you tried to use Coulomb's law to determine the speed. What mathematical difficulties would arise?

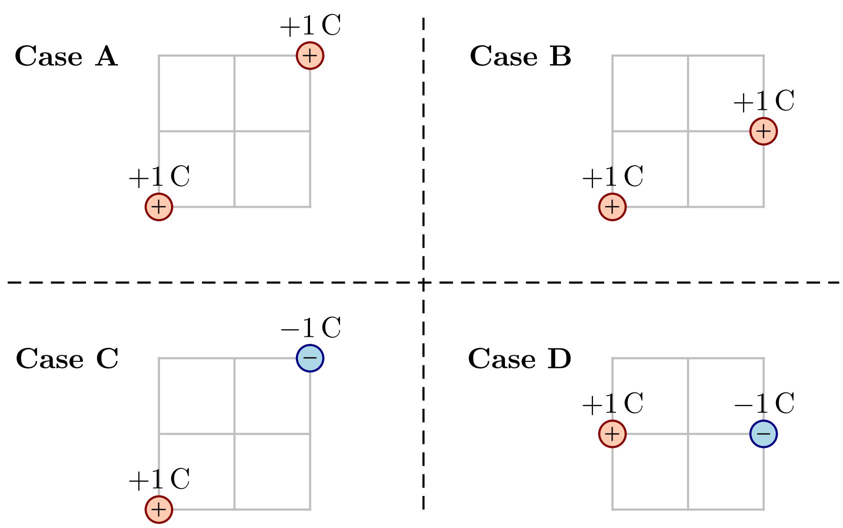
b) Use energy conservation to determine the velocity of the proton when it is 0.0025 m from the fixed charge.

Second an electron, initially 0.50 m from the fixed charge, is launched directly away.

c) Determine the minimum launch speed of the electron such that it escapes the fixed charge.

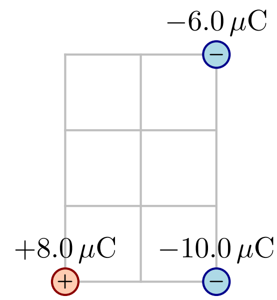
47 Electric potential energy for two charges

Four arrangements of two charged particles are as illustrated. Rank the these in terms of increasing electric potential energy, indicating equal cases where applicable. Explain your answer. *(132S22)*



48 Electric potential energy for three charges

Three charged particles are held fixed as illustrated; the grid units are each 0.020 m. *(132S22)*



a) Determine the electric potential energy of the arrangement.

b) Suppose that the $+8.0\,\mu$C charge were replaced by a $-8.0\,\mu$C. Would the electric potential energy increase, decrease or stay constant? Explain your answer.

49 Escape speed of a charge

A small 0.040 m diameter sphere carries charge +20.0 nC. A proton is released from rest at the surface of the sphere. *(132S22)*

a) Determine the speed with which the proton travels when it is infinitely far from the sphere.

b) If the radius of the sphere were decreased while the charge stays constant would the ultimate speed of the proton be larger, smaller or the same as before? Explain your answer.

50 Proton fleeing a positive source charge

A point particle has charge +20.0 nC and is held fixed. A proton is released at a distance of 0.050 m from the charge. *(132S22)*

a) Determine the speed with which the proton travels when it is 0.10 m from the charge.

b) Determine the speed with which the proton travels when it is 10.0 m from the charge.

c) Determine the speed with which the proton travels when it is 100.0 m from the charge. Does its speed change much as it continues beyond this point?

51 Alpha particle and a gold nucleus

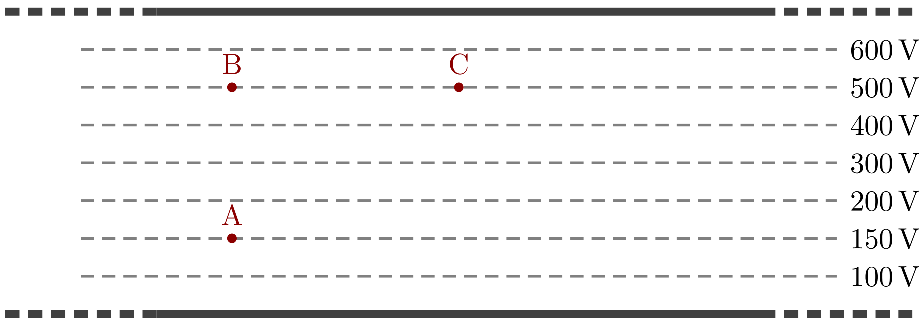
An alpha particle consists of two protons and two neutrons. Suppose that an alpha particle is directly toward a nucleus of a gold atom (the nucleus contains neutrons and 79 protons). Assume that the alpha particle is fired with speed 3.0×10^6 m/s. This speed is sufficient that the force exerted by the electrons can be ignored to same extent; assume that it is negligible. *(132S22)*

a) If the alpha particle is fired from infinitely far away, determine the closest that it approaches the center of the nucleus (ignore the electrons in the gold atom).

b) Clearly the alpha particle cannot be fired from infinitely far away. Assume that it is fired from 2.0 m with the same speed toward the nucleus. Determine the closest that it approaches the center of the nucleus. Does this differ much from the result if it were fired from infinitely far away?

52 Electrostatic potential between plates

Two parallel plates are charged. The electrostatic potential is such that it is constant along lines that are parallel to the plates. Several of these are as illustrated. Positively charged particles are moved between the two points. *(132S22)*

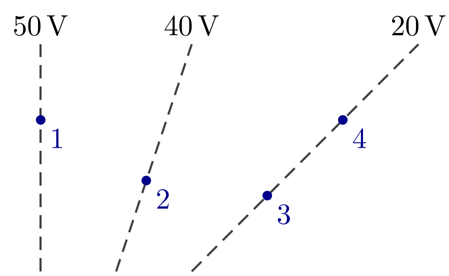


a) A charge is released at A with an initial velocity in the vertical direction and moves to B in a straight line. In a separate experiment the same charge is released at A with velocity of the same magnitude, but angled away from the vertical. It moves in a curved path, arriving at C. How does the change in electrostatic potential energy in the A → B case compare to that of the A → C case? How does the final speed in the A → B case compare to that of the A → C case?

b) The particle is made to move in two paths from B to C at a constant speed (an external force will be required to do this). One path is a straight line from B to C and the other a semicircular path within the plates from B to C. Compare the changes in total energy and the net work done by the external force in the two cases. Do your answers depend on the path taken? Would they be valid for any path taken?

53 Motion in a potential

Hidden source charges produce an electric potential as illustrated. Other probe charges move between various locations as illustrated. Let v_1 be the speed of the probe at point 1, v_2 be the speed at point 2, etc. *(132S22)*



a) A proton moves from point 1 to point 2. Is v_2 the same as, larger than or smaller than v_1?

b) An electron moves from point 1 to point 2. Is v_2 the same as, larger than or smaller than v_1?

c) Two electrons are released from rest at points 3 and 4 in separate experiments. Both reach point. How do their speeds compare when they reach point 1?

d) A proton is released from rest at point 1 and moves to point 4. Separately another proton is released from point 2 and it moves to point 3. Which proton moves faster when it reaches the 20 V line, or are their speeds the same at this stage?

54 Accelerating electrons

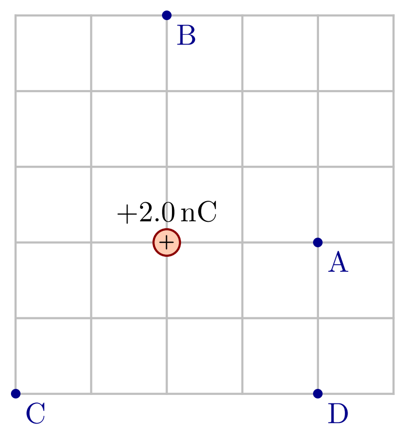
Electrons can be made to accelerate by creating an electrostatic potential difference across a region of space. *(132S22)*

a) Suppose that an electron is initially at rest and moves through a region where the potential difference is 100 V. Determine the speed of the electron at the end of this region.

b) Through what potential difference must an electron be accelerated so that it reaches a speed of 1.5×10^7 m/s if it was initially at rest?

55 Electric potential at various locations

A charged particle is held fixed as illustrated; the grid units are each 0.010 m. *(132S22)*

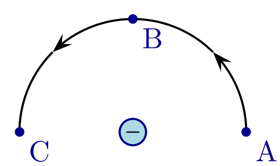


a) Determine the electric potential at each point A, B, C and D.

b) Determine and describe all possible locations at which the electric potential is 900 V. Use the diagram (or a copy of the diagram) to illustrate these locations.

56 Motion around a point charge

A negatively charged particle is held fixed as illustrated. A proton moves in the illustrated circular trajectory. Explain your answers for the following. *(132S22)*



a) Does the electric potential energy of the proton increase, decrease or stay the same as it moves from A to B?

b) Is the speed of the proton at B larger, smaller or the same as at A?

c) Does the electric potential energy of the proton increase, decrease or stay the same as it moves from B to C?

d) Is the speed of the proton at C larger, smaller or the same as at B?

e) Is the speed of the proton at C larger, smaller or the same as at A?

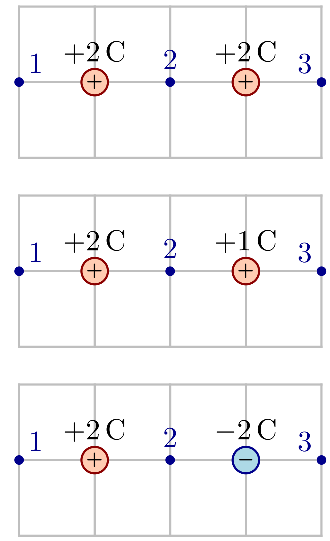
57 Electric potential for two charges

Various arrangements of two charged particles are as illustrated. In each case rank the potentials at the indicated points in order from smallest to largest. Explain your answer. *(132S22)*

Case a)

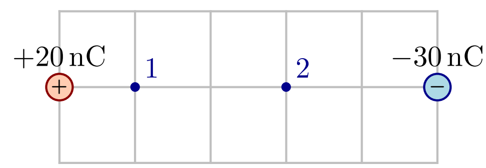
Case b)

Case c)



58 Electric potential difference produced by two point charges

Two point charges are held at rest as illustrated. The grid units are 0.10 m. *(132S22 Class)*

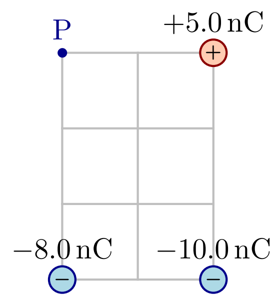


a) Determine the electric potential at point 1 and also at point 2.

b) A 3.0×10^{-4} kg probe charge with charge 800 nC is held at rest at point 1. Determine its speed when it reaches point 2.

59 Potential from multiple point source charges

Various source charges are held at rest as illustrated. The grid units are 0.010 m. Determine the electric potential at point P. *(132S22)*



60 Electric potential energy and motion

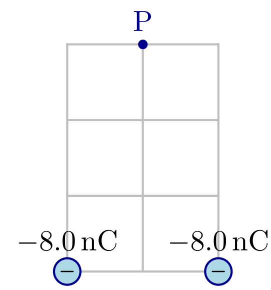
The electric field between the plates of a parallel plate capacitor is $4.0 \times 10^4\,\mathrm{N/C}$. A proton is released from rest at a point between the plates. *(132S22)*

a) Determine the change in electric potential energy of the proton after it has moved 0.0025 m.

b) Determine the speed of the proton after it has moved 0.0025 m for the point where it was released.

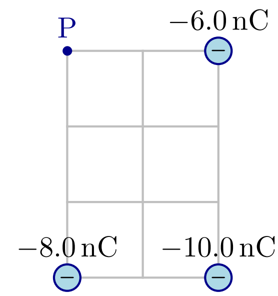
61 Escape speed of a particle

A 0.0050 kg sphere with charge −0.0080 C is held at rest at the point labeled P in the vicinity of various fixed charges as illustrated. The grid units are 0.010 m. The sphere is released. Determine the speed of the sphere when it is infinitely far from the illustrated charges. *(132S22)*



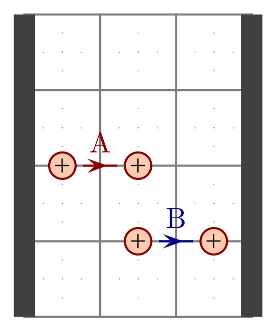
62 Escape speed of a particle

A 0.0050 kg sphere with charge −0.0080 C is held at rest at the point labeled P in the vicinity of various fixed charges as illustrated. The grid units are 0.010 m. The sphere is released. Determine the speed of the sphere when it is infinitely far from the illustrated charges. *(132S22)*



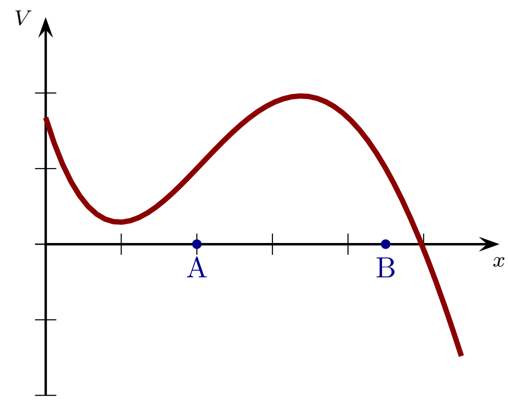
63 Protons in a capacitor

Two protons are released from rest at different points inside a parallel plate capacitor. Assume that the plates are infinite. The right plate of the capacitor is negatively charged, the left plate positively. They travel as illustrated. How does the speed of A compare (smaller, larger, or same as) to that of B after each has traveled the same distance? Explain your answer. *(132S22)*



64 Particles in a potential

Hidden source charges produce the illustrated potential. Various probe charges are, in separate experiments, released from rest at the indicated locations. *(132S22)*



a) A positive charge is released from A. In which direction and how far will it move?

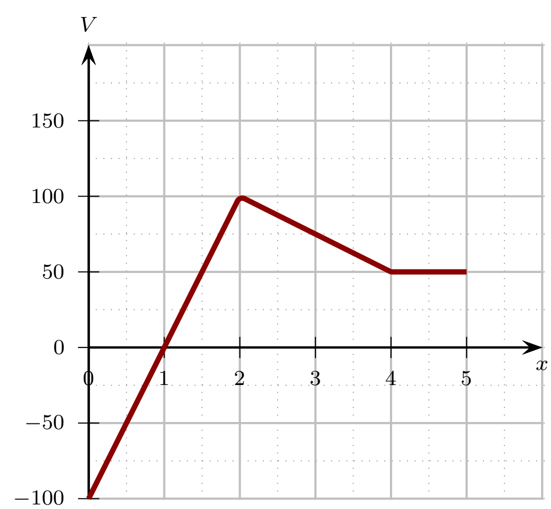
b) A positive charge is released from B. In which direction and how far will it move?

c) A negative charge is released from A. In which direction and how far will it move?

d) A negative charge is released from B. In which direction and how far will it move?

65 Particles in a potential

Various metal plates that are perpendicular to the x axis produce the illustrated electric potential. The horizontal axis units are meters and the vertical axis units are Volts. *(132S22)*



a) A sphere with mass 0.020 kg and charge -8.0×10^{-2} C is released from rest at $x = 0$ m. Determine its speed when it is at $x = 2.0$ m and again at $x = 5.0$ m.

b) A sphere with mass 0.40 kg and charge $+7.5 \times 10^{-2}$ C, moves left at $x = 5$ m with speed 5.0 m/s. Determine its subsequent minimum speed.

66 Parallel plate capacitor and charge

A parallel plate capacitor has two square plates with sides 0.04 m. These are placed a distance 2.5×10^{-3} m apart. These carry equal but opposite charges. Assume that the charges are uniformly distributed (on each plate) and that the plates can be regarded as infinite. Suppose that the electric potential difference from one plate to the other is 12 V. *(132S22)*

a) Determine the charge per surface area on each plate.

b) Determine the charge on each plate.

67 Parallel plate capacitor and electron acceleration

A parallel plate capacitor has two circular plates with radius 0.125 m. These are placed a distance 5.0×10^{-3} m apart. These carry equal but opposite charges with magnitudes 45 nC. Assume that the charges are uniformly distributed (on each plate) and that the plates can be regarded as infinite. *(132S22)*

a) Determine the charge density (charge per area) on each plate.

b) Determine the electric potential difference from one plate to the other.

c) An electron is held at rest on the negative plate. Determine its speed just before it hits the positively charge plate.

68 Electrostatic potential produced by a solid sphere

A solid sphere with radius R carries total charge Q uniformly distributed throughout it. The electric field produced by the charge points radially outwards and along the x-axis it is

$$\vec{\mathbf{E}} = \begin{cases} \dfrac{1}{4\pi\epsilon_0} \dfrac{Qx}{R^3} \hat{\mathbf{i}} & \text{if } x < R \\ \dfrac{1}{4\pi\epsilon_0} \dfrac{Q}{x^2} \hat{\mathbf{i}} & \text{if } x > R. \end{cases}$$

Determine the electric potential difference between the center and the edge of the sphere. *(132S22 Class)*

69 Electron tunneling through a charged sphere

A solid sphere with radius R carries total charge $Q > 0$ uniformly distributed throughout it. The electric field produced by the charge points radially outwards and along the x-axis it is

$$\vec{\mathbf{E}} = \begin{cases} \dfrac{1}{4\pi\epsilon_0} \dfrac{Qx}{R^3} \hat{\mathbf{i}} & \text{if } x < R \\ \dfrac{1}{4\pi\epsilon_0} \dfrac{Q}{x^2} \hat{\mathbf{i}} & \text{if } x > R. \end{cases}$$

Suppose that a very narrow tunnel was drilled diametrically through the center of the sphere and that this did not affect the electric field produced by the rest of the sphere. An electron is released from rest at the edge of this tunnel. *(132S22)*

a) Determine an expression for the electron's velocity when it reaches the center of the sphere.

b) Suppose that the radius of the sphere is 1.0 cm. Determine the maximum charge such that the electron's velocity at the center is less than $0.01c$ where c is the speed of light.

70 Electrostatic potential produced by a line of charge

An infinite line of charge lies along the z axis. The linear charge density (charge per meter) is λ. The electric field produced by the charge points radially outwards and along the x-axis it is

$$\vec{\mathbf{E}} = \frac{1}{2\pi\epsilon_0}\,\frac{\lambda}{x}\,\hat{\mathbf{i}}.$$

Suppose that $\lambda = +7.5 \times 10^{-9}\,\mathrm{C/m}$. *(132S22)*

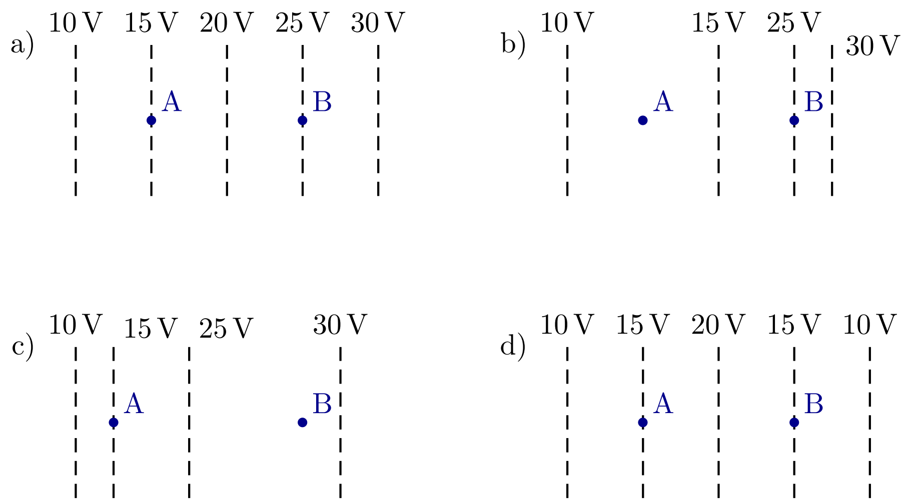
a) Determine the electric potential difference from $x = 0.5\,\mathrm{m}$ to $x = 1.0\,\mathrm{m}$.

b) If a proton were released at $x = 0.5\,\mathrm{m}$, determine its speed at $x = 1.0\,\mathrm{m}$.

c) Determine the electric potential difference from $x = 1.0\,\mathrm{m}$ to $x = 1.5\,\mathrm{m}$.

