Weds: Discussion /quiz

Ex 179, 180, 181, 182, 183, 184, 185

Thurs: Warm Up 11

Waves in one dimension Ch 16

We saw that a snapshot of waves in one dimension can be represented by

$$\boxed{y = A \sin(kx + \phi)}$$

where A = amplitude

k = wavenumber $= 2\pi/\lambda$ λ is wavelength

ϕ = phase.

We now consider the temporal evolution of waves.

Temporal evolution of waves

If we consider the animation we observe that, as time passes, the pattern appears to move from left to right. We also observe that individual points move up + down

Demo: W.o.a.S

* Set up with $f = 1.25\ \text{Hz}$
* observe single bead
* observe crests

There are equivalent ways of describing certain aspects of the temporal evolution.

Observe a single point

Point oscillates up/down

Period of oscillation

$T :=$ time for one complete cycle

Units: s

Frequency of wave

$f := \frac{1}{T}$

Hertz.

Units: Hz

~ Number of cycles completed per second

Observe crests moving past a single point

Period of wave

$T :=$ time between passage of successive crests past point

same

Number of crests passing a given point per second

In both cases the angular frequency is

$$\boxed{\omega := 2\pi f}$$

Quiz 1 80% – 100%

If we were to observe the vertical position at one location, we could describe it via

$$A \sin(\omega t) \quad \text{OR} \quad A \sin(\omega t + \phi).$$

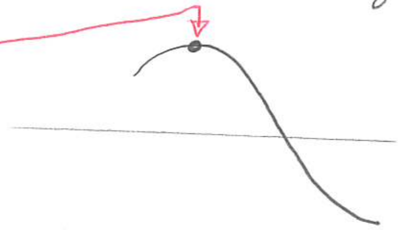
Spatial and temporal description of sinusoidal waves

Combining the spatial and temporal descriptions eventually gives a displacement

$$\boxed{y = A\sin(kx - \omega t + \phi)}$$

We can see by the following that this corresponds to a travelling wave. Suppose that

$$kx - \omega t + \phi = \pi/2$$



This represents a point for which $y = A$. As t increases, x would increase so as to follow this point. Then we get

$$kx = \omega t + \pi/2 - \phi$$

$$x = \frac{\omega}{k} t + \frac{\pi/2 - \phi}{k}.$$

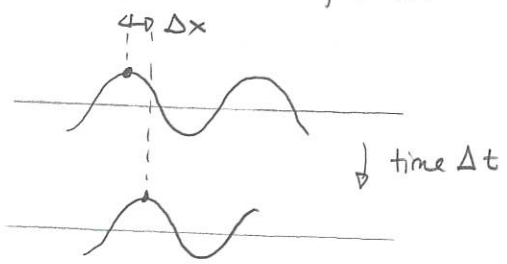
We can see that x increases linearly with t. This describes the crest moving to the right.

Demo: W.o.a.S

* show crest traveling

We define the wavespeed as the speed with which the pattern moves. Thus

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$



Then we obtain

$$\frac{dx}{dt} = \frac{\omega}{k} \quad \Rightarrow \quad v = \omega / k$$

Thus

$$\boxed{\begin{array}{l}\text{For any sinusoidal wave, the wavespeed satisfies} \\ \quad v = \omega/k \quad \Rightarrow \quad v = \lambda f\end{array}}$$

Quiz 2 80%

Electromagnetic Waves

Maxwells equations describe how source charges and currents produce electric + magnetic fields. They predict that electric and magnetic fields can exist in a complete vacuum. One can take Maxwell's equations and manipulate these mathematically in a vacuum region. We find that the resulting equations describe waves of electric and magnetic fields. They also describe the speed of the waves as

oscillating charge

produces oscillating fields where there is no matter.

$$\text{speed} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

where $\mu_0 = 4\pi \times 10^{-7}\ Tm/A$

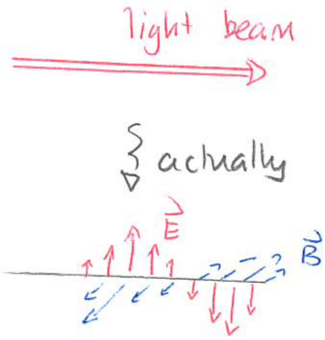
$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/N{\cdot}m^2$

The value that this predicts is roughly $3.0 \times 10^8 m/s$. This is the speed of light. So

$$c = 1/\sqrt{\mu_0 \epsilon_0}$$

An immediate conclusion is:

$\boxed{\text{Light is an electromagnetic wave}}$



We should thus be able to describe properties of light in terms of such electromagnetic waves.

Demo: PSU-S waves

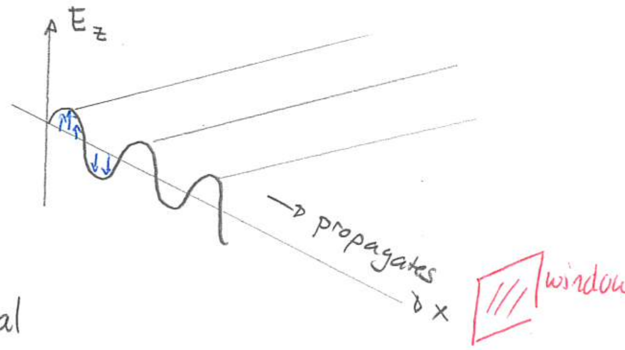
Note that there are a wide range of electromagnetic waves. They differ by

1) their frequencies and wavelengths
2) how they are produced and detected.

Intensity of electromagnetic waves

We can ask what the wave actually transports. Classical electromagnetic theory predicts that it transports energy and it describes precisely how to determine this from the electric and magnetic fields. The simplest situation is a wave of the form

$$\vec{E} = \vec{E}_0 \sin(kx - \omega t)$$



We consider the energy passing through a window with area A perpendicular to the direction of propagation. Then classical electromagnetic theory predicts that

The rate at which energy passes through the window

$$\equiv \text{Power} = P = c\epsilon_0 \vec{E}\cdot\vec{E} A$$

$$\Rightarrow P = c\epsilon_0 \vec{E}\cdot\vec{E} A$$

$$P = c\epsilon_0 E_0^2 A \sin^2(kx - \omega t)$$

This clearly fluctuates and we aim to resolve that fluctuation.