Phys 132 2022

Lecture 35

Weds: Discussion / quiz

Ex: 201, 202, 205, 206

208, 209, 210, 211

Thurs: Warm Up 13

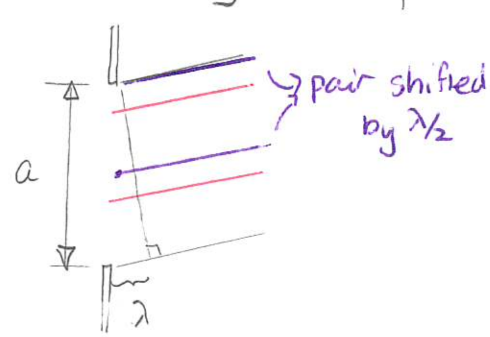
Ex: Class Ave 71% Q2, Q4, Q9

Single slit diffraction

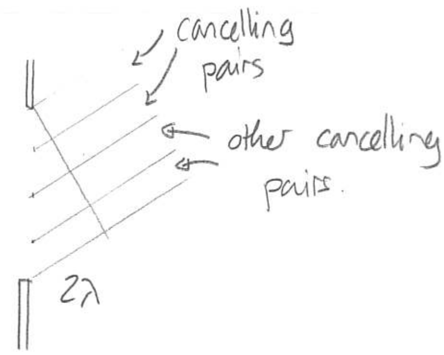
Interference phenomena occur with light incident on just one aperture, such as a single slit. This is called diffraction. The analysis of this is based on

From each point in the aperture, there is a wave that propagates spherically outward.

We can then consider how this multitude of waves overlaps and interferes. The easiest conditions to check are for complete cancellation, which occurs when waves shifted by $\lambda/2$ "pair up".



The collection of waves is divided into 2. Upper half shifted by $\lambda/2$ versus lower half



Divide collection into four. Each shifted by $\lambda/2$

Then geometry / trigonometry give

$$\boxed{\begin{array}{l}\text{Perfect cancellation when} \\ \left.\begin{array}{l} a \sin \theta_1 = \lambda \\ a \sin \theta_2 = 2\lambda \\ \vdots \end{array}\right\} \quad a \sin \theta_p = p\lambda \qquad p = \pm 1, \pm 2, \pm 3, \ldots \\ \text{where } a = \text{slit width}\end{array}}$$

Quiz1 20% → 80%

Quiz2

There is a formalism which allows one to evaluate the intensities at intermediate locations and this readily yields a plot of intensity versus position on the screen.

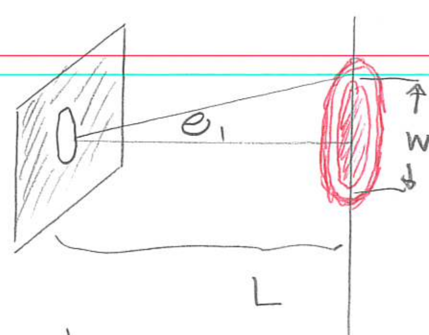
Demo: Double / Single Slit Intensity Profiles

Diffraction from a circular aperture / obstacle

Now consider light passing through a circular aperture. The same interference phenomena occur. The pattern will have circular symmetry

Demo: Hyperphysics

image



The detailed analysis reveals that first order dark fringes occur when

$$\theta_1 \approx 1.22 \frac{\lambda}{D} \qquad \text{(radians)}$$

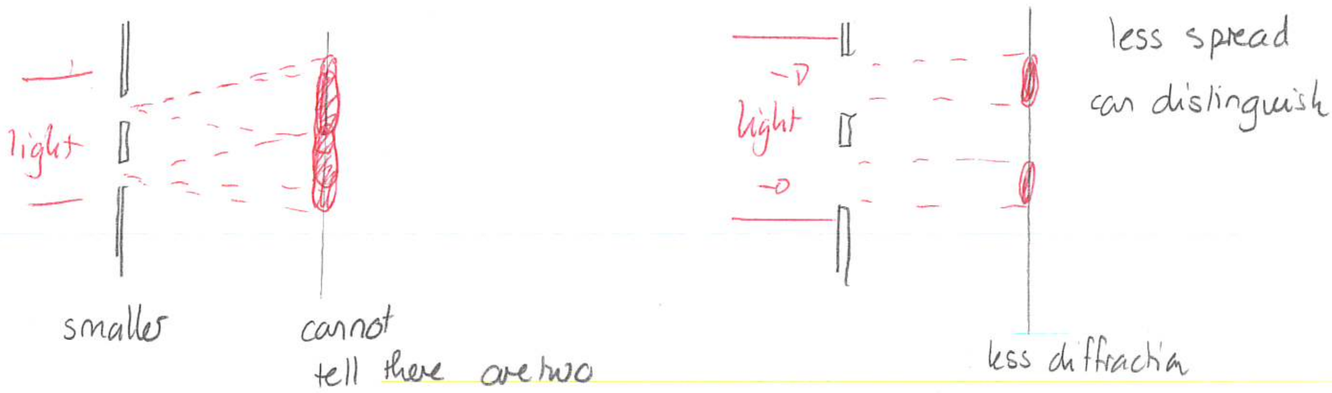
where D is the aperture diameter

Then approximately the width of the central max is:

$$w \approx 2.44 \lambda \frac{L}{D}$$

where L is the distance to the screen.