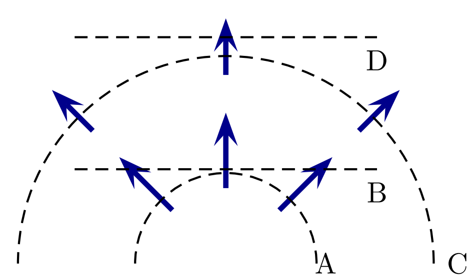
71 Equipotentials and fields

Each of the illustrations shows a set of equipotentials produced by hidden charges. Describe whether the magnitude of the electric field at A is large than, smaller than or the same size as that at B. Explain your answers. *(132S22)*



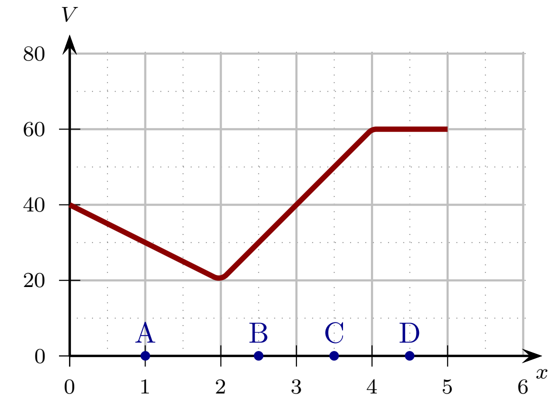
72 Fields and equipotentials

An electric field that radiates outward is partly illustrated. Indicate which of the dashed lines is a possible equipotential. For those which are, indicate which is at a higher potential. *(132S22)*



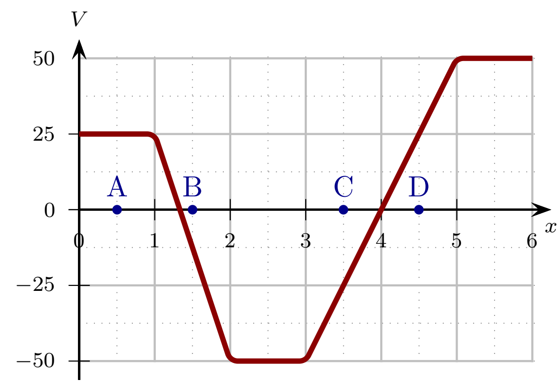
73 Potentials and fields, 1

Various metal plates that are perpendicular to the x axis produce the illustrated electric potential. The horizontal axis units are meters and the vertical axis units are Volts. Determine the electric field at points A,B,C and D. *(132S22)*



74 Potentials and fields, 2

Various hidden sources produce the illustrated electric potential. The horizontal axis units are meters and the vertical axis units are Volts. Determine the electric field at each indicated locations: A, B, C, and D. *(132S22)*



75 Non-uniformly charged sphere

A solid sphere is charged in such a way that the potential that it produces is spherically symmetric (thus the associated electric fields point radially out or in from the center of the sphere). In a certain region the electric potential along the x axis is

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{\alpha}{x} + \beta x^2\right]$$

where $\alpha > 0$ is a constant with units of Coulomb's and $\beta > 0$ is a constant with Coulomb/m^3. *(This is a real situation that arises for a solid uniformly charged sphere surrounded by a solid shell that is uniformly charged with a different charge. The two constants relate to the charges and the radii of the spheres.)* *(132S22)*

a) Determine an expression for the electric field in this region.

b) Determine the location where the electric field is zero. Is the potential zero at the same location?

76 Potential within a charged sphere

A solid sphere with radius R is uniformly charged with total charge Q. With the origin at the center of the sphere, electric potential inside the sphere in the xy plane is

$$V = -\frac{Q}{8\pi\epsilon_0}\frac{x^2+y^2}{R^3}.$$

(132S22)

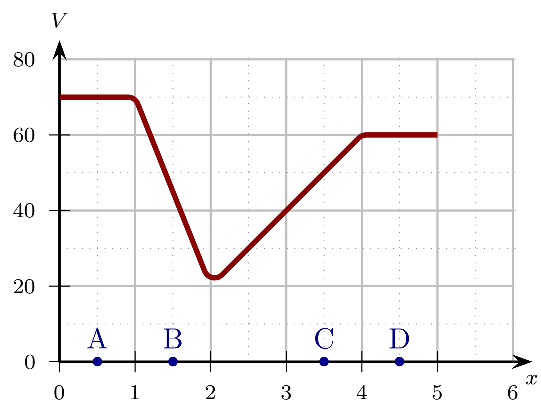
a) Describe the shape of any equipotential inside the sphere. Sketch several of these as accurately as possible.

b) Use the equipotentials to describe the direction of the electric field inside the sphere.

c) Use the potential to determine an expression for the electric field anywhere along the x axis.

77 Potentials and motion

Various metal plates that are perpendicular to the x axis produce the illustrated electric potential. The horizontal axis units are meters and the vertical axis units are Volts. In separate experiments, electrons are released from rest at the indicated locations, A, B, C, and D. *(132S22)*

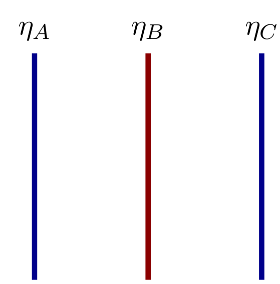


a) At each location, describe whether the electron moves and, if so, in what direction.

b) Rank the magnitudes of the accelerations of the electrons that are placed at the four locations.

78 Parallel sheets

Three infinitely large sheets are parallel to each other as illustrated. The separation between adjacent plates is 5.0 cm. The charge densities on the sheets are :



$$\eta_A = 4.0 \times 10^{-8}\,\mathrm{C/m^2}$$
$$\eta_B = -4.0 \times 10^{-8}\,\mathrm{C/m^2}$$
$$\eta_C = 2.0 \times 10^{-8}\,\mathrm{C/m^2}$$

a) Determine the electric field in the gaps between the sheets.

b) An proton is released from the right surface of plate A. It passes through a minute hole in the middle plate. Determine the speed of the electron at the instant before it hits plate C. *(132S22)*

Capacitors

79 Parallel plate capacitance

Consider the parallel plate capacitor as demonstrated in class. Estimate its capacitance. *(132S22 Class)*

80 Parallel plate capacitors

Suppose that you aim to construct a 20 nF capacitor from two circular disks that are spaced 0.50 mm apart. Determine the radius of the plates that will accomplish this. *(132S22)*

81 Charge and capacitors in parallel

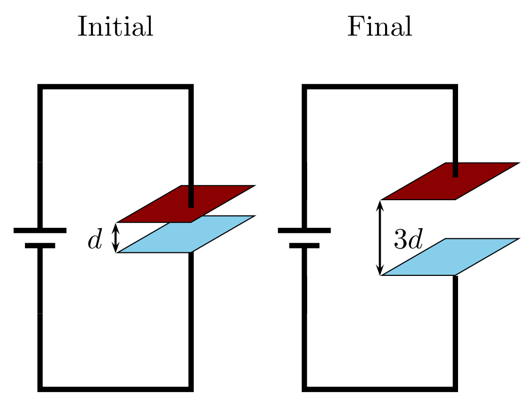
A $4.0\,\mu$F and a $12.0\,\mu$F capacitor are connected in parallel to a 5.0 V battery. *(132S22)*

a) Determine the total charge supplied by the battery to the combination.

b) Determine the charge on each capacitor.

82 Capacitors and energy

A parallel plate capacitor is connected to a battery that provides electric potential difference ΔV. The capacitor is allowed to charge completely. While the capacitor is connected to the battery, the plates are separated so that the distance between them is three times what it had been. *(132S22)*



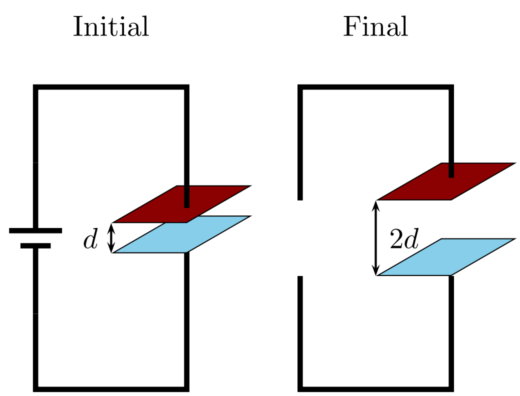
a) Does the electric potential difference across the capacitor plates change from the beginning to the end of this process? If so by what factor? Explain your answer.

b) Does the charge stored on the capacitor plates change from the beginning to the end of this process? If so by what factor? Explain your answer.

c) Does the energy stored in the capacitor change from the beginning to the end of this process? If so by what factor? Explain your answer.

83 Capacitors and energy

A parallel plate capacitor is connected to a battery that provides electric potential difference ΔV. The capacitor is allowed to charge completely. The battery is disconnnected. After this has occured, the plates are separated so that the distance between them is twice times what it had been. Does the energy stored in the capacitor change from the beginning to the end of this process? If so by what factor? Explain your answer. *(132S22)*



84 Energy and capacitors in parallel

Two capacitors, one with capacitance 30 nF and the other 50 nF are connected in parallel to a 15 V battery. *(132S22)*

a) Determine the charge stored in each capacitor.

b) Determine the energy stored in each capacitor.

c) Determine the total charge stored on both capacitors and the total energy stored in both.

85 Capacitors in series

Two capacitors, one with capacitance C and the other $3C$ are connected in series to a battery that provides potential difference ΔV. *(132S22)*

a) Determine an expression for the equivalent capacitance.

b) Determine an expression for the charge supplied by the battery. How is the charge on each capacitor related to this?

c) Determine an expression for the potential difference across each capacitor.

86 Flash capacitor

Camera flashlights often operate by storing charge in a capacitor and then rapidly discharging it through a bulb. Suppose that a camera flash has to produced 25 W of light over a period of 0.5 ms. *(132S22)*

a) Determine the energy that needs to be stored in the capacitor.

b) Suppose that the potential difference across the fully charged capacitor is 200 V. Determine the capacitance of the capacitor and the charge stored in the capacitor.

87 Capacitor and dielectric

A parallel plate capacitor is attached to a battery and then disconnected. *(132S22)*

a) After the battery is disconnected, a dielectric is inserted between the plates. Does the charge on the plates remain constant, increase or decrease while the dielectric is being inserted? Does the potential difference across the plates remain constant, increase or decrease while the dielectric is being inserted? Explain your answer.

b) Does the energy stored in the capacitor remain constant, increase or decrease while the dielectric is being inserted? Explain your answer.

c) Does the capacitor exert a force on the dielectric while it is being inserted? If so, in which direction? Explain your answer.

A parallel plate capacitor is attached to a battery and is left connected.

d) While the battery is connected, a dielectric is inserted between the plates. Does the charge on the plates remain constant, increase or decrease while the dielectric is being inserted? Does the potential difference across the plates remain constant, increase or decrease while the dielectric is being inserted? Explain your answer.

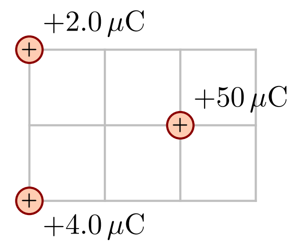
e) Does the energy stored in the capacitor remain constant, increase or decrease while the dielectric is being inserted and the battery is connected? Explain your answer.

f) Does the capacitor exert a force on the dielectric while it is being inserted and the battery is connected? If so, in which direction? Explain your answer.

Additional Electrostatics Problems

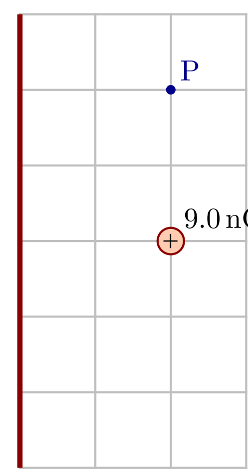
88 Two dimensional charge arrangements: force, 3

Three charged particles are held fixed as illustrated; the grid units are each 0.20 m. Determine the magnitude and direction of the net force on charge $+50\,\mu\text{C}$ charge. *(132S22 Class)*



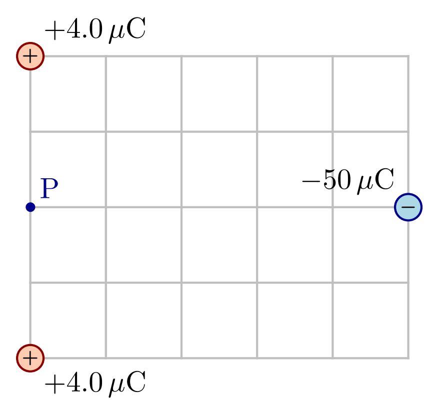
89 Field produced by sheet and point charge

A sheet with area $5.0\,\text{m}^2$ carries total charge 250 nC. A point particle with charge 9.0 nC is 0.0050 m to the right of the sheet. Determine the net electric field at point P, a distance 0.10 m from the point charge. *(132S22 Class)*



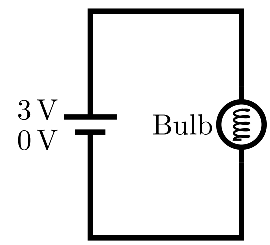
90 Fleeing point charge

Three charged particles are held fixed as illustrated; the grid units are each 0.010 m. A $-6.0\,\mu\text{C}$ particle with mass 0.030 kg is released from rest at point P. Determine its speed when it is infinitely far from the three fixed charges. *(132S22 Class)*



Currents and Circuits

91 Current and particle flow



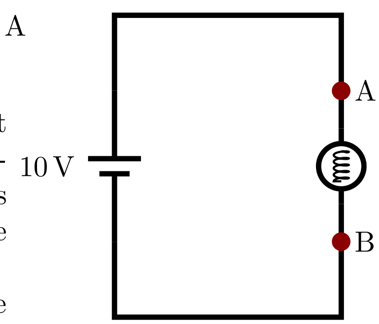
A battery is connected to a bulb as illustrated. Explain your answers to the following questions. *(132S22)*

a) Suppose that the current consists of positively charged particles that flow around the circuit. Consider one such particle. Will it flow from the 3 V terminal to the 0 V or the other way round? Based on your answer, is current direction clockwise or counterclockwise around the circuit?

b) Suppose that the current consists of negatively charged particles that flow around the circuit. Consider one such particle. Will it flow from the 3 V terminal to the 0 V or the other way round? Based on your answer, is current direction clockwise or counterclockwise around the circuit?

c) Does the direction of the current around this circuit depend on whether the particles that move are positive or negative?

92 Energy and currents



A battery and bulb are connected in the illustrated circuit. A steady current of 2.0 A flows through the bulb. *(132S22)*

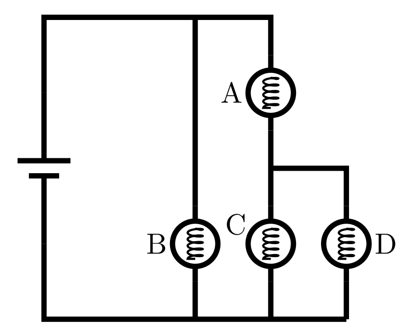
a) Assume that positive charge moves from point A to point B. Determine the charge that passes point B in 5.0 s. Determine the change in electrostatic potential energy of this charge as it moves from point A to point B. Does the charge gain or lose electrostatic energy?

b) Using the same data as above, assume that negative charge moves from point B to point A. Determine the change in electrostatic potential energy of this charge as it moves from point B to point A. Does the charge gain or lose electrostatic energy?

c) Suppose that the battery can provide 9.6 kJ of energy. Determine the amount of time for which the battery could operate in this circuit.

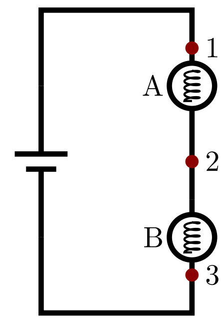
93 Currents at junctions

A battery and bulbs are connected in the illustrated circuit. The current in C is the same as in D. Suppose that the current in A is 6 A and the current though the battery is 10 A. Determine the currents through the other bulbs. *(132S22)*



94 Currents in bulbs, 1

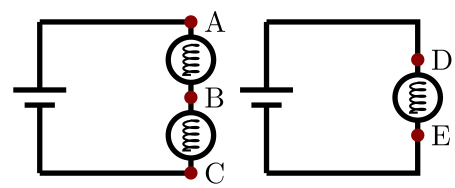
Various bulbs are connected to a battery in the illustrated circuit. *(132S22)*



a) Suppose that bulbs A and B are identical. Ranks the currents at points 1, 2, and 3, in order of decreasing current, indicating equality whenever it occurs. Explain your answer.
b) Suppose that bulb A has a greater resistance than bulb B. Ranks the currents at points 1, 2, and 3, in order of decreasing current, indicating equality whenever it occurs. Explain your answer.
c) Suppose that bulb A has a smaller resistance than bulb B. Ranks the currents at points 1, 2, and 3, in order of decreasing current, indicating equality whenever it occurs. Explain your answer.

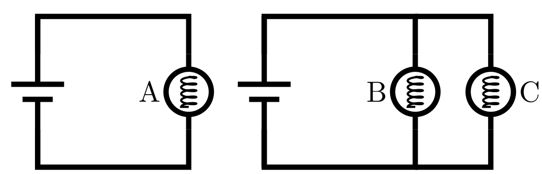
95 Currents in bulbs, 2

Batteries and bulbs are connected in the illustrated circuits. The two batteries are identical and the three bulbs are identical. Rank the currents at locations, A, B, C, D and E in order of increasing magnitude. Explain your answer. *(132S22)*



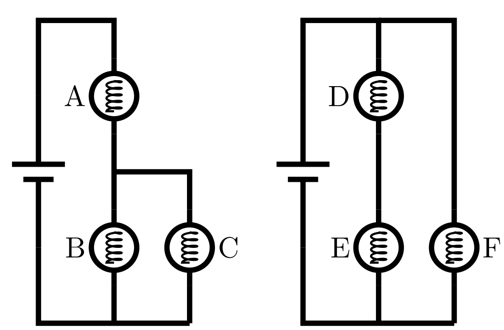
96 Currents in bulbs, 3

Several identical bulbs are connected to identical batteries in the illustrated circuits. Rank bulbs A, B and C in order of increasing brightness. Explain your answer. *(132S22)*



97 Currents in bulbs, 4

Various identical bulbs are connected to identical batteries in the illustrated circuits. *(132S22)*



a) Rank the bulbs A, B, and C in order of increasing brightness. Explain your answer.
b) Rank the bulbs D, E, and F in order of increasing brightness. Explain your answer.

98 Air conditioner power

A window unit air conditioner uses power 1400 W and has a thermostat that turns it on and off. During the summer it runs for 9.0 hr every day. The air conditioner is connected to the 120 V mains outlet. *(132S22)*

a) Determine the total energy consumed by the air conditioner in a day.

b) Determine the electrical current that flows into the air conditioner as it runs.

99 Toaster resistance

A toaster provides power 1200 W when it is connected to a 120 V mains outlet. *(132S22)*

a) Determine the resistance of the toaster.

b) Suppose that the toaster is connected to a 240 V outlet. Determine the power that it provides, assuming that its resistance stays constant.

100 Heater element

A simple electric heater consists of coil of wire attached to a power supply. Suppose that the power supply produces voltage ΔV, the power that the heater produces is P, the length of the coil is L and its cross-sectional area is A. *(132S22)*

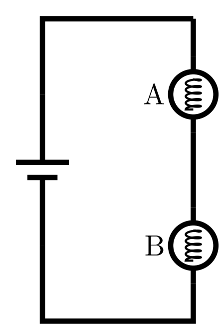
a) Determine an expression for the ratio A/L in terms of the voltage, power and resistivity of the material.

b) If the heater is to provide 1000 W when connected to a 120 V outlet and the coil is constructed of a 10.0 m strand of copper with circular cross-section, what should the diameter of the copper wire be?

c) A better alternative for the coil might be a metal alloy called nichrome. Explain in as much detail as possible how and why this might be better.

101 Bulbs in series

A battery and bulbs are connected in the illustrated circuit. The resistance of A is three times the resistance of B. *(132S22)*



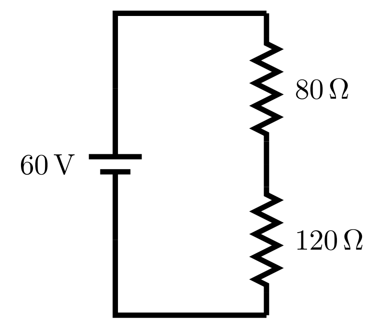
a) Determine the ratio of the currents through the bulbs, $I_{\mathrm{B}}/I_{\mathrm{A}}$.

b) Determine the ratio of the potential differences across the bulbs, $\Delta V_{\mathrm{B}}/\Delta V_{\mathrm{A}}$.

c) Determine the ratio of the powers dissipated by the bulbs, $P_{\mathrm{B}}/P_{\mathrm{A}}$.

102 Two resistors in series, 1

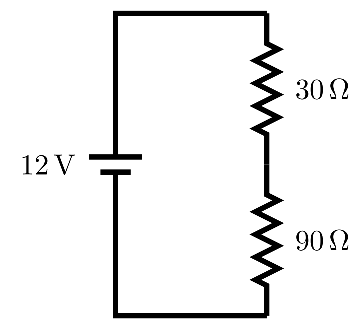
Two resistors are connected in the illustrated circuit. *(132S22)*



a) Determine the current in the circuit.
b) Determine the potential difference across each resistor.
c) Determine the power delivered to each resistor.

103 Two resistors in series, 2

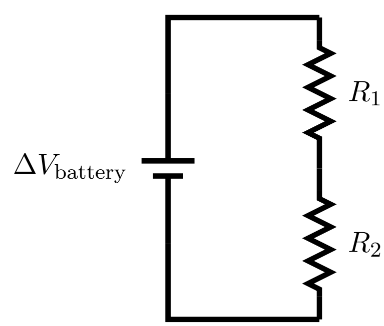
Two resistors are connected in the illustrated circuit. *(132S22)*



a) Determine the current in the circuit.
b) Determine the potential difference across each resistor.
c) Determine the power delivered to each resistor.

104 Voltage divider

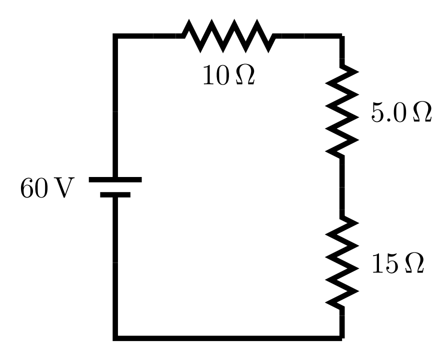
Two resistors are connected in the illustrated circuit. *(132S22)*



a) Determine an expression for the potential difference, in terms of R_1, R_2 and $\Delta V_{\text{battery}}$, across each resistor.
b) How would the two resistance have to be related in order to divide the battery voltage into two parts, one of which is five times the other? Explain your answer.

105 Three resistors in series

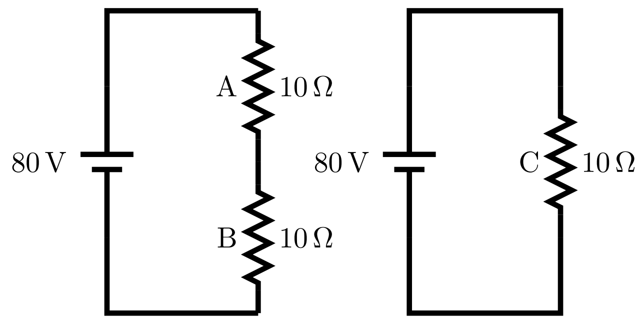
Three resistors are connected in the illustrated circuit. *(132S22)*



a) Determine the current in the circuit.
b) Determine the potential difference across each resistor.
c) Determine the power delivered to each resistor.

106 Power in resistors in series

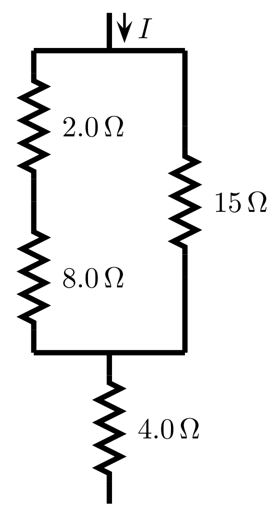
Consider the resistors in the illustrated circuits. Let P_A be the power delivered to A, etc. Which of the following is true? Explain your answer. *(132S22)*



i) $P_A = P_B = \frac{1}{4} P_C$
ii) $P_A = P_B = \frac{1}{2} P_C$
iii) $P_A = P_B = P_C$
iv) $P_A = P_B = 2P_C$
v) $P_A = P_B = 4P_C$
vi) $P_A > P_B = P_C$

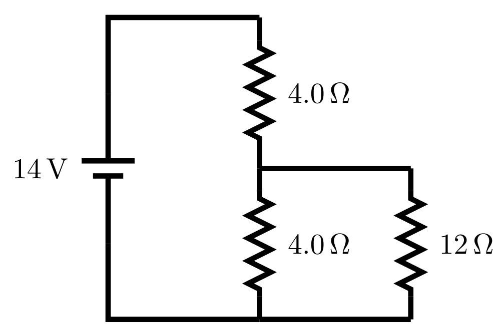
107 Equivalent resistance

Determine the equivalent resistance of the illustrated resistor combination. *(132S22)*



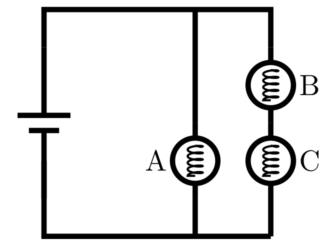
108 Resistors in series and parallel

Determine the currents through, voltages across and the power dissipated by each resistor in the illustrated circuit. *(132S22)*



109 Currents in bulbs: combination 1

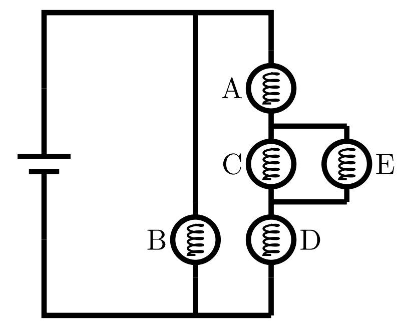
A battery and identical bulbs are connected in the illustrated circuit. Rank the bulbs in order of increasing brightness. *(132S22)*



a) Rank the bulbs in order of increasing brightness. Explain your answer.
b) Suppose that bulb C is replaced by a wire. Describe the effect of this change on the brightness of bulb A and B. Explain your answer.

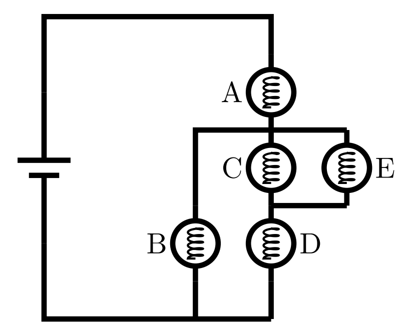
110 Currents in bulbs: combination 2

A battery and identical bulbs are connected in the illustrated circuit. Rank the bulbs in order of increasing brightness. Explain your answer. *(132S22)*



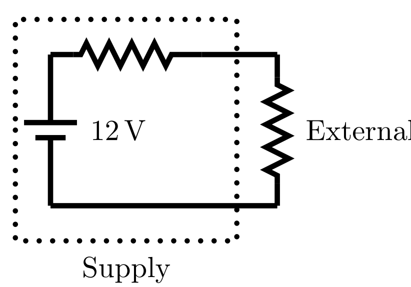
111 Currents in bulbs: combination 3

A battery and identical bulbs are connected in the illustrated circuit. Rank the bulbs in order of increasing brightness. Explain your answer. *(132S22)*



112 Voltage supply

A voltage supply consists of a perfect 12 V battery in series with a resistor. This is connected to an external resistor provided by the user of the supply. When the supply is connected to a 100 Ω external resistor the current is 100 mA. *(132S22)*



a) Determine the current when the supply is connected to a 68 Ω external resistor.
b) Suppose that the supply is connected to a parallel combination of a 100 Ω external resistor and a 68 Ω external resistor. Is the current provided by the supply the sum of the currents when connected to the individual resistors? Explain your answer.

113 Resistor combinations

You are given three $4\,\Omega$ resistors and can connect them in various ways. List all possible effective/equivalent resistances that you can attain by combining all three resistors. *(132S22)*

114 Resistor combinations

You are given three identical resistors, each with resistance R, and can connect them in various ways. List all possible effective/equivalent resistances that you can attain by combining all three resistors. *(132S22)*

115 Power produced by devices

An electric heater has a stated power of $800\,\text{W}$; this means that when connected to a $120\,\text{V}$ power supply or outlet, it will use power $800\,\text{W}$. In the following assume that the heater obeys Ohm's law. *(132S22)*

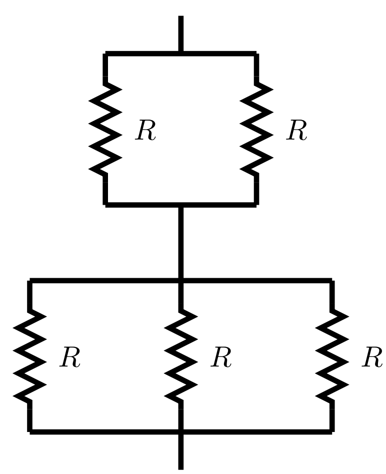
a) Suppose that two such devices are connected in parallel to $120\,\text{V}$ outlets. Determine the total power consumed.

b) Suppose that two such devices are connected in parallel to a single $120\,\text{V}$ outlets. Determine the total power consumed. *This would be very difficult in practice. Don't try this!*

c) Suppose that one such device is connected to a $240\,\text{V}$ outlet (these occur in other countries). Describe as precisely as possible how the power produced is related to that when it is connected to a $120\,\text{V}$ outlet.

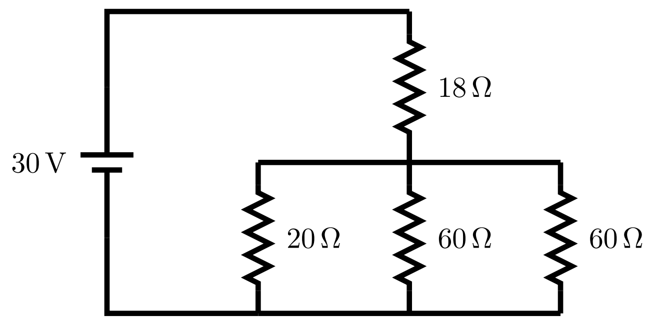
116 Equivalent resistance

Determine the equivalent resistance of the illustrated resistor combination. *(132S22)*



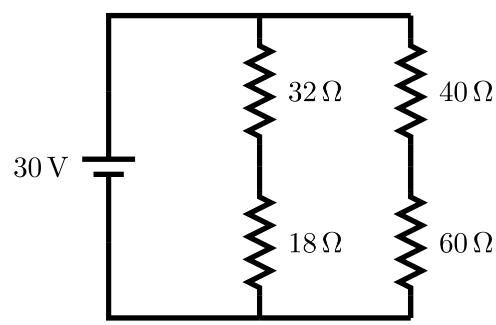
117 Resistors in series and parallel, 1

Determine the currents through, voltages across and the power dissipated by each resistor in the illustrated circuit. *(132S22)*



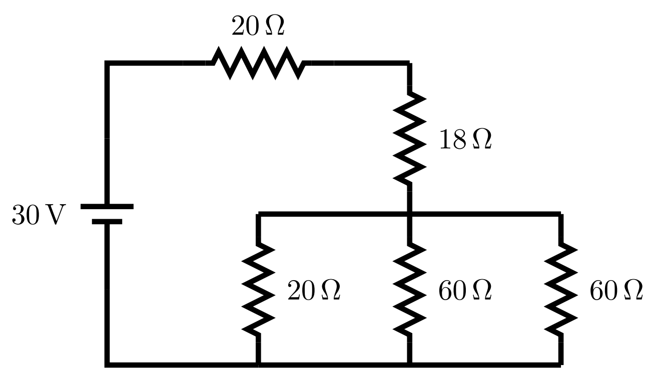
118 Resistors in series and parallel, 2

Determine the currents through, voltages across and the power dissipated by each resistor in the illustrated circuit. *(132S22)*



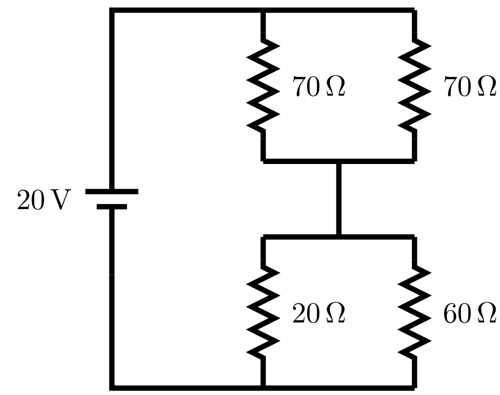
119 Resistors in series and parallel, 3

Determine the currents through, voltages across and the power dissipated by each resistor in the illustrated circuit. *(132S22)*



120 Resistors in series and parallel, 4

Determine the currents through, voltages across and the power dissipated by each resistor in the illustrated circuit. *(132S22)*



121 Real battery: current for parallel circuits

A particular real battery has internal resistance 30 Ω and EMF 120 V.

(132S22)

a) A single device with resistance 240 Ω is connected to the battery. Determine the power delivered.

b) A second device with resistance 240 Ω is connected in parallel with the first device. Determine the power that the first device now delivers. Explain how this relates to your household devices and electrical network.

122 Real batteries: optimal power

A real battery with internal resistance r and EMF $\mathcal{E}$ is connected to a resistor with resistance R. *(132S22)*

a) Explain whether the voltage across the (external) resistor increases, decreases or stays constant as the external resistance increases.

b) Show that the power provided to the external resistor is

$$P = \frac{R}{(r+R)^2}\,\mathcal{E}^2.$$

c) How much power is delivered to the external resistor as $R \to 0$?

d) How much power is delivered to the external resistor as $R \to \infty$?

e) Determine the value of the external resistance so that the power delivered to this is a maximum.

123 Discharging capacitor

A capacitor with capacitance, C is connected to a resistor with resistance R. At time $t = 0$ the charge on the capacitor is Q_0. Subsequently the charge satisfies

$$\frac{dQ}{dt} = -\frac{1}{RC}\,Q. \qquad (1)$$

The aim of this exercise is to solve this for Q at all later times.

a) A solution to Eq. (1) is a function $Q(t)$ such that when it is substituted into both sides of equation Eq. (1) the resulting statement is *true for all times t.* To test this consider the possibility $Q(t) = t^3$. Substitute into both sides of Eq. (1) and carry out the relevant mathematics. Is the resulting equality *true for all times t?*

b) Consider the possible solution $Q = Ae^{\alpha t}$ where A and α are constants. Substitute into both sides of Eq. (1) and carry out the relevant mathematics. Is the resulting equality *true for all times t?* What condition on α would guarantee this?

c) Determine a value for A such that the charge at $t = 0$ is Q_0.

d) The resulting expression is written as

$$Q = Q_0 e^{-t/\tau} \tag{2}$$

where τ is a constant called the capacitive time constant. Determine an expression for τ in terms of R and C.

The capacitive time constant quantifies the rate at which the capacitor charge decays. In the following, suppose that $R = 2.0 \times 10^5\,\Omega$ and $C = 5.0 \times 10^{-7}\,\mathrm{F}$.

e) Determine the capacitive time constant. What fraction of the initial charge remains at $t = 0.1\,\mathrm{s}$? What fraction of the initial charge remains at $t = 0.2\,\mathrm{s}$?

f) In experiments data for Q versus t is conveniently represented by plotting $\ln(Q)$ versus t. Determine an expression for $\ln(Q)$ in terms of t and constants. What sort of plot would this yield? What is the slope of this plot?

124 Capacitor discharge rate

A capacitor with capacitance, C is connected to a resistor with resistance R. The charge on the capacitor at time t is

$$Q(t) = Q_0 e^{-t/\tau}$$

where $\tau = RC$ and Q_0 is the charge at time $t = 0$. The purpose of this exercise is to compare the charge at any two later times that are separated by exactly τ. Denote these times t_i and $t_i + \tau$.

a) Determine expressions for $Q(t_i)$ and $Q(t_i + \tau)$.

b) Determine the ratio of the charges

$$\frac{Q(t_i + \tau)}{Q(t_i)}.$$

Does this ratio depend on the earlier time t_i?

c) You should be able to convert this into a statement: "In a time interval of duration τ the fraction of charge (at the start of the interval) that remains is ...." Complete this statement.

d) You should also be able to convert this into a statement: "In a time interval of duration τ the fraction of charge that disappears is ...." Complete this statement.

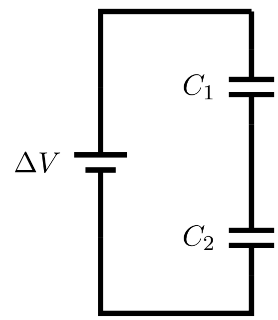
These mean that τ describes the rate of discharge in a fractional sense.

Additional Circuits Problems

125 Capacitors in Series: Energy

Two capacitors are connected in series. Determine an expression for or the energy stored in each capacitor for the case where $C_2 = 2C_1$.

(132S22)



126 Electrons in a circuit

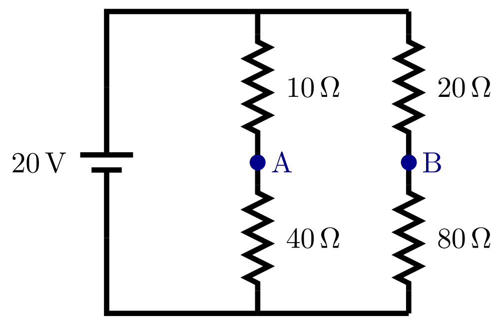
A $220\,\Omega$ resistor is connected to a $9.0\,\text{V}$ battery.

(132S22)

a) Determine the number of electrons that flow past any point in the circuit in $30\,\text{s}$.

b) Determine the amount of energy needed to push a single electron through the resistor.

127 Resistors in series and parallel, 2



a) Determine the currents through and voltages across each resistor in the illustrated circuit.

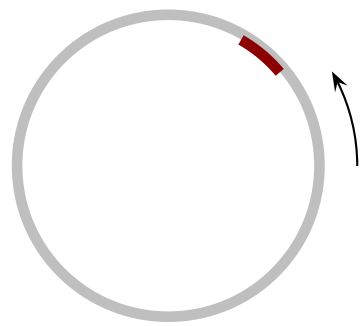
b) Determine the potential difference between points A and B. *(132S22)*

Currents and Magnetic Fields

128 Magnetic field at the center of a circular current loop

A circular loop with radius R carries current I as illustrated. The aim of this exercise is to use the Biot-Savart law to determine the magnetic field at the center of the loop. First consider the contribution to the field from the shaded section. This is

$$\vec{\mathbf{B}}_{\text{segment}} = \frac{\mu_0}{4\pi} I \frac{\Delta\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}. \tag{3}$$



a) Use the diagram to indicate the vectors $\vec{\mathbf{r}}, \hat{\mathbf{r}}$ and $\Delta\vec{\mathbf{s}}$.

b) Determine the direction of $\Delta\vec{\mathbf{s}} \times \hat{\mathbf{r}}$. Would the direction be different for a different segment of the current in the loop?

c) Let the magnitude of $\Delta\vec{\mathbf{s}}$ be Δs. Determine an expression for the magnitude of $\Delta\vec{\mathbf{s}} \times \hat{\mathbf{r}}$ and use this and Eq. (3) to determine an expression for the magnitude of $\vec{\mathbf{B}}_{\text{segment}}$.

d) What is the value of r in this situation? Use your answer to determine an expression for the magnitude of $\vec{\mathbf{B}}_{\text{segment}}$.

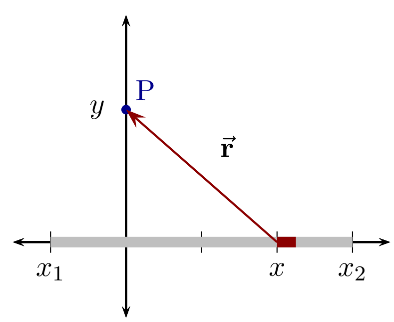
e) Add the contributions from all segments to obtain an expression for the field at the center of the loop.

f) Suppose that you aimed to use this to determine the magnetic field at a point that is not at the center of the loop. What mathematical difficulties would complicate the calculation compared to that for the center of the loop?

129 Magnetic field produced by a straight section of current

A straight section of wire carries current I flowing to the right. The aim of this exercise is to use the Biot-Savart law to determine the magnetic field at the illustrated point. First consider the contribution to the field from the shaded section, whose length is $\mathrm{d}x$. This is

$$\vec{\mathbf{B}}_{\text{segment}} = \frac{\mu_0}{4\pi} I \frac{\Delta\vec{\mathbf{s}} \times \vec{\mathbf{r}}}{r^3}. \tag{4}$$



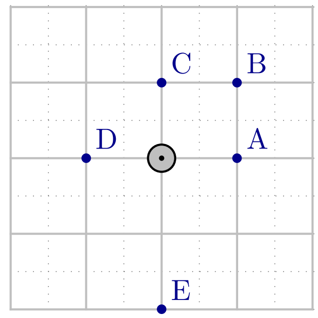
a) Express $\vec{\mathbf{r}}$ and $\Delta\vec{\mathbf{s}}$ in terms of $x, y, \mathrm{d}x$ and $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

b) Use these to determine expressions for $\vec{\mathbf{s}} \times \vec{\mathbf{r}}$ and r in terms of $x, y, \mathrm{d}x$ and $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$.

c) Set up an integral for the field produced by all segments.

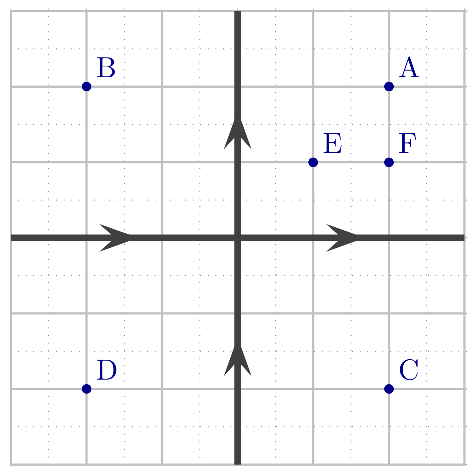
130 Field produced by a straight current

An infinitely long wire carries a 25 A current out of the page. Determine the magnetic field at each illustrated point. The solid grid blocks are each 1.0 cm wide.