This will establish the resolution limits for optics. Consider two small adjacent holes



Quiz 3

Demo: Poisson Spot

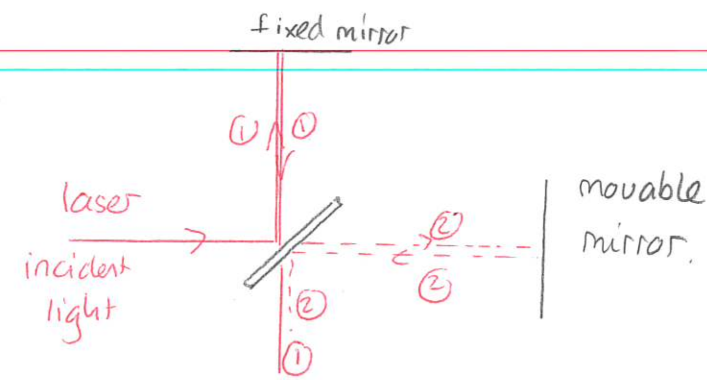
Interferometry

An interferometer is an optical device that splits and recombines light. The two recombining parts will interfere.

Slide: MZ Slide

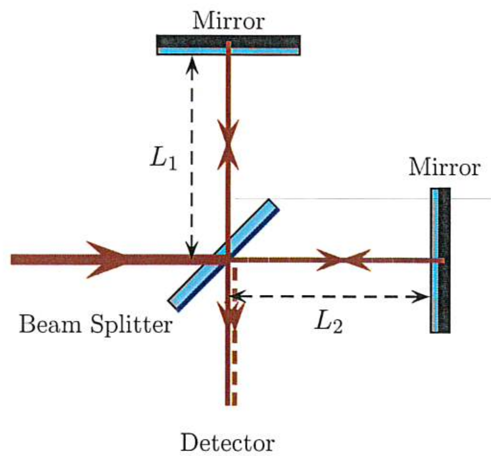
A Michelson Interferometer uses a different configuration

Waves ① and ② interfere and how this occurs depends on the different distances traveled by ① and ②



220 Michelson interferometer

A Michelson interferometer is as illustrated. Initially the light used in the interferometer is produced by a HeNe laser with wavelength 632 nm. When the mirrors are adjusted so that $L_1 = L_2$ it is observed that the intensity of the light at the detector is a maximum.



a) Suppose that initially $L_1 = L_2$. What is the smallest distance that the moveable mirror must be shifted so that the light at the detector appears dark?

b) Suppose that initially $L_1 = L_2$. What is the smallest distance that the moveable mirror must be shifted so that the light at the detector again appears bright?

c) The light at the detector is initially bright. By how far must the moveable mirror be shifted so that the light at the detector cycles through 50 bright fringes?

d) Suppose that light from a different source is incident on the interferometer and when the moveable mirror is moved by 0.011 mm the light at the detector cycles through 40 bright fringes. Determine the wavelength of the light.

e) The LIGO observatory uses a Michelson inteferometer to detect gravitational waves. Such a wave would stretch the length of one "arm" by $\Delta L \approx 10^{-22} L$ where L is the length of the "arm." By repeatedly reflecting light back and forth down the arm the apparatus attains an arm length of 1000 km. Determine ΔL when a gravitational wave passes LIGO. Does it seem plausible to measure such a change in position?

f) Analysis of the interference of the two waves predicts that the intensity of the light at the detector is

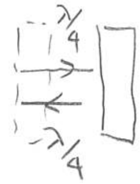
$$I = I_0 \cos^2 \left(\frac{2\pi \Delta L}{\lambda} \right).$$

Let $\Delta I = I_0 - I$ be the change in intensity as the gravitational wave passes. Determine the fractional change in intensity at the detector $\Delta I / I_0$. For small angles $\cos\theta \approx 1 - \theta^2/2$. Assuming $\lambda = 10^{-6}$ m determine $\Delta I / I_0$. Does it seem plausible to measure such a change?

Answer a) Wave 2 must be shifted by $\lambda/2$

$\Rightarrow$ extra distance traveled is $\lambda/2$

$\Rightarrow$ mirror shifted by $\lambda/4$



$$\Rightarrow \frac{632\,nm}{4} = 158\,nm$$

b) Shifted by $\lambda/2$ $\Rightarrow$ $\frac{632\,nm}{2} = 316\,nm$.

c) exactly 50 of the shifts in b $\Rightarrow$ $50 \times \lambda/2 = 25\lambda$

$\Rightarrow 25 \times 632\,nm = 15.8\,\mu m$

d) shift $= 40 \times \lambda/2 = 20\lambda$

$\Rightarrow 0.011 \times 10^{-3}\,m = 20\lambda$

$$\Rightarrow \lambda = \frac{0.011 \times 10^{-3}}{20} = 550\,nm$$

e) $\Delta L = 10^{-22} \times 10^{3}\,km = 10^{-22} \times 10^{6}\,m = 10^{-16}\,m$ smaller than an atom

f) $\Delta I = I_0 - I = I_0\left[1 - \cos^2(\ldots)\right]$

$$\frac{\Delta I}{I_0} = 1 - \cos^2\left(\frac{2\pi \Delta L}{\lambda}\right) \approx 1 - \left[1 - \left(\frac{2\pi\Delta L}{\lambda}\right)^2 \frac{1}{2}\right] = \frac{4\pi^2 \Delta L^2}{2\lambda^2}$$

$$\Rightarrow \frac{\Delta I}{I_0} = \frac{2\pi^2 \Delta L^2}{\lambda^2} = \frac{2\pi^2 (10^{-16}\,m)^2}{(10^{-6}\,m)^2} = 1.9 \times 10^{-19}$$

very small change.