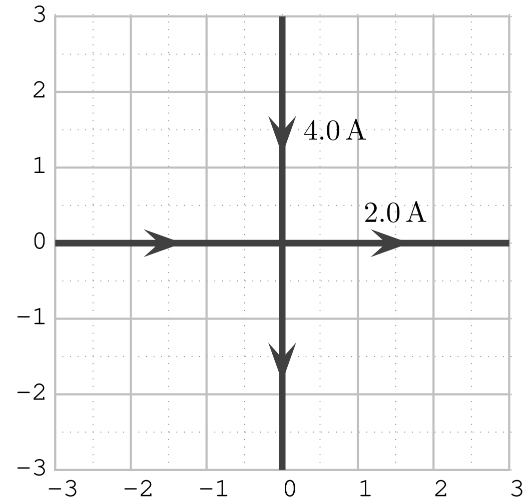
131 Field produced by perpendicular pairs of straight currents

Two infinitely long wires are oriented as illustrated and each carry the same current. Rank the magnitudes of the net magnetic fields produced at the indicated points. Explain your answer.



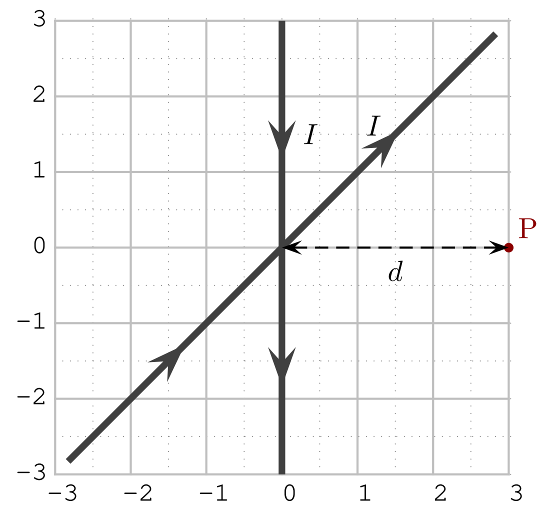
132 Field produced by perpendicular pairs of straight currents

Two infinitely long wires are oriented as illustrated and carry currents as illustrated. Indicate the locations at which the net magnetic field is zero. Explain your answer.



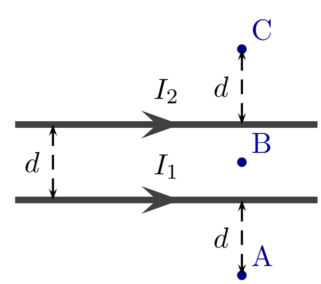
133 Field produced by angled pairs of straight currents

Two infinitely long wires are oriented as illustrated and carry current with the same magnitude I. Determine an expression for the field at the illustrated point P in terms of I and d.



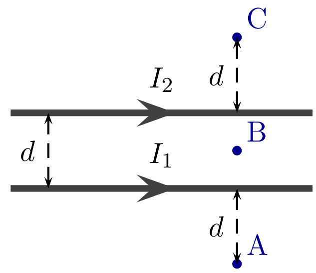
134 Magnetic fields produced by parallel currents, 1

Two infinitely long straight parallel wires carry the illustrated currents with $I_1 > I_2$. Determine an expression for the magnetic field at the points A, B (midway between the wires) and C.



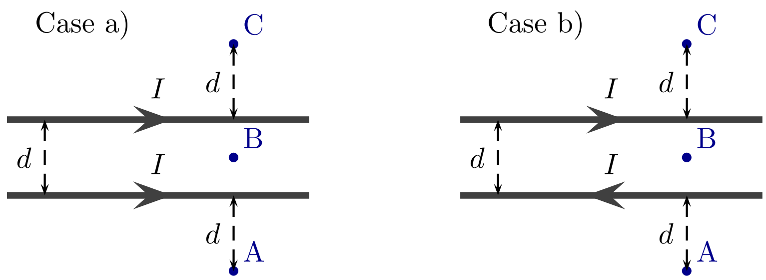
135 Magnetic fields produced by parallel currents, 2

Two infinitely long straight parallel wires carry the illustrated currents with $I_1 = 50\,\text{A}$ and $I_2 = 30\,\text{A}$. Suppose that $d = 0.0020\,\text{m}$. Determine the magnetic field at the points A, B (midway between the wires) and C.



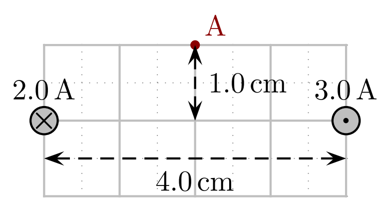
136 Magnetic fields produced by parallel currents, 3

Infinitely long straight parallel wires carry the illustrated currents with identical currents. Consider the magnetic field at the points A, B (midway between the wires) and C. In each case rank the magnetic fields at the indicate locations in order of increasing magnitude. Indicate equality whenever it occurs.



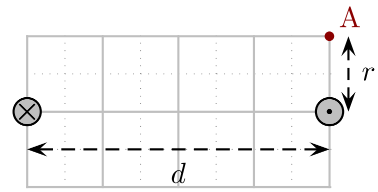
137 Field produced by two parallel straight currents, numeric

Two infinitely long wires point perpendicular to the page.
Determine the magnetic field at the point A.



138 Field produced by two parallel straight currents, algebraic

Two infinitely long wires point perpendicular to the page. Each carries the same current, I. Show that the net magnetic field at point A is

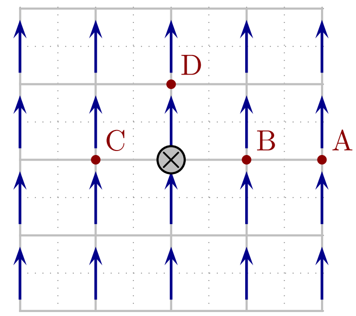


$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi}\left[\left(-\frac{1}{r} + \frac{r}{d^2+r^2}\right)\hat{\mathbf{i}} - \frac{d}{d^2+r^2}\hat{\mathbf{j}}\right].$$

Does this reduce to the correct expression when $d = 0$?

139 Field produced by a straight current in a uniform background field

An infinitely long wire is placed in the illustrated 6.0×10^{-3} T uniform magnetic field (created by something else). The wire carries a 300 A current into the page. The solid grid blocks are each 1.0 cm wide. Determine the magnitude of the net magnetic field at each illustrated point.



140 Magnetic fields produced by a circular loop

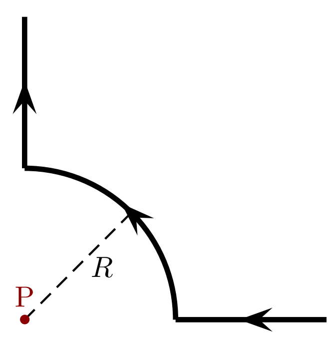
A circular loop with a single turn carries a constant current.

a) Suppose that the loop has radius 0.50 cm. Determine the current that it must carry in order to produce a 25 T magnetic field at the center of the loop.

b) Suppose that the loop carries current 40 A. Determine the radius of the loop in order to produce a 25 T magnetic field at the center of the loop.

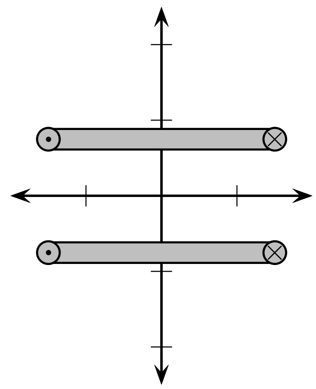
141 Magnetic fields produced by a circular current fragment

Two infinitely long straight wires make an arc of radius R. Determine an expression for the magnetic field at point P.



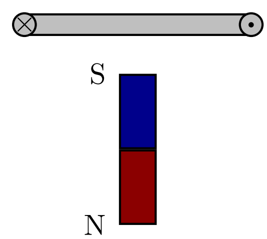
142 Fields produced by a Helmholtz coil

Two current loops are situated as illustrated; both are perpendicular to the plane of the page. The distance between the loops is exactly equal to the radius of each loop and the loops carry the same current. This combination is called a Helmholtz coil. Determine an expression for the field produced by the two coils at the point midway between them and along the axis that connects them.



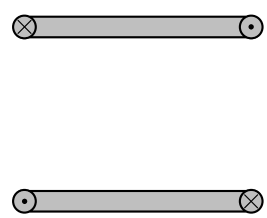
143 Forces exerted by one current loop on a bar magnet.

A current loop is situated above a bar magnet as illustrated. The loop is perpendicular to the plane of the page. Indicate the effective north and south ends of the loop and use these to describe the direction of the force that the loop exerts on the magnet. Indicate the direction of the force that the magnet exerts on the loop.

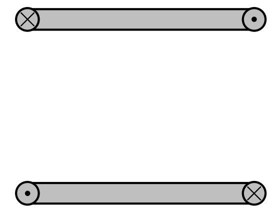


144 Force exerted by one current loop on another

Two current loops are situated as illustrated; both are perpendicular to the plane of the page. Indicate the effective north and south ends of each loop and use these to describe the direction of the force that the upper loop exerts on the lower loop.



145 Forces exerted by one current loop on another



Two current loops are situated as illustrated; both are perpendicular to the plane of the page.

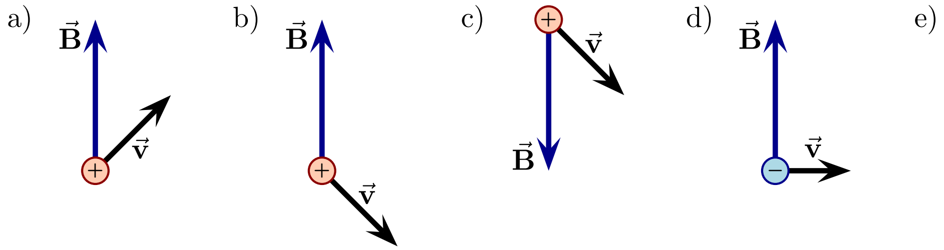
a) Indicate the direction of the magnetic dipole moment produced by each loop.

b) Use the magnetic dipole moments to describe the direction of the force exerted by one loop on the other.

Magnetic Forces

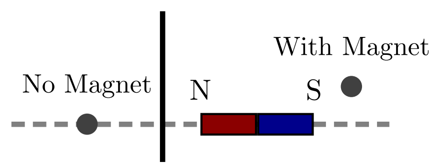
146 Force on a charged particle

Each of the following diagrams illustrates a charged particle moving in an external magnetic field. Indicate the direction of the force exerted by the magnetic field on the charged particle for each case. Explain your answer.



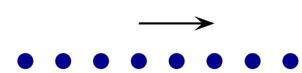
147 Force on a beam of particles

A beam of particles is fired at high speed and travels in a straight line. You view the beam traveling toward you and it approaches at the indicated (no magnet) direction. A magnet is held as illustrated and the beam is deflected as illustrated (with magnet). What charge do the particles have? Explain your answer.



148 Force on a beam of electrons

A beam of electrons moves from left to right. The electrons are placed in a uniform magnetic field.



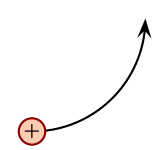
a) If initially the electrons are to be deflected downwards, what is the direction of the magnetic field? Explain your answer.

b) As the electrons continue to move in the uniform field, does the magnitude of the force on the electrons increase, decrease or stay the same? Explain your answer.

c) As the electrons continue to move in the uniform field, does the speed of the electrons increase, decrease or stay the same? Explain your answer.

149 Charged particle in a field

A proton follows the illustrated trajectory in a magnetic field. What is the direction of the magnetic field? Explain your answer.



150 Mass spectrometer

The set up of a mass spectrometer is illustrated in Fig P29.64 (page 864). Suppose that the particles are accelerated from rest through a potential difference of ΔV. Show that the radius of orbit is of a particle with charge q is

$$r = \frac{1}{B}\sqrt{\frac{2m|\Delta V|}{|q|}}$$

where B is the magnitude of the field and m is the mass of the particle.

151 Cyclotron

A charged particle in a cyclotron orbits in a circle perpendicular to a known uniform magnetic field. Suppose that the magnetic field strength is B, the charge of the particle is q, its mass is m and the radius of orbit is R.

a) Starting with and using Newton's Second Law determine an expression for the time taken to complete on orbit, T, in terms of B, q and m. This is called the period of orbit.

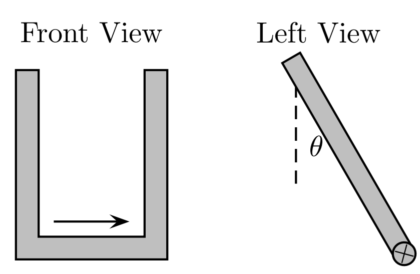
b) The frequency of orbit is $f := 1/T$. Determine an expression for f in terms of B, q and m.

c) The charged particle will radiate an electromagnetic signal at frequency f which is easily measured. Suppose that the magnetic field strength is known. Explain how a cyclotron could be used to determine the charge to mass ratio q/m for the particle.

d) Suppose that protons are inserted into a cyclotron whose magnetic field is 1.2500 T. Determine the cyclotron frequency for these. How is it related to the cyclotron frequency for alpha particles in the same field?

152 Force on a current beam

A current beam is shaped as shown and carries the illustrated current. The beam can pivot about its two upper points. The beam is placed in a uniform vertical magnetic field which causes it to deflect and remain stationary at an angle of θ with respect to the vertical.

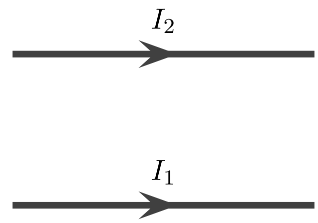


a) Suppose that the length of the horizontal section is l, the current is I, the mass of the entire beam is m and the magnetic field has magnitude B. Determine an expression for θ in terms of these quantities.

b) In a lab the beam has mass 10 g and length 15 cm and the power supply provides a maximum current of 3.0 A. Determine the magnetic field strength needed to deflect the beam at an angle of 5.0° from the vertical.

153 Interacting parallel wires

Two parallel wires each carry currents in the illustrated directions. Each wire has length L and they are separated by distance d.



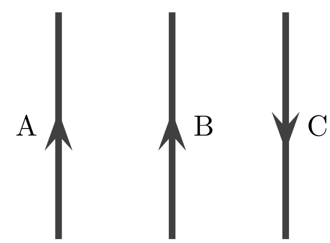
a) Determine an expression for the magnitude of the field produced by the upper wire at the location of the lower wire and use this to determine an expression for the magnitude of the force exerted by the upper wire on the lower wire.

b) Use the field to determine the direction of the force exerted by the upper wire on the lower wire.

c) Suppose that the wires are each 1.5 m long and are separated by 3.0 mm. Each carries current 2.5 A. Determine the force exerted by the upper wire on the lower wire.

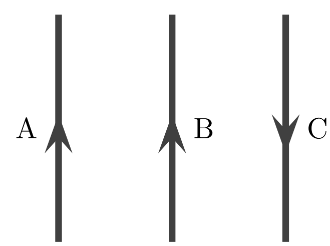
154 Forces on wires, 1

Three wires each carry currents of identical magnitudes in the illustrated directions. The distance between adjacent wires are equal. Rank the wires in order of the magnitudes of the net magnetic force on each.



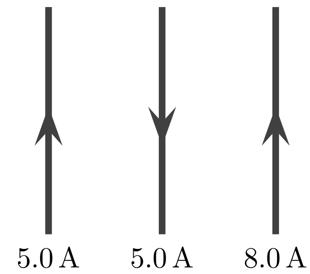
155 Forces on wires, 2

Three wires each carry currents of identical magnitudes, I in the illustrated directions. The distance between adjacent wires, d are equal. Determine an expression for the net force on each wire and rank the wires in order of the magnitudes of the net magnetic force on each.



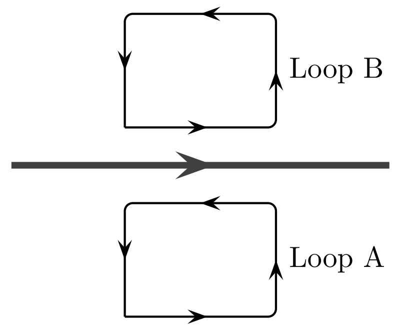
156 Forces on wires, 3

Three wires each carry different currents as illustrated. The length of each wire is 0.75 m and the distance between adjacent wires is 0.020 m. Determine the net force on each wire.



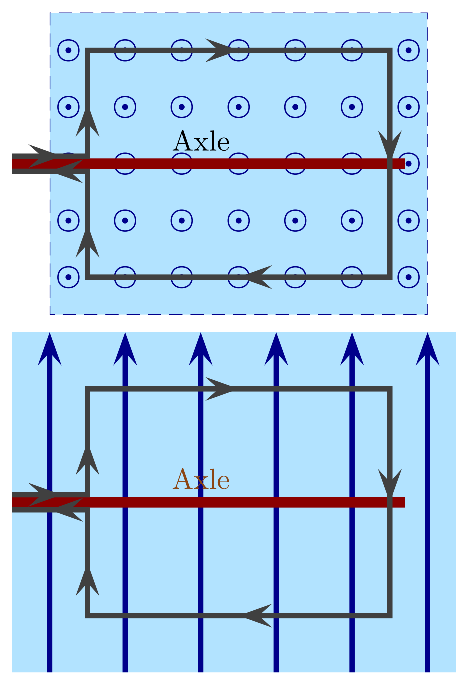
157 Loop and wire

A long straight wire carries a constant current as illustrated. Two loops carry current as illustrated. Describe the direction of the force exerted by the long straight wire on each loop.



158 Current loop in a uniform magnetic field: force directions

A loop is placed in a uniform magnetic field and a current flows as illustrated. Initially the loop lies perpendicular to the magnetic field as illustrated.



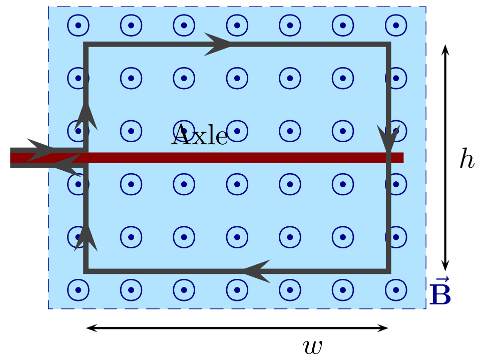
a) For the initial configuration, determine the direction of the force on each side of the loop. Use this to describe how the loop will move if it is released from rest in this position.

Suppose that later the loop lies in the plane of the magnetic field as illustrated.

b) In this configuration, determine the direction of the force on each side of the loop. Use this to describe how the loop will move if it is released from rest in this position.

159 Current loop in a uniform magnetic field: force magnitude and directions

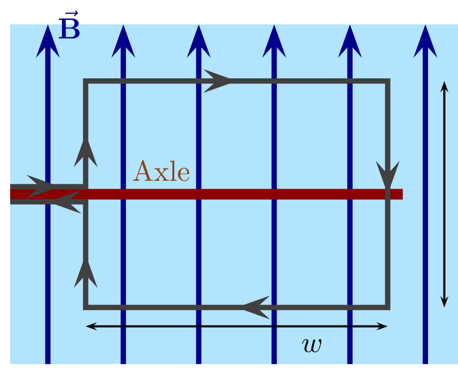
A loop is placed in a uniform magnetic field and a current flows as illustrated. The magnetic field strength is B and the magnitude of the current is I. Initially the loop lies perpendicular to the magnetic field as illustrated.



a) For the initial configuration, determine an expression for the force on each side of the loop. Determine an expression for the net force on the loop.

b) For the initial configuration, determine an expression for the torque (about the axle) on each side of the loop and the net torque on the loop.

Suppose that the loop lies in the plane of the magnetic field as illustrated.



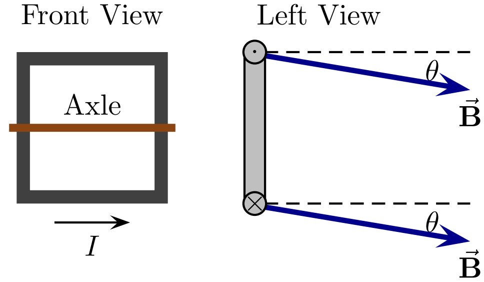
c) In this configuration, determine an expression for the force on each side of the loop. Determine an expression for the net force on the loop.

d) In this configuration, determine an expression for the torque (about the axle) on each side of the loop and the net torque on the loop. Rewrite the expression in terms of the area of the loop.

e) Describe how the loop would begin to move if it were released from this position.

160 Current loop in a uniform magnetic field

A square loop with sides of length L carries current I is placed in a uniform magnetic field with magnitude B. Viewed from the left side the field lies in the plane of the page and is angled as illustrated. Viewed from this side the current in the upper end of the loop is out of the page and in the lower end of the loop it is into the page.

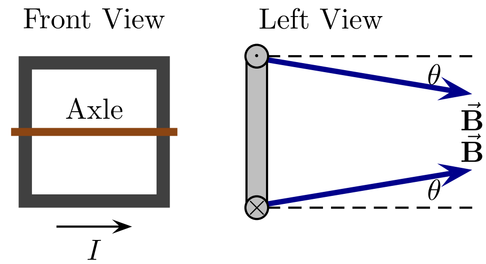


a) Determine an expression for the net force that acts on the loop in terms of L, I, B and θ.

b) Determine an expression for the net torque on the loop (about an axle that is out of the page in the left view).

161 Current loop in a non-uniform magnetic field

A square loop with sides of length L carries current I is placed in a non-uniform magnetic field with magnitude B. Viewed from the left side the field lies in the plane of the page and is angled as illustrated. Viewed from this side the current in the upper end of the loop is out of the page and in the lower end of the loop it is into the page.

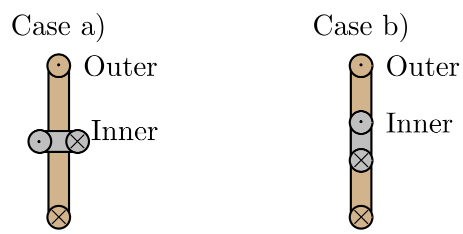


a) Determine an expression for the net force that acts on the loop in terms of L, I, B and θ.

b) Is the net torque on the loop (about an axle that is out of the page in the left view) zero or non-zero? Explain your answer.

162 Nested current loops

Two circular current loops, one much smaller than the other, are nested in either of the configurations. Their centers coincide. Let r_o be the radius of the outer loop, r_i be the radius of the inner loop, I_o be the current in the outer loop and I_i be the current in the inner loop.



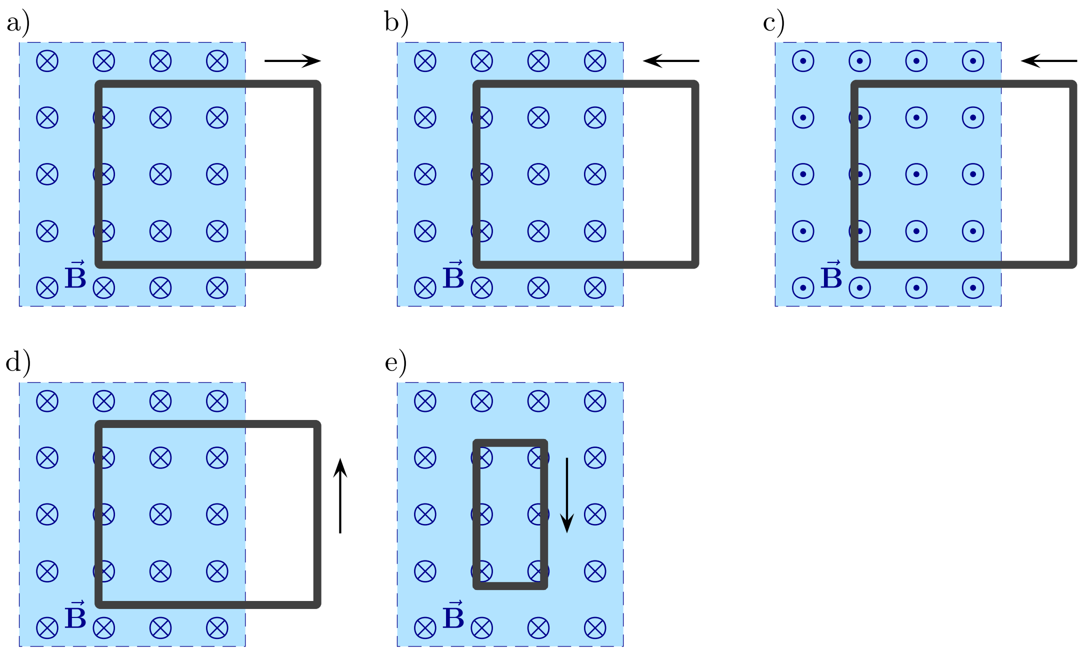
a) In each case, determine an expression for the net torque (including direction) exerted by the outer loop on the inner loop. The inner loop is small enough that the field within it can be approximated as constant.

b) In each case, describe how the inner loop will begin to move.

Magnetic Induction

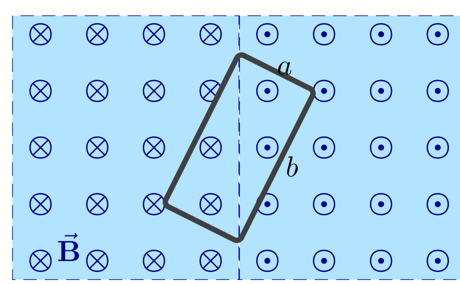
163 Motional EMF and current

In each of the following a loop lies partly in a uniform magnetic field and is dragged as illustrated. In each case, determine if the dragging produces any current and, if so, its direction. Explain your answers.

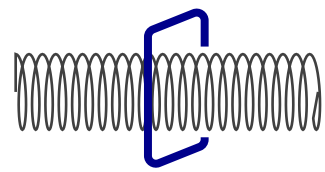


164 Rectangular loop in two magnetic field regions

The illustrated regions each have a uniform magnetic field. The magnitude of the field on the left is $2.5 \times 10^{-4}\,\mathrm{T}$ and on the right it is $3.5 \times 10^{-4}\,\mathrm{T}$. A rectangular loop is oriented with its diagonal along the boundary between the two regions. Its longer side has length $b = 3.0\,\mathrm{cm}$ and the shorter side $a = 1.0\,\mathrm{cm}$. Determine the magnetic flux through the loop.



165 Flux: square loop around a solenoid

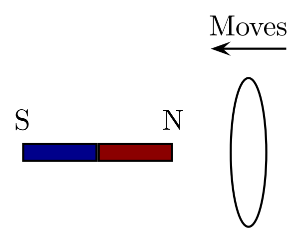


A square loop, with sides 40 cm long, is placed around a solenoid, which can be approximated as infinitely long. The solenoid has radius 5.0 cm, has 100 turns per meter and carries current 7.0 A. The square loop and the solenoid are arranged so that their axes are along the same line. When viewed from the right, the current in the solenoid flows counterclockwise.

a) Determine the magnetic flux through the square loop.

b) Suppose that the radius of the solenoid were decreased but everything else was kept the same. How would this affect the magnetic flux through the square loop? Explain your answer.

166 Loop moved near a permanent magnet



A circular loop is initially held at rest near to a bar magnet. The bar magnet lies along the axis of the loop. The loop is then pushed toward the magnet. In each of the following explain your answers.

a) While the loop is initially at rest, is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)?

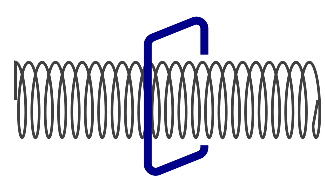
b) While the loop is initially at rest, does the magnet exert a force on the loop? If so, what is its direction?

c) While the loop approaches the magnet, is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)?

d) While the loop approaches the magnet, does the magnet exert a force on the loop? If so, what is its direction?

e) The loop eventually passes "across" the magnet and moves left away from the magnet, During this period is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)? Does the magnet exert a force on the loop? If so, what is its direction?

167 Square loop around a solenoid



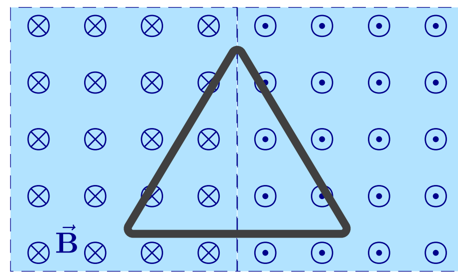
A square loop is placed around a solenoid, which can be approximated as infinitely long. The square loop and the solenoid are arranged so that their axes are along the same line. When viewed from the right, the current in the solenoid flows clockwise. In each of the following explain your answers.

a) Suppose that the current in the solenoid is constant. Is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)?

b) Suppose that the current in the solenoid is increasing. Is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)?

c) Suppose that the current in the solenoid is decreasing. Is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)?

168 Triangular loop in a magnetic field



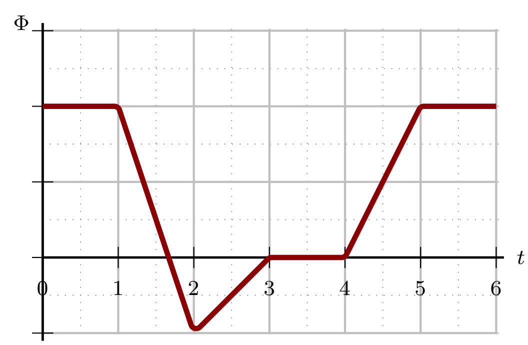
The illustrated regions each have a uniform magnetic field with the same magnitude but opposite directions. A triangular loop is initially at the illustrated position. The loop can be dragged in various directions from this initial position. In each of the following explain your answers.

a) Suppose that the loop is dragged vertically. Just after it starts to move, is there a current in the loop? If so, what is its direction?

b) Suppose that the loop is dragged to the left. Just after it starts to move, is there a current in the loop? If so, what is its direction?

c) Suppose that the loop is dragged to the right. Just after it starts to move, is there a current in the loop? If so, what is its direction?

169 Flux through a loop

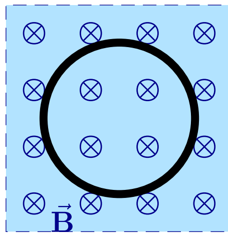


A solenoid produces a flux through a loop of wire. A graph of the flux versus time is as illustrated. During which time intervals will there be a non-zero current in the loop? Explain your answer.

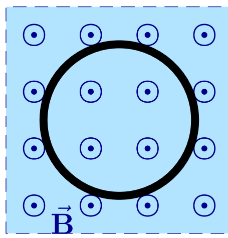
170 Changing magnetic fields and loops

In each of the following a loop lies in a uniform magnetic field which changes as time passes. Determine the direction of the induced current in the loop. Explain your answers.

a) Field increases

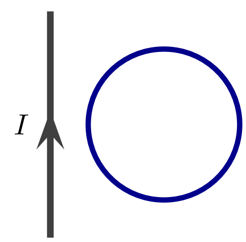


b) Field decreases



171 Induced currents

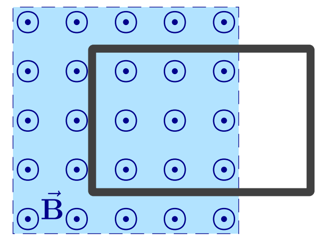
A conducting loop is in the vicinity of a long straight wire. The wire carries a current, I, pointing in the illustrated direction. In each of the following situations describe whether there is a current in the loop and, if so, what it's direction is. Explain your answers.



a) The loop is fixed and the current in the straight wire steadily increases.
b) The loop is fixed and the current in the straight wire steadily decreases.
c) The current in the wire stays constant and the loop is pushed to the right.
d) The current in the wire stays constant and the loop is pushed to the left.
e) The current in the wire stays constant and the loop is pushed up (the page).

172 Loops partly in a field

A rectangular loop lies partly in a region of uniform magnetic field as illustrated. In the following, explain your answers.

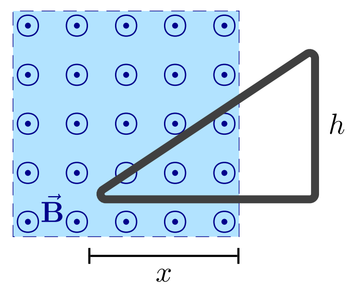


a) The magnetic field increases as time passes. Determine the direction of the induced current in the loop. Determine the direction of the force exerted by the field on the loop.

b) The magnetic field decreases as time passes. Determine the direction of the induced current in the loop. Determine the direction of the force exerted by the field on the loop.

173 Loop dragged through a field

A triangular loop lies partly in a region of uniform magnetic field as illustrated. The vertical edge has height h and the horizontal edge has length w. The loop is dragged to the right with constant speed v.

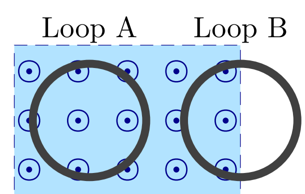


a) Determine an expression for the EMF around the loop at the moment when the length of the horizontal section within the field is x. Is the current constant as the loop moves?

b) Determine the direction of the current around the loop.

174 Current around loops

Two circular loops, each with radius 0.040 m, are placed in a region of uniform magnetic field. Exactly half of loop B is in the magnetic field. The field strength increases from 0 T to 2.0×10^{-3} T over a period of 0.005 s.



a) The resistance of loop A is 0.025 Ω. Determine the EMF around the loop and the current through the loop.

b) The resistance of loop B is 0.005 Ω. Determine the EMF around the loop and the current through the loop.

175 Inductive loading

Electronic circuits often contain loops of current that produce magnetic fields. Whenever the current in one loop changes, the resulting magnetic field changes and this can induce an EMF in an adjacent loop. As a simple model to illustrate such effects consider two closely spaced parallel circular loops, each with radius 3.0 cm. Suppose that one loop (loop A) contains a steady current of 0.50 A. Initially there is no current in the adjacent loop (loop B). Loop B contains a circuit device that will be destroyed if the voltage across it exceeds 10 V.

a) Determine the magnetic field at the center of loop A.

b) Assume that the magnetic field throughout loop B is the same as that at the center of loop A. Determine the flux through loop B.

c) Suppose that the circuit is instantly turned off and the current in loop A drops to zero in 0.5×10^{-9} s. Determine the EMF induced in loop B while this happens. Will the device in loop B survive?

176 Induced EMF

A circular loop with radius 10 cm lies in the xy plane. An electromagnetic produces a uniform magnetic field

$$\vec{\mathbf{B}} = 0.0030t^2\,\hat{\mathbf{j}} + 0.0040t^3\,\hat{\mathbf{k}}$$

where the constants have units such that the field is in Teslas.

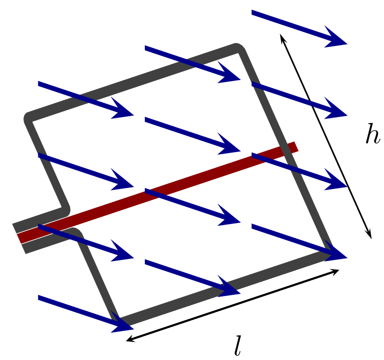
a) Determine an expression for the EMF in the loop at any time.

b) Determine the EMF in the loop at $t = 0.0$ s.

c) Determine the EMF in the loop at $t = 10.0$ s.

177 Generator

A generator consists of a loop which is forced to rotate about a fixed axle while it is in a uniform magnetic field as illustrated. Suppose that initially the loop is oriented vertically and then is forced (by a hand or something else external) to rotate with constant angular velocity, ω, about the illustrated axle. The aim of this exercise is to determine an expression for the EMF produced around the loop.



a) Sketch a side view of the loop, indicate the normal and the angle between the normal and the field, θ. Express θ in terms of ω and time.

b) Determine an expression for the flux through the loop as a function of time.

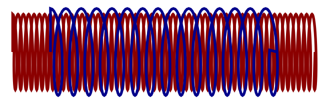
c) Determine an expression for the EMF around the loop as a function of time.

d) Does the EMF remain constant?

e) Suppose that the loop is connected to a resistor with resistance R. Determine an expression for the power delivered to the resistor.

178 Transformer

A simple model of a transformer consists of two solenoids, whose axes lie along the same straight line. Suppose that the inner solenoid has N_1 turns and the outer solenoid has N_2 turns. The two solenoids are arranged so that their radii are essentially the same.



a) Consider any single loop of each solenoid. How does the flux through a single loop of the inner solenoid compare to that through a single loop of the outer solenoid.

b) Let Φ_1 be the total flux through the inner solenoid and Φ_2 be the total flux through the outer solenoid. Find an expression relating these and N_1 and N_2.

c) Let $\mathcal{E}_1$ be the EMF across the inner solenoid and $\mathcal{E}_2$ be the EMF across the outer solenoid. Find an expression relating these and N_1 and N_2.

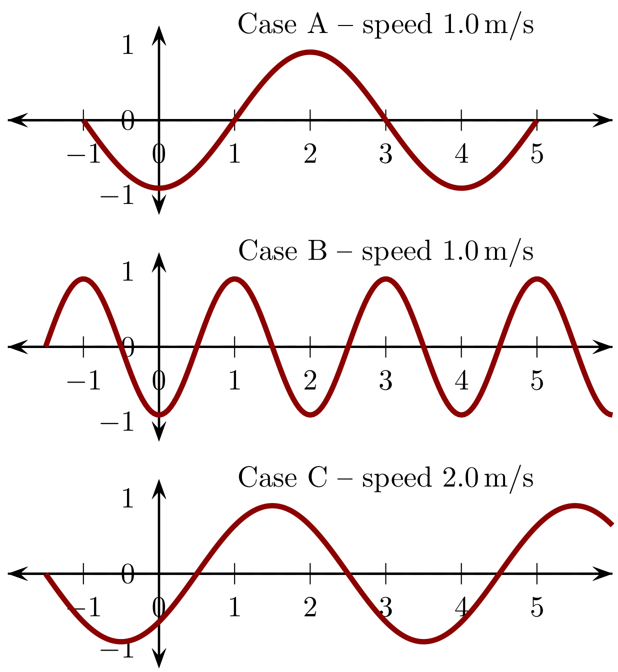
Waves

179 Wavelength and frequency

Various waves on strings are as illustrated. The wave speeds are provided for each case. The units of the axes are meters. *(132S22)*

a) Rank the waves in order of increasing wavelength. Indicate equality whenever it occurs. Explain your answer.

b) Rank the waves in order of increasing frequency. Indicate equality whenever it occurs. Explain your answer.

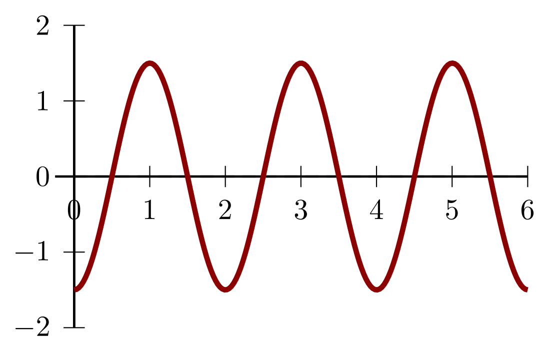


180 Waves on a string

A snapshot of a wave on a string is illustrated. The units of the axes are meters. *(132S22)*

a) Determine the wavelength of the wave.

b) The wave is observed as time passes and it is found that 20 crests pass the 4 m mark in 5.0 s. Determine the wave speed.



181 Waves on a string with an oscillating end

The PhET animation "Waves on a String" allows you to visualize transverse waves on a string. Find the animation at

http://phet.colorado.edu/en/simulation/wave-on-a-string

and open it. Adjust the settings as follows:

1. Check the button "No end" at the upper right.

2. Check the button “Oscillate” at the upper left.
3. In the control panel at the bottom, adjust “Damping” to none.
4. In the control panel at the bottom, adjust “Tension” to low.
5. In the control panel at the bottom, check the “Rulers” and “Timer” buttons.

(132S22)

a) Set the frequency to 0.30 Hz. Using the rulers and timer, determine the speed of the wave. Using the ruler, determine the wavelength of the wave. Determine the frequency of the wave by counting crests that pass a given point.

b) Set the frequency to 0.60 Hz. Repeat part a).

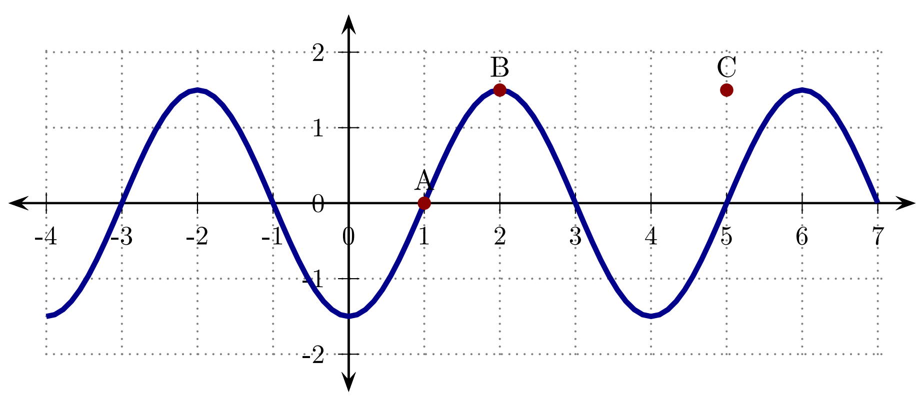
c) Is the speed the same in both cases?

d) Check that $v = \lambda f$ in both cases.

e) For one of the frequencies, vary the amplitude and show that it does not affect the wave speed or wavelength.

182 Sinusoidal waves

A snapshot of a segment of a wave on a string at a particular instant is illustrated. The distances are measured in meters. *(132S22)*



a) The portion of the wave at A takes 0.0050 s to complete one cycle. Determine the frequency of the wave.

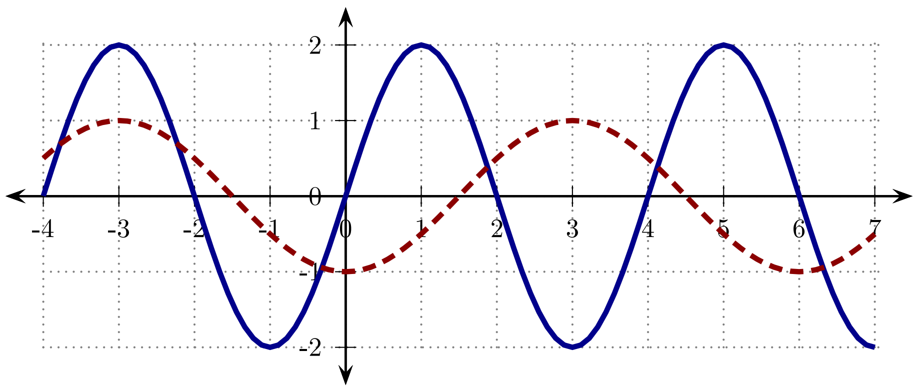
b) Determine the wavelength and speed of the wave.

c) Determine how long it will take the crest labeled B to reach the point C.

d) Using the graph above, sketch the wave at an instant 0.0025 s after that illustrated above.

183 Wave displacement at one instant

A snapshot of two waves on a string at a particular instant ($t = 0$) are illustrated. The distances are measured in meters. *(132S22)*

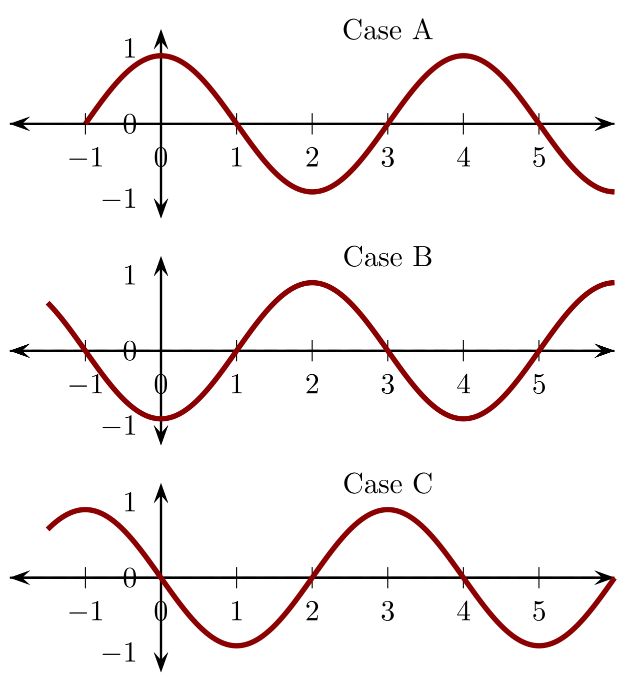


a) Determine an expression of the form $y = A\sin(kx)$, for the wave illustrated by the solid line (i.e. find numbers for A and k).

b) Determine an expression of the form $y = A\sin(kx + \phi)$, for the wave illustrated by the dashed line (i.e. find numbers for A, k and ϕ).

184 Wave phase

Snapshots of waves on strings at one instant are as illustrated. The units of the axes are meters. Each wave can be expressed as $y = A\sin(kx + \phi)$ where $A > 0$. Determine the phase ϕ in each case. *(132S22)*



185 Wave displacement function

A wave travels along a string. At one instant a snapshot is taken. The resulting pattern is described by the displacement function

$$y = A \sin(kx).$$

The definition of wavelength means that the displacement at x_0 must be the same as at $x_0 + \lambda$ regardless the value of x_0. Using the expression above, show that if $k = 2\pi/\lambda$, then y evaluated at x_0 must be the same as y evaluated at $x_0 + \lambda$ (regardless of x_0). *(132S22)*

186 Wave displacement

a) The displacement for a wave on a string is

$$y = 4.0 \sin(20x - 100t).$$

Determine the amplitude, wavelength, frequency and wavespeed of the associated wave.

b) The general displacement for a sinusoidal wave is

$$y = A \sin(kx - \omega t).$$

Consider this at $t = 0$. List the four values of x (left and right nearest to $x = 0$) at which a maximum occurs.

c) At $t = 0$ determine expressions for all locations where the displacement is a (positive) maximum. Use this to determine an expression, in terms of k, for the horizontal distance between any successive maxima. *(132S22)*

187 Wave speeds

Waves on different strings (A, B) are described by displacement functions

$$y_A = 3.0\,\mathrm{m} \sin\left(8.0\,\mathrm{m}^{-1}x - 40\,\mathrm{s}^{-1}t\right),$$
$$y_B = 3.0\,\mathrm{m} \sin\left(8.0\,\mathrm{m}^{-1}x + 40\,\mathrm{s}^{-1}t\right)$$

a) Determine the wavespeed for A.
b) Determine the wavespeed for B.
c) What do the $\pm$ signs inside the argument describe physically? *(132S22)*

188 Wave speeds: comparison

Waves on different strings (A, B, C and D) are described by displacement functions

$$y_A = 5 \sin(4x - 20t),$$
$$y_B = 5 \sin(20x - 4t),$$
$$y_C = 10 \sin(8x - 40t), \text{ and}$$
$$y_D = 10 \sin(4x - 8t).$$

Rank these in order of increasing wavespeed, indicating equality whenever it occurs. Explain your answer. *(132S22)*

Standing Waves

189 Standing waves on a string

A 0.800 m string is configured so that waves on the string travel with speed 410 m/s. *(132S22 Class)*

a) Sketch the fundamental ($n = 1$) and determine its wavelength and frequency.
b) Sketch the second harmonic ($n = 2$) and determine its wavelength and frequency.
c) Sketch the third harmonic ($n = 3$) and determine its wavelength and frequency.
d) Determine formulas for the wavelengths and frequencies of *all* possible standing waves.

A general expression for the displacement of the string, with length L and both of whose ends are fixed, is

$$y(x,t) = A\sin(kx)\cos(\omega t)$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi f$. These satisfy $\omega = kv$.

e) Show that this expression satisfies the requirement for a fixed end at $x = 0$.
f) Since the other end is fixed, at $x = L$, the displacement must be zero. Substitute this into the general expression above and show that it can only be satisfied if k takes certain specific values. Determine what these values are (in terms of L) and use this to obtain expressions for the wavelengths of all possible standing waves.

190 Standing waves on a string

Waves on a particular stretched string travel with speed the string travel with speed 250 m/s. Both ends of the string are held fixed. The fundamental frequency is 512 Hz. *(132S22)*

a) Determine the length of the string.
b) Determine the frequency of the second harmonic.
c) Determine the frequency of the third harmonic.
d) Is there a harmonic with a frequency of 1250 Hz. If so which is it?
e) Is there a harmonic with a frequency of 2560 Hz. If so which is it?

191 Standing waves: laser

A HeNe laser partly operates by producing standing waves between two mirrors, spaced 14.871 cm apart. The standing waves have nodes at each mirror. The wavelength of the light emitted by this laser is ideally 632.8 nm. *(132S22)*

a) Determine the number of nodes in the standing wave (you will need to round off).
b) Determine the wavelength of the standing waves with ten more nodes.
c) The stated wavelength is only accurate to 0.1 nm. Determine the number of nodes in the standing waves with wavelength 632.7 nm and 632.9 nm. What does this let you say about the number of nodes for this laser?

EM Waves: Intensity and Polarization

192 LED intensity

A LED produces light radiates outwards uniformly in all directions. Let I_1 be the intensity at a distance r_1 from the LED. *(132S22)*

a) Let I_2 be the intensity at a distance $r_2 > r_1$ from the LED. Determine an expression for I_2 in terms of I_1, r_1 and r_2.

b) How is r_2 related to r_1 such that the intensity at r_2 is 1% of the intensity at r_1? Explain your answer.

193 Laser intensity

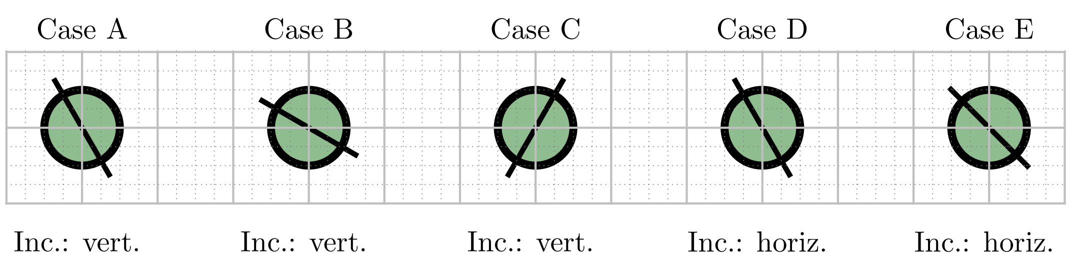
A Helium-Neon laser produces light with power 50 mW. The beam diameter is 2.00 mm. *(132S22)*

a) Determine the amplitude of the electric field produced by the laser.

b) Suppose that the beam passes though a lens which focuses the beam to a narrower diameter. Determine the diameter such that the laser would cause air to breakdown (produce sparks); the necessary electric field is about 3×10^6 N/C.

194 Polarization filters

Linearly polarized light is incident on polarizing filters whose transmission axes are oriented as illustrated. The diagram describes the polarization axis of the incident light in each case. Rank the intensities of the transmitted light, assuming that the intensity of the incident light is the same in each case. Explain your answers. *(132S22)*



195 Two polarization filters

Unpolarized light, with intensity I_0 is incident on a polarization filter whose transmission axis is vertical. After passing through this filter it is incident on a second filter. Determine the angle from the horizontal at which the transmission axis of the second polarizer must be so that the intensity of the light transmitted by the second filter is $\frac{3}{8}I_0$? *(132S22)*

196 Sequence of polarization filters

Light that is vertically polarized is incident on a succession of polarization filters. The angle between the transmission axes of any two successive filters is the same and the transmission axis of the final filter is horizontal. Let I_0 be the intensity of the light incident on the first filter and I_f be the light transmitted by the final filter. *(132S22)*

a) Suppose that there are three filters. Thus the angles between successive transmission axes is 30°. Determine the ratio I_f/I_0.

b) Suppose that there are six filters. Thus the angles between successive transmission axes is 15°. Determine the ratio I_f/I_0.

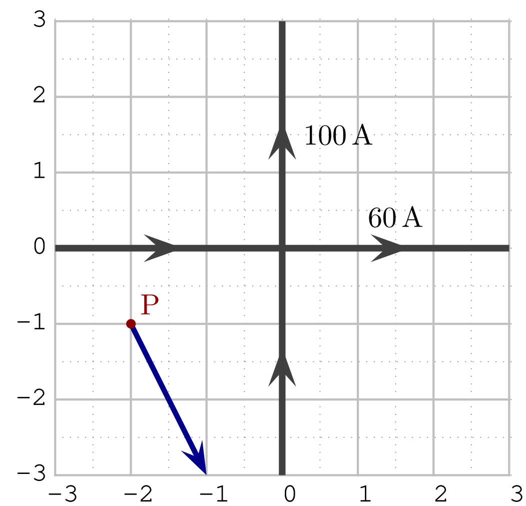
c) Suppose that there are 90 filters. Thus the angles between successive transmission axes is 1°. Determine the ratio I_f/I_0.

d) As more filters are added in this way, what does the ratio I_f/I_0 approach?

Magnetism and Electromagnetic Waves: Additional Questions

197 Proton near two straight wires

Two infinitely long wires are oriented as illustrated and carry currents as illustrated. A proton is at the location labeled P and travels with velocity 6.0×10^5 m/s in the illustrated direction. The grid units are 0.0010 m. *(132S22 Class)*



a) Determine the net magnetic field at P.

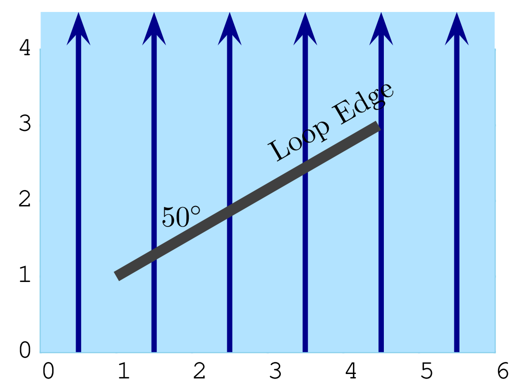
b) Determine the net force acting on the proton when it is at P.

198 Loop in a time-varying field

A square loop is placed in a region of uniform magnetic field. The loop axis lies in the plane of the page and one edge is visible. The sides of the loop are 30 cm long and the field varies with time according to

$$\vec{\mathbf{B}}(t) = \left(20\,\mathrm{T/s^2}t^2 - 20\,\mathrm{T/s^3}t^3\right)\hat{\mathbf{j}}.$$

Determine the EMF produced around the loop at $t = 1.0\,\mathrm{s}$. *(132S22 Class)*



199 Traveling sinusoidal wave

A sinusoidal wave on a string is described by the displacement

$$y(x,t) = 5.0\,\mathrm{mm}\sin\left(5.0\,\mathrm{m^{-1}}x - 30\,\mathrm{s^{-1}}t + \pi/2\right).$$

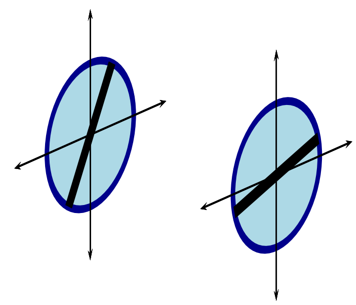
A snapshot at $t = 0\,\mathrm{s}$ shows a crest, labeled A, at $x = 0\,\mathrm{m}$. *(132S22 Class)*

a) Determine the speed of the wave.

b) Determine the time taken for the crest A to reach the $x = 24\,\mathrm{m}$ mark.

200 Two polarizers: incident light polarized

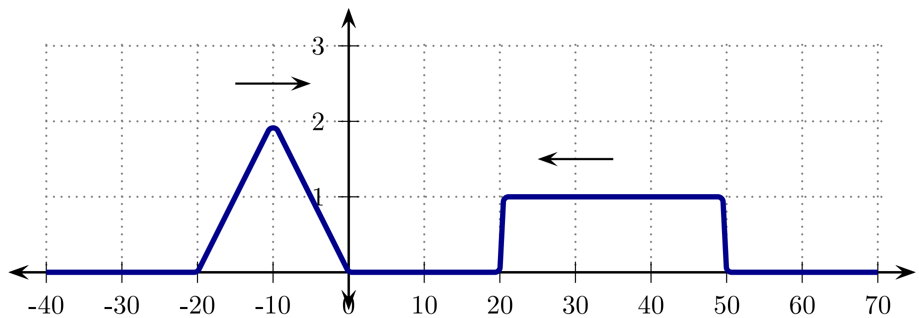
Horizontally polarized light with intensity I_0 is incident from the left on a polarizer, whose transmission axis 60° from the horizontal. This transmitted light is later incident on a second polarizer, whose transmission axis 15° from the horizontal. Determine an expression for the intensity of the light transmitted by the second polarizer (in terms of I_0). *(132S22 Class)*



Interference of Waves

201 Interference of pulses

Two pulses on a string approach each other as illustrated; a snapshot at one instant is illustrated. The grid units are each 10 cm and the pulses travel with speed 10 cm/s. *(132S22)*



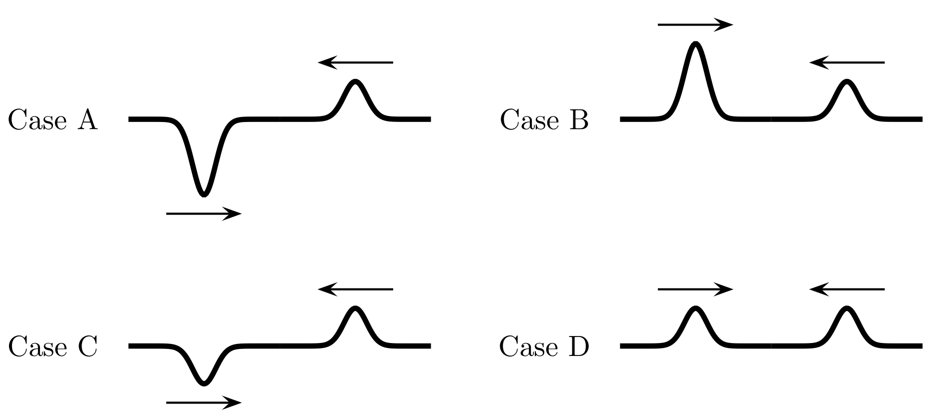
a) Sketch the string at a moment 1.0 s after that illustrated in the figure.

b) Sketch the string at a moment 2.0 s after that illustrated in the figure.

c) Sketch the string at a moment 3.0 s after that illustrated in the figure.

202 Interference of pulses

Various pulses approach each other as illustrated. The pulses overlap and interfere; when each does so there will be a point of maximum displacement away from the horizontal. *(132S22)*



Rank the situations in order of increasing maximum displacement away from the horizontal during interference (indicate any ties in the ranking). Explain your answer.

203 Adding sinusoidal waves: in phase

Two sources separated by distance Δr produce one dimensional sinusoidal traveling waves. These are described by

$$y_1(x,t) = A\sin(kx - \omega t)$$
$$y_2(x,t) = A\sin(k(x-\Delta r) - \omega t).$$

In the following note that

$$\sin A + \sin B = 2\cos\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right).$$

(132S22)

a) Show that the superposition is represented by

$$y(x,t) = B\sin(kx - \omega t + \phi)$$

where B is independent of location and time and ϕ a phase constant. Find an expression for B.

b) Use the previous part to determine conditions where constructive interference occurs.

c) Use the previous part to determine conditions where destructive interference occurs.

204 Adding sinusoidal waves: out of phase

Two sources separated by distance Δr produce one dimensional sinusoidal traveling waves. The sources oscillate out of phase. These are described by

$$y_1(x,t) = A\sin(kx - \omega t)$$
$$y_2(x,t) = A\sin(k(x-\Delta r) - \omega t + \pi).$$

In the following note that

$$\sin A + \sin B = 2\cos\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)$$

(132S22)

a) Show that the superposition is represented by

$$y(x,t) = B\sin(kx - \omega t + \phi)$$

where B is independent of location and time and ϕ a phase constant. Find an expression for B.

b) Use the previous part to determine conditions where constructive interference occurs.

c) Use the previous part to determine conditions where destructive interference occurs.

205 Interference of waves

a) Two sources produce waves that travel (along one direction) with speed 32 m/s. The sources oscillate with frequency 8.0 Hz. By what distances must the sources be separated to produce constructive interference? By what distances must the sources be separated to produce destructive interference?

b) Two sources produce waves that travel in a medium with speed 300 m/s. It is found that the closest separation between the sources that results in destructive interference is 0.050 m. Determine the wavelength and frequency of the waves. *(132S22)*

Wave Optics

206 Double slit interference: wavelength and spacing dependence

Light with wavelength λ is incident on a double slit with spacing d. A pattern is observed on a screen which is far from the slits (compared to the separation between the slits). *(132S22)*

a) Explain whether each of the following is correct or not.

   i) The pattern looks like a wave with bright fringes separated by a distance λ.

   ii) The pattern looks like a wave with bright fringes separated by a distance d.

   iii) The pattern looks like a wave with the locations of the bright fringes determined by the wavelength only.

   iv) The pattern looks like a wave with the locations of the bright fringes determined by the slit separation only.

b) The wavelength is doubled to 2λ and the slit spacing is also doubled to $2d$. Which of the following is true? Explain your answer.

   i) The separation between the fringes is a quarter of what it was before.

   ii) The separation between the fringes is a half of what it was before.

   iii) The separation between the fringes is the same as before.

   iv) The separation between the fringes doubles.

   v) The separation between the fringes increases by four times.

207 Double slit interference pattern: small angles

Light from a HeNe laser has wavelength 632.8 nm and is incident on a pair of slits separated by 40 μm = 40000 nm. A pattern is produced on a screen a distance of 1.25 m from the laser. *(132S22)*

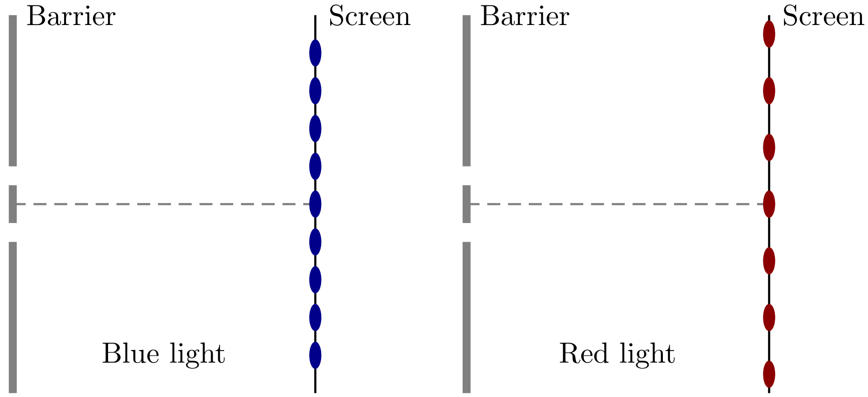
a) Determine the angles at which the $m = 1$, $m = 2$ and $m = 3$ bright fringes appear.

b) Determine the distance from the central bright fringe to each of the first three fringes.

c) Determine the separation between adjacent bright fringes.

208 Double slit interference patterns: different colors

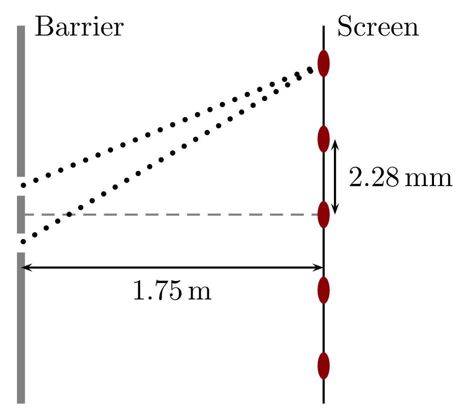
Light with different colors is incident on a pair of double slits. The interference patterns are illustrated with the colored dots indicating bright fringes.



Based on the illustrated patterns how does the wavelength of the red light compare (same, larger, smaller, ...) to that of the blue light? Explain your answer, **using information from the patterns**. *(132S22)*

209 Double slit interference patterns: determining wavelength

Light is incident on two slits whose separation is 0.500 mm. The distance from the slits to the screen is 1.75 m. The colored dots in the figure illustrate the centers of the bright fringes. *(132S22)*



a) Determine the wavelength of the light.

b) Determine the distance from the central bright fringe to the $m = 2$ bright fringe.

c) The dotted lines indicate the paths taken by the waves from each slit to one particular bright fringe. Determine the difference in the length of these paths.

210 Double slit interference pattern

a) Light with wavelength 650 nm illuminates a double slit where the spacing between the slits is 3500 nm. Determine the angles at which all the bright fringes appear.

b) Light with wavelength λ illuminates a double slit with slit spacing 8.1λ. Determine the number of bright fringes that appear. *(132S22)*

211 Hydrogen emission spectrum

Light from hydrogen is passed through a diffraction grating. It is observed that the spectrum consists of three colors (red, turquoiose, dark blue). It is observed that the first-order bright red fringe occurs at an angle of $7.53°$; the wavelength of this light is 656 nm. Then the first-order turquoise fringe is observed at $5.54°$ while the first-order dark blue fringe is observed at $4.96°$. *(132S22)*

a) Determine the wavelengths of the turquoise and dark blue fringes.

b) There is actually a fourth color that is too faint to be observed accurately but whose wavelength is 397 nm. Determine the angles at which the first-order fringe for this would occur.

212 Diffraction grating: determining wavelength

Light with known wavelength λ_{known} is incident on a diffraction grating. It is found that the first order bright fringe occurs at angle $\theta_{1\text{known}}$. The same diffraction grating is used to determine the wavelength of light with a mystery wavelength. It is observed that the first order bright fringe occurs at angel $\theta_{1\text{mystery}}$. *(132S22)*

a) Determine an expression for the mystery wavelength in terms of the angles and λ_{known}.

b) Filtered green light from mercury has wavelength 546.1 nm. The first order bright fringe occurs at $10.05°$. Light from a different source is incident on the same grating. The first order bright fringe occurs at $11.99°$. Determine the wavelength of the light from the different source.

This strategy involving "comparisons" is common in spectroscopy.

213 Single slit

Laser light is incident on a single slit whose width can be varied. The diffraction pattern is projected onto a distant screen. The slit with is initially a_0 and is then narrowed to $\frac{a_0}{5}$. Which of the following is true regarding the width of the central maximum on the distant screen? Explain your answer. *(132S22)*

i) Width reduces to 1/5 of original width.

ii) Width is halved.

iii) Stays the same.

iv) Width doubles.

v) Width increases by a factor of 5.

214 Single slit and white light

White light contains a mixture of light of all colors. Suppose that white light is incident on a single slit with width 0.25 mm. *(132S22)*

a) Determine the angle at which the first minimum in the diffraction pattern occurs for red light with wavelength 700 nm.

b) Determine the angle at which the first minimum in the diffraction pattern occurs for violet light with wavelength 380 nm.

c) Describe the appearance in terms of colors of the central bright spot in the diffraction pattern.

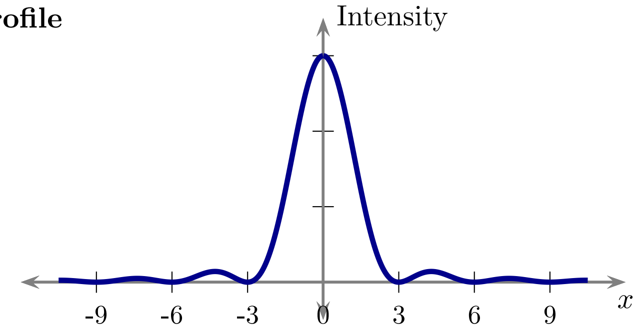
215 Single slit diffraction

A single slit with width 0.020 mm is illuminated with light of various wavelengths. A pattern is produced on a screen 0.90 m from the slit. *(132S22)*

a) Determine the width of the central maximum if the wavelength of the light is 700 nm.

b) Determine the width of the central maximum if the wavelength of the light is 400 nm.

216 Interference/diffraction intensity profile



Light from a laser with wavelength 632.8 nm is incident on arrangement of barriers and slits. The resulting pattern is projected onto a screen 2.25 m from the slit arrangement. The diagram illustrates a plot of the intensity as a function of horizontal position, indicated in centimeters. *(132S22)*

a) Is the slit arrangement a single slit or a double slit? Explain your answer.

b) Determine the slit width or the distance between the slits (whichever applies in this case).

217 Laser beam divergence

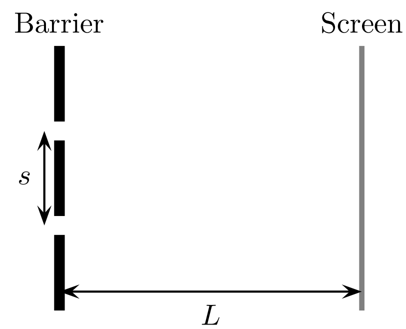
A laser generally produces a narrow beam. Ideally this beam would emerge from a circular aperture in the body of a laser and travel outwards without diverging. Consider a Helium-Neon laser with wavelength 632.8nm. This emerges from a circular aperture with non-zero diameter. *(132S22)*

a) Explain why it is impossible to avoid a spreading beam after it emerges from the aperture.

b) Suppose that you would like to focus the laser beam to a small point 1.5 m from the laser. Would decreasing the aperture diameter improve the beam focus.

c) Determine the diameter to which the laser beam spreads after traveling 1.5 m if the aperture has a diameter of 2.0 mm.

218 Diffraction from two circular openings

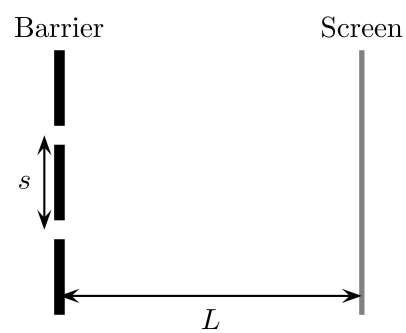


Light with wavelength 650 nm is incident on a barrier that contains two circular openings, each with diameter 1.0 mm. The separation between the centers of the two openings is s. *(132S22)*

a) Sketch the two central diffraction maxima produced by the two apertures and use this to provide the mimimum value of s in terms of the width of each maximum, w, such that the two maxima *do not overlap.*

b) Determine the closest spacing between the holes so that they can be distinguished at a distance of 5.0 m.

219 Diffraction from two circular openings



Light with wavelength λ is incident on a barrier that contains two circular openings, each with diameter D. The separation between the centers of the two openings is s. *(132S22)*

a) Sketch the two central diffraction maxima produced by the two apertures and use this to provide the mimimum value of s in terms of the width of each maximum, w, such that the two maxima *do not overlap.*

b) Use this result to provide a minimum value of s in terms of λ, D and L such that the two maxima *do not overlap.*

c) Red light with wavelength 650 nm is incident on a piece with two small holes which each have diameter 1.0 mm. Determine the closest spacing between the holes so that they can be distinguished at a distance of 5.0 m.

220 Optical resolution and wavelength

A piece of foil has two closely spaced, small circular holes. These are illuminated from behind – the light that passes through is used to form an image of the holes. Which of the following is true? *(132S22)*

i) One would be less likely to be able to distinguish the images of the holes by using red light rather than blue light.

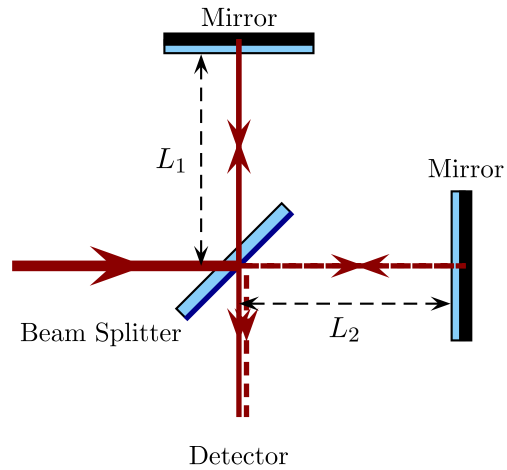
ii) One would be less likely to be able to distinguish the images of the holes by using blue light rather than red light.

iii) One would be equally likely to be able to distinguish the images of the holes regardless of the color of the light used.

Explain your answer.

221 Michelson interferometer

A Michelson interferometer is as illustrated. Initially the light used in the interferometer is produced by a HeNe laser with wavelength 632 nm. When the mirrors are adjusted so that $L_1 = L_2$ it is observed that the intensity of the light at the detector is a maximum. *(132S22 Class)*



a) Suppose that initially $L_1 = L_2$. What is the smallest distance that the moveable mirror must be shifted so that the light at the detector appears dark?

b) Suppose that initially $L_1 = L_2$. What is the smallest distance that the moveable mirror must be shifted so that the light at the detector again appears bright?

c) The light at the detector is initially bright. By how far must the moveable mirror be shifted so that the light at the detector cycles through 50 bright fringes?

d) Suppose that light from a different source is incident on the interferometer and when the moveable mirror is moved by 0.011 mm the light at the detector cycles through 40 bright fringes. Determine the wavelength of the light.

e) The LIGO observatory uses a Michelson inteferometer to detect gravitational waves. Such a wave would stretch the length of one "arm" by $\Delta L \approx 10^{-22} L$ where L is the length of the "arm." By repeatedly reflecting light back and forth down the arm the apparatus attains an arm length of 1000 km. Determine ΔL when a gravitational wave passes LIGO. Does it seem plausible to measure such a change in position?

f) Analysis of the interference of the two waves predicts that the intensity of the light at the detector is

$$I = I_0 \cos^2\left(\frac{2\pi \Delta L}{\lambda}\right).$$

Let $\Delta I = I_0 - I$ be the change in intensity as the gravitational wave passes. Determine the fractional change in intensity at the detector $\Delta I / I_0$. For small angles $\cos\theta \approx 1 - \theta^2/2$. Assuming $\lambda = 10^{-6}\,\text{m}$ determine $\Delta I / I_0$. Does it seem plausible to measure such a change?

222 Michelson interferometer

Light with wavelength λ is incident on a Michelson interferometer and the resulting pattern at the detector displays a dark spot. The single moveable mirror is then moved so that the pattern cycles exactly once from a dark spot to a bright spot. Which of the following is the distance moved by the mirror? Explain your answer. *(132S22)*

i) λ

ii) $\frac{3\lambda}{4}$

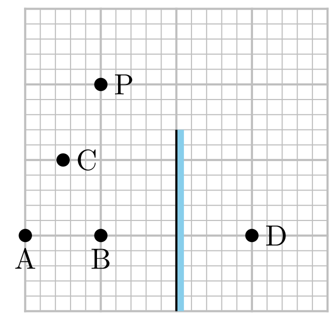
iii) $\frac{\lambda}{2}$

iv) $\frac{\lambda}{4}$

Geometric Optics

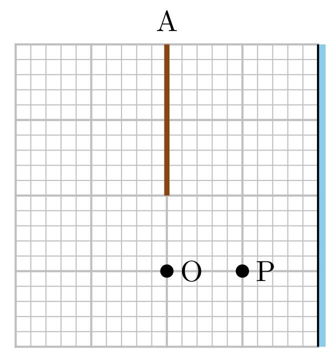
223 Image formation by a flat mirror

An object labeled P is held in front of a flat mirror as illustrated. Small observers are located at the points labeled A, B, C and D. Indicate the location of the image of P produced by the mirror. Which observers will be able to see the image? Explain your answers. *(132S22)*



224 Visible image in a mirror

A stick labeled A is held parallel to a flat mirror as illustrated. Two small observers are located at the points labeled O and P. In the following, explain your answers. *(132S22)*



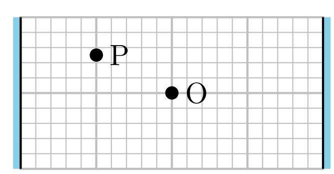
a) Suppose that the upper half of the mirror surface is covered by an opaque material. How much of the image of the stick will each observer be able to see?

b) Suppose that the lower half of the mirror surface is covered by an opaque material. How much of the image of the stick will each observer be able to see?

c) Suppose that the lower half of the mirror surface is covered by an opaque material and O moves away from the mirror (in a direction perpendicular to the mirror). As O moves away from the mirror does more or less of the image become visible?

225 Image formation by parallel mirrors

Two mirrors are parallel as illustrated. The distance between them is 4.0 m. An object O is placed midway between the mirrors. Another object is placed at a quarter of the distance between the two mirrors. *(132S22)*



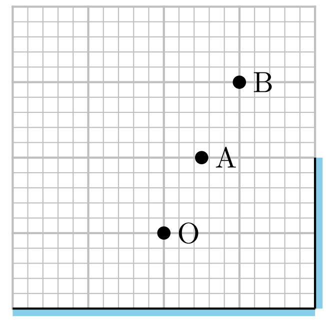
a) Determine the distance between successive images of O on either side.

b) Are images of P on the right evenly spaced? Explain your answer.

c) Are images of P on the left evenly spaced? Explain your answer.

If you can determine a formula for the locations of all the images for point P that will be impressive!

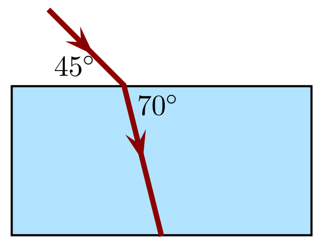
226 Image formation by multiple mirrors



Two mirrors are oriented at right angles as illustrated. An object O is placed at the indicated point. Two observers, A and B are located as illustrated. *(132S22)*

a) Recreate the diagram accurately and indicate the locations of all the images of the object O. Explain your answer.
b) Which of the images can observer A see? Explain your answer.
c) Which of the images can observer B see? Explain your answer.

227 Index of refraction

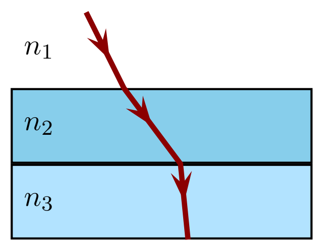


A laser beam in air is incident on a transparent rectangular block as illustrated. The angles between the surfaces and the rays are as illustrated. *(132S22)*

a) Determine the index of refraction of the material from which the block is made.

b) Determine the speed of light in the block.

228 Refraction through multiple materials



Two rectangular materials are stacked as illustrated. The indices of refraction of the materials and the surrounding medium are as illustrated. A light beam follows the indicated path. Which of the following is true regarding the indices of refraction? Explain your answer. *(132S22)*

i) $n_3 < n_1 < n_2$

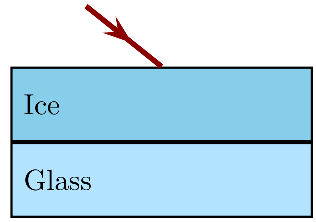
ii) $n_1 < n_3 < n_2$

iii) $n_1 < n_2 < n_3$

iv) $n_3 < n_2 < n_1$

v) $n_2 < n_1 < n_3$

229 Refraction through ice and glass



A rectangular slab of ice, with index of refraction 1.33 lies on top of a rectangular block of glass with index of refraction 1.55. Light traveling through the air is incident in the ice at an angle of 50° from the normal. *(132S22)*

a) Determine the angle at which the light travels through the glass.
b) Determine the angle at which the light emerges from the glass.

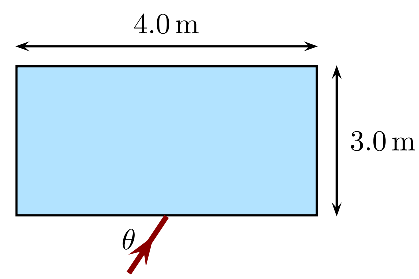
230 Optical fiber

An optical fiber contains a cylindrical core with radius 13 mm. This is surrounded by a inner cladding with radius 14 mm and this is surrounded by an outer cladding with radius 18 mm. The index of refraction of the core is 1.522, that of the inner cladding is 1.343 and that of the outer cladding is 1.484. Consider light traveling through the core that is incident on the inner cladding. This light can be scattered off impurities in the inner core and hit the boundary between the core and inner cladding at various angles. *(132S22)*

a) Determine the minimum angle between the core and the inner cladding such that no light will be transmitted into the inner cladding.

b) Consider light that emerges from the outer cladding. Determine the range of possible angle (from the surface) at which it can emerge.

231 Refraction at multiple surfaces

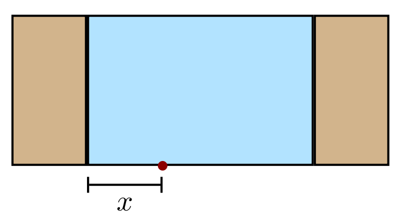


A rectangular block of glass with index of refraction 1.55 is placed in air. The block has height 3.0 m and length 4.0 m. A light ray is incident midway along the length of the block at an angle θ with respect to the surface. *(132S22)*

a) Determine where the light emerges back into the air and the direction in which it emerges the block if $\theta = 70°$.

b) Determine where the light emerges back into the air and the direction in which it emerges the block if $\theta = 10°$. Remember that light can be reflected and refracted at each interface.

232 Bees looking into a pool



A small light source (a fluorescent fish) sits on the bottom of a rectangular pool, whose depth is d and length is L. The distance from the fish to the edge of the pool is x. A swarm of bees hovers over ground to the left of the pool. *(132S22)*

a) Indicate, using angles from the surface, the region in which the bees must be in order to see the fish. Derive an expression for the limiting angle such that beyond this, no bee can see the fish.

b) Suppose that the depth of the pool is 1.5 m. Determine the minimum distance x such that any bee can see the fish.

c) For a general depth, determine the minimum distance x (in terms of d) such that any bee can see the fish.

233 Critical angle

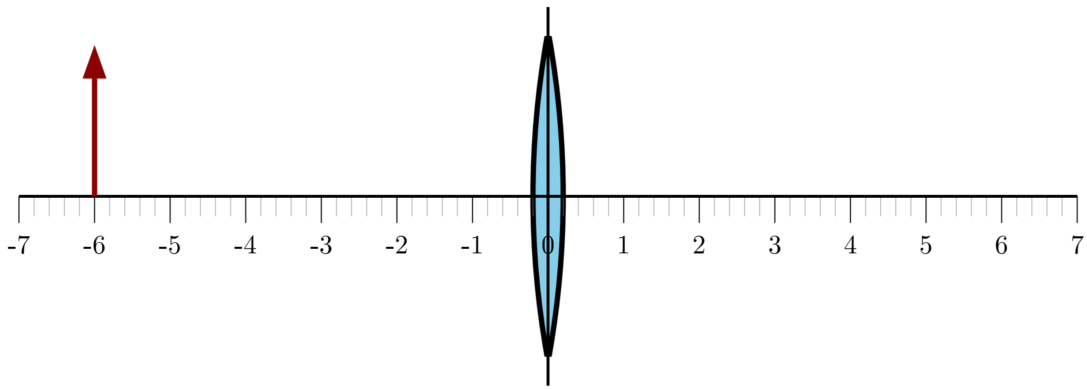
Light passes from water, with index of refraction 1.33, into air, with index of refraction 1.00. *(132S22 Class)*

a) Determine the angle between the refracted ray and the normal for the following angles between the incident ray and the normal: $30^\circ, 45^\circ, 60^\circ$.

b) Determine the maximum angle between the normal and the incident ray such that some light is refracted into the air.

234 Image formation by a convex lens: object beyond focal point

A convex lens has focal length 2.0 cm. An arrow with height 2.0 cm is placed 6.0 cm left of the lens. *(132S22 Class)*



a) Trace two rays from the tip of the arrow to determine where the image of the tip is produced.

b) Determine the distance from the lens plane to the image of the arrow.

c) Determine the height of the image of the arrow. Determine the magnification

$$m := \frac{h'}{h}$$

where h is the height of the object and h' is the height of the image.

The thin lens equation relates the positions of the object and the image via

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

where s is the distance from the lens to the object and s' is the distance from the lens to the image.

d) Use the thin lens equation to predict the location of the image. Check this against your diagram.

e) The magnification equation predicts

$$m = -\frac{s'}{s}.$$

Use this to predict the magnification and the height of the image. Check this against your diagram.

235 Image formation by a convex lens: object beyond focal point

An object with height 2.0 cm is placed 5.0 cm to the left of a convex lens with focal length 3.0 cm. *(132S22)*

a) Determine the location and height of the image using an accurate ray-tracing diagram.

b) Determine the exact location and height of the image by using equations.

236 Image formation by a convex lens: thin-lens equation and magnification

An object is placed beyond the focal point of a convex lens with focal length f. *(132S22)*

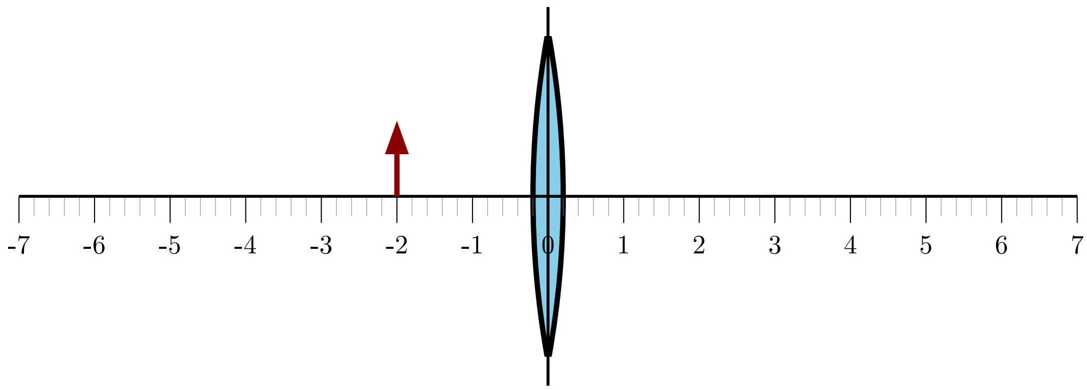
a) Use the thin-lens equation to determine an expression for the location of the image in terms of the location of the object and the focal length of the lens.

b) Using *the equation from the previous part* describe what happens to the distance from the image to the lens as the distance from the object to the lens increases.

c) Using *the equation from the previous part* predict what happens to the distance from the image to the lens as the object recedes infinitely far from the lens.

237 Image formation by a convex lens: object between lens and focal point

A convex lens has focal length 6.0 cm. An arrow with height 1.0 cm is placed 2.0 cm left of the lens. *(132S22 Class)*



a) Trace two rays from the tip of the arrow to determine where the image of the tip is produced.

b) Determine the distance from the lens plane to the image of the arrow.

c) Determine the height of the image of the arrow. Determine the magnification

$$m := \frac{h'}{h}$$

where h is the height of the object and h' is the height of the image.

The thin lens equation relates the positions of the object and the image via

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

where s is the distance from the lens to the object and s' is the distance from the lens to the image.

d) Use the thin lens equation to predict the location of the image. Check this against your diagram.

e) The magnification equation predicts

$$m = -\frac{s'}{s}.$$

Use this to predict the magnification and the height of the image. Check this against your diagram.

238 Image formation by a convex lens: object between lens and focal point

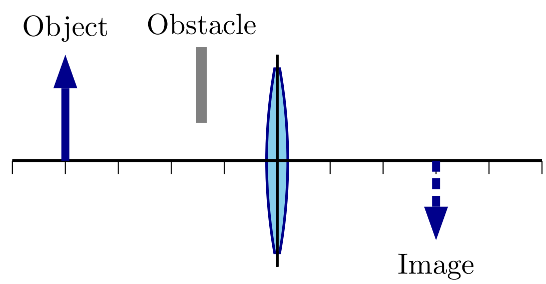
An object with height 2.0 cm is placed 1.0 cm to the left of a convex lens with focal length 3.0 cm. *(132S22)*

a) Determine the location and height of the image using an accurate ray-tracing diagram.

b) Determine the exact location and height of the image by using equations.

239 Obstructed lens

An object is placed beyond the focal point of a converging lens and an image is produced as illustrated. An rectangular piece of opaque material obstacle is placed in front of the lens, obscuring the lens to various degrees. Suppose that the obstacle covers the top half of the lens. How will this affect the appearance of the image compared to when there is no obstacle present? Explain your answer.



240 Images formed by a convex lens: object between lens and focal point

An object is placed to the left of a convex lens. The object is between the lens and the focal point. *(132S22)*

a) The object is moved closer to the lens. Which of the following is true? Explain your choice.

   i) The image stays in the same place as the object is moved.
   ii) The image moves towards the lens as the object is moved.
   iii) The image moves away from the lens as the object is moved.
   iv) The image is always at the focal point.

b) The object is moved closer to the lens. Which of the following is true? Explain your choice.

   i) The image height increases as the object is moved.
   ii) The image height decreases as the object is moved.
   iii) The image stays constant as the object is moved.

c) Explain whether it is possible for the image to be located between the object and the lens.

d) Explain whether it is possible for the image to be larger than the object (while the object is between the lens and the focal point).

241 Convex lens magnification possibilities

An inventor proposes to use a convex lens to produce an upright image of an object with a magnification less than 1. Use a ray tracing diagram to explain whether this is possible or impossible. *(132S22)*

242 Convex lens magnification

Consider a convex lens with focal length $f > 0$ and object located at $s > 0$.

a) Show that the magnification produced by the lens is:

$$m = \frac{f}{f - s}.$$

b) Determine the maximum value of the magnification and the location of the object when this occurs.

c) Determine the magnification as the object approaches the lens. *(132S22)*

243 Magnified images produced by convex lenses, 1

An object is placed left of a convex lens with focal length f. *(132S22)*

a) Determine, in terms of f, where the object must placed in order to produce an image with magnification -1.

b) Determine, in terms of f, where the object must placed in order to produce an image with magnification $\frac{1}{2}$.

244 Magnified images produced by convex lenses, 2

An object is placed left of a convex lens with focal length f. *(132S22)*

a) Determine, in terms of f, where the object must placed in order to produce an image with magnification -3.

b) Determine, in terms of f, where the object must placed in order to produce an image with magnification 3.

c) Determine, in terms of f, where the object must placed in order to produce an image with magnification $-\frac{1}{3}$.

245 Projector, 1

A projector uses a convex lens to form an image of an object that is 4.0 cm tall. The image is 1.2 m tall. The distance from the lens to the screen is 3.0 m. Determine the focal length of the lens needed to do this. *(132S22)*

246 Projector, 2

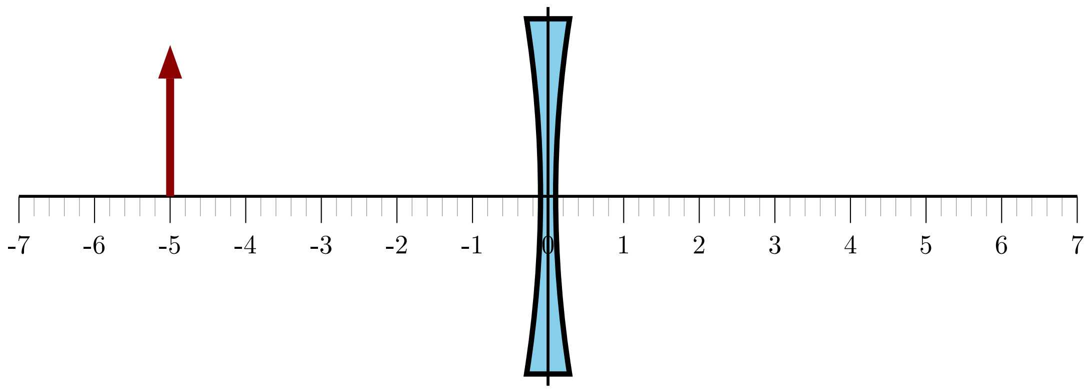
A projector uses a convex lens with focal length 15 cm to form an image of an object, that is inside the projector behind the lens, on a distant screen. The distance between the object and image is fixed at 2.2 m. The lens can be shifted back and forth to produce a clear image. *(132S22)*

a) Determine the distance between the object and lens so that a clear image is produced on the screen.

b) Determine the magnification of the image.

247 Image formation by a concave lens

A concave lens has focal length $-3.0\,\text{cm}$. An arrow with height $2.0\,\text{cm}$ is placed $5.0\,\text{cm}$ left of the lens. *(132S22 Class)*



a) Trace two rays from the tip of the arrow to determine where the image of the tip is produced.

b) Determine the distance from the lens plane to the image of the arrow.

c) Determine the height of the image of the arrow. Determine the magnification

$$m := \frac{h'}{h}$$

where h is the height of the object and h' is the height of the image.

The thin lens equation relates the positions of the object and the image via

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

where s is the distance from the lens to the object and s' is the distance from the lens to the image.

d) Use the thin lens equation to predict the location of the image. Check this against your diagram.

e) The magnification equation predicts

$$m = -\frac{s'}{s}.$$

Use this to predict the magnification and the height of the image. Check this against your diagram.

248 Images formed by a concave lens

An object is placed to the left of a concave lens. *(132S22)*

a) The object is moved closer to the lens. Which of the following is true? Explain your choice.

   i) The image stays in the same place as the object is moved.
   ii) The image moves towards the lens as the object is moved.
   iii) The image moves away from the lens as the object is moved.
   iv) The image is always at the focal point.

b) The object is moved closer to the lens. Which of the following is true? Explain your choice.

   i) The image height increases as the object is moved.
   ii) The image height decreases as the object is moved.
   iii) The image stays constant as the object is moved.

c) Explain whether it is possible for the image to be located left of the object.

d) Explain whether it is possible for the image to be larger than the object (while the object is between the lens and the focal point).

249 Concave lens: image formation

A concave lens is used to form an image of an upright object. *(132S22)*

a) The following refer to the focal point that is on the same side of the lens as the object. Which of the following is true? Explain your choice.

   i) The image will be upright regardless of where the object is located.
   ii) The image will be inverted regardless of where the object is located.
   iii) The image will be upright when the object is beyond the focal point of the lens and inverted when the object is between the focal point and the lens.
   iv) The image will be inverted when the object is beyond the focal point of the lens and upright when the object is between the focal point and the lens.

b) The following refer to the focal point that is on the same side of the lens as the object. Which of the following is true? Explain your choice.

   i) The image will be between the focal point and the lens regardless of where the object is located.
   ii) The image will be beyond the focal point regardless of where the object is located.
   iii) The image could be either side of the focal point depending on where the object is located.

250 Image formation by a concave lens

An object with height 2.0 cm is placed 6.0 cm to the left of a concave lens with focal length −8.0 cm. *(132S22)*

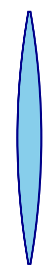
a) Determine the location and height of the image using an accurate ray-tracing diagram.

b) Determine the exact location and height of the image by using equations.

251 Lens in water

A convex lens is constructed using glass for which the index of refraction is 1.50 and when placed in air it has a focal length of 48 mm. Suppose that the lens is placed in water. Does the focal length remain the same or change? If it changes, describe whether it is larger than or smaller than 48 mm. Explain your answer. *(132S22)*

252 Focal length of a bi-convex lens



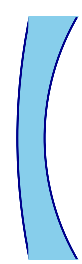
Consider the illustrated lens which is made of glass, for which the index of refraction is 1.50. The lens is surrounded by air. *(132S22)*

a) Suppose that the magnitude of the radius of curvature of the left side is 30 cm and of the right side it is 60 cm. Determine the focal length of the lens for an object placed to its left.

b) Suppose that the lens is flipped so that the magnitude of the radius of curvature of the left side is 60 cm and of the right side it is 30 cm. Determine the focal length of the lens for an object placed to its left.

c) Suppose that the *magnitude* of the radius of curvature of the left side is R_{left} and the *magnitude* of the radius of curvature of the right side is R_{right}. Does the focal length of the lens depend on whether the object is left or right of the lens? Explain your answer.

253 Focal length of a convex/concave lens



Consider the illustrated lens, for which the index of refraction is 1.50. The lens is surrounded by air. *(132S22)*

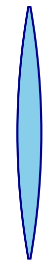
a) Suppose that the magnitude of the radius of curvature of the left side is 30 cm and of the right side it is 20 cm. Determine the focal length of the lens for an object placed to its left.

b) Suppose that the lens is flipped so that the magnitude of the radius of curvature of the left side is 20 cm and of the right side it is 30 cm. Determine the focal length of the lens for an object placed to its left.

c) Suppose that the *magnitude* of the radius of curvature of the left side is R_{left} and the *magnitude* of the radius of curvature of the right side is R_{right}. Does the focal length of the lens depend on whether the object is left or right of the lens? Explain your answer.

254 Focal length of a lens in different media

Consider the illustrated lens which is made of glass, for which the index of refraction is 1.50. The radius of curvature of each side is 50 cm. *(132S22)*



a) Suppose that the lens is in air (index of refraction 1.00). Determine the focal length of the lens.
b) Suppose that the lens is in water (index of refraction 1.33). Determine the focal length of the lens.

255 Nearsightedness

A nearsighted person has a far point of 1.5 m and a normal near point of 0.25 m. A single corrective lens is used to allow that person to view objects that are infinitely distant. Assume that the lens is placed against the eye.

a) Determine the focal length of the lens so that the person can view an object that is infinitely distant. Is the image of this object (created by the corrective lens) larger or smaller than the object?
b) Determine the location of the closest object that the person can see clearly.
c) With this corrective lens, what is the range of vision of the person?

256 Farsightedness

A farsighted person has a far point of 0.35 m and a far point at infinity. A single corrective lens is used to allow that person to view objects that are at the normal near point, 0.25 m. Assume that the lens is placed against the eye.

a) Determine the focal length of the lens so that the person can view an object that is at the normal near point. Is the image of this object (created by the corrective lens) larger or smaller than the object?
b) Determine the location of the furthest object that the person can see clearly.
c) With this corrective lens, what is the range of vision of the person?

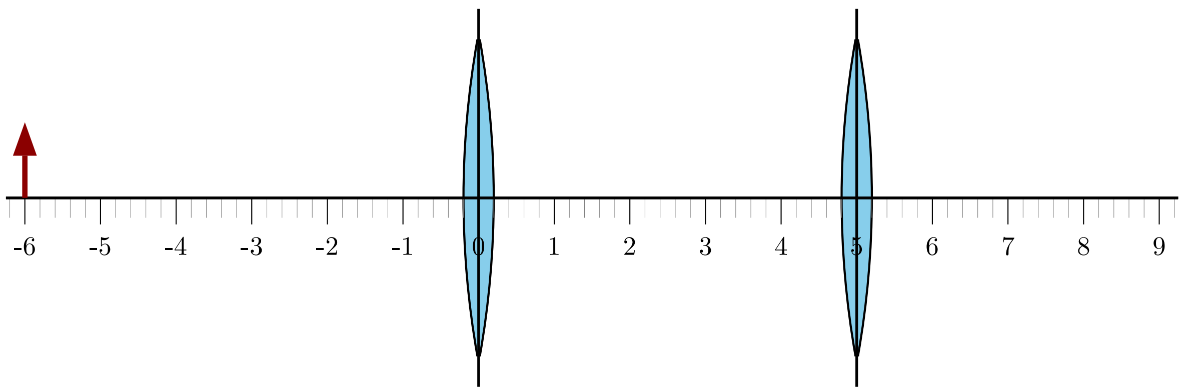
257 Magnifier

A magnifier produces an angular magnification of 10 for a person with a normal near point.

a) Determine the focal length of the lens.
b) Determine the angular magnification for someone with a near point of 0.40 m.

258 Image formation by two convex lenses

Two convex lenses are arranged as illustrated. The lens on the left has focal length 2.0 cm and the lens on the right has focal length 4.0 cm. The lenses are 5.0 cm apart. An arrow with height 1.0 cm is placed 6.0 cm left of the lens on the left.



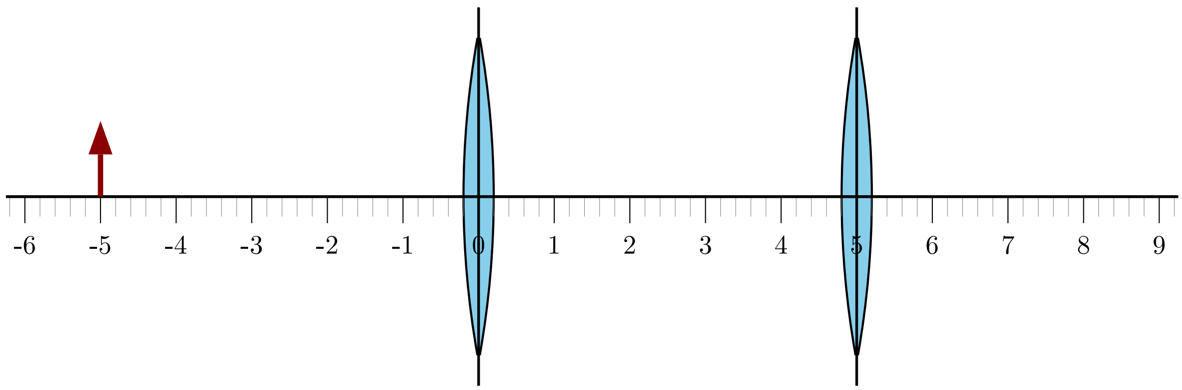
a) Trace two rays from the tip of the arrow to determine the image produced by the lens combination.

b) Use equations to determine the location of the image produced by the lens combination.

c) Use equations to determine the height of the image produced by the lens combination.

259 Image formation by two convex lenses

Two convex lenses are arranged as illustrated. The lens on the left has focal length 1.0 cm and the lens on the right has focal length 1.5 cm. The lenses are 5.0 cm apart. An arrow with height 1.0 cm is placed 5.0 cm left of the lens on the left.



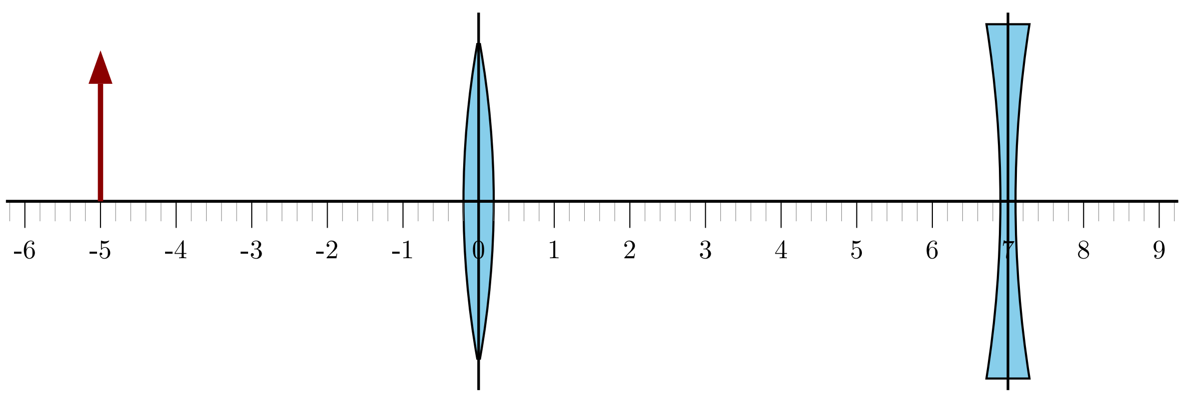
a) Trace two rays from the tip of the arrow to determine the image produced by the lens combination.

b) Use equations to determine the location of the image produced by the lens combination.

c) Use equations to determine the height of the image produced by the lens combination.

260 Image formation by a convex/concave lens combination

Two lenses are arranged as illustrated. The lens on the left has focal length 2.0 cm and the lens on the right has focal length −3.0 cm. The lenses are 7.0 cm apart. An arrow with height 2.0 cm is placed 5.0 cm left of the lens on the left.



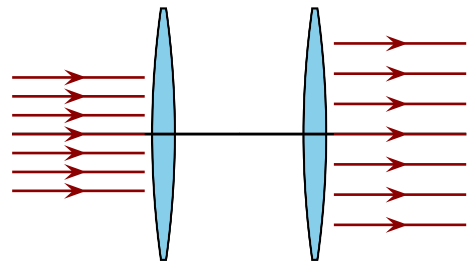
a) Trace two rays from the tip of the arrow to determine the image produced by the lens combination.

b) Use equations to determine the location of the image produced by the lens combination.

c) Use equations to determine the height of the image produced by the lens combination.

261 Beam expander

A beam expander consists of two convex lenses. It is configured so that light rays that enter the expander parallel to the optical axis from the left emerge parallel to the optical axis on the right. Suppose that the focal length of the lens on the left is f_1 and the focal length of the lens on the right is f_2.



a) Determine an expression for the separation of the lenses in terms of f_1 and f_2 so that the incoming and outgoing beams are both parallel to the optical axis.

b) Determine an expression for the factor by which the beam width increases (provided that the incoming and outgoing beams are both parallel to the optical axis).