Phys 132   2022

Lecture 22

Weds: Review

Thurs: Exam II Ch 26.5 → 26.6

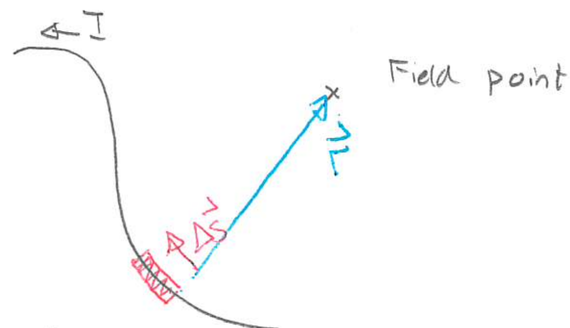
Ch 27, 28

Biot-Savart Law

The Biot-Savart Law gives a prescription for determining the magnetic field produced by a steady current by breaking the current into segments. Then

$$\vec{B}_{segment} = \frac{\mu_0}{4\pi} I \frac{\Delta\vec{s} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi} I \frac{\Delta\vec{s} \times \vec{r}}{r^3}$$



where $\hat{r}$ is the unit vector along $\vec{r}$.

The field produced by the entire current is

$$\vec{B} = \sum_{\text{all segments}} \vec{B}_{segment}$$

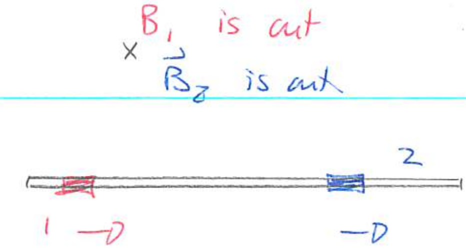
We now use this to determine the field produced by a straight section of current.

Quiz 1   100%

× $\vec{B}$ here is what?

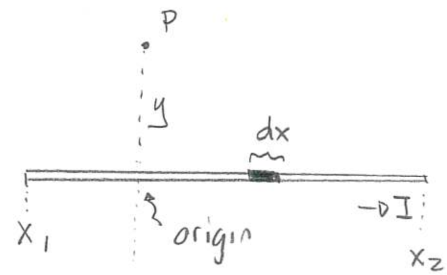
Warm Up 1

We see that the contributions at a given point from all segments are along the same direction.



Quiz 2 90%-

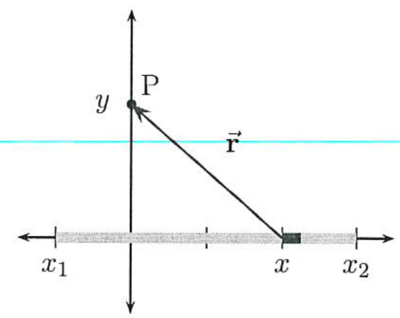
We consider the field produced at an arbitrary point above a segment



We set up the co-ordinates as illustrated. We consider the contribution from the shaded segment.

126 Magnetic field produced by a straight section of current

A straight section of wire carries current I flowing to the right. The aim of this exercise is to use the Biot-Savart law to determine the magnetic field at the illustrated point. First consider the contribution to the field from the shaded section, whose length is dx. This is



$$\vec{\mathbf{B}}_{\text{segment}} = \frac{\mu_0}{4\pi} I \frac{\Delta\vec{\mathbf{s}} \times \vec{\mathbf{r}}}{r^3}. \tag{4}$$

a) Express $\vec{\mathbf{r}}$ and $\Delta\vec{\mathbf{s}}$ in terms of x, y, dx and $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

b) Use these to determine expressions for $\vec{\mathbf{s}} \times \vec{\mathbf{r}}$ and r in terms of x, y, dx and $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$.

c) Set up an integral for the field produced by all segments.

Answer: a) $\vec{r} = -x\hat{i} + y\hat{j}$

$\Delta\vec{s} = dx\,\hat{i}$

b) $\Delta\vec{s} \times \vec{r} = dx\,\hat{i} \times [-x\hat{i} + y\hat{j}] = -x\,dx\,\cancel{\hat{i}\times\hat{i}}\ 0 + y\,dx\,\underbrace{\hat{i}\times\hat{j}}_{\hat{k}}$

$= y\,dx\,\hat{k}$

$r = \sqrt{x^2 + y^2}$

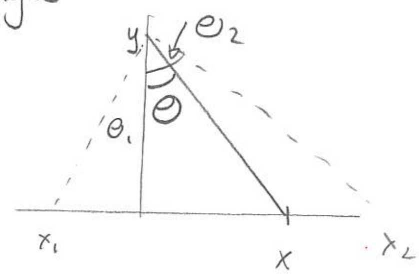
c) $\vec{B}_{\text{segment}} = \dfrac{\mu_0}{4\pi} I \dfrac{y\,dx\,\hat{k}}{(x^2+y^2)^{3/2}}$

Thus

$$\vec{B} = \frac{\mu_0}{4\pi} I \int_{x_1}^{x_2} \frac{y\,dx}{(x^2+y^2)^{3/2}} \hat{k}$$

$$= \frac{\mu_0}{4\pi} I y \hat{k} \int_{x_1}^{x_2} \frac{dx}{(x^2+y^2)^{3/2}}$$

The integral can be done using the illustrated angle



$$\frac{x}{y} = \tan\theta$$

$$x = y\tan\theta$$

$$\Rightarrow \quad dx = d\theta \frac{dx}{d\theta} = d\theta \; y \frac{1}{\cos^2\theta}$$

$$\vec{B} = \frac{\mu_0}{4\pi} I y \int_{\theta_1}^{\theta_2} \frac{1}{(y^2\tan^2\theta + y^2)^{3/2}} \, y \frac{1}{\cos^2\theta} \, d\theta$$

$$= \frac{\mu_0}{4\pi} I y^2 \int_{\theta_1}^{\theta_2} \frac{1}{y^3(1+\tan^2\theta)^{3/2}} \frac{1}{\cos^2\theta} \, d\theta$$

$$= \frac{\mu_0}{4\pi} \frac{I}{y} \int_{\theta_1}^{\theta_2} \frac{1}{(1+\tan^2\theta)^{3/2}} \frac{1}{\cos^2\theta} \, d\theta$$

Then $\quad 1+\tan^2\theta = 1+\frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

So

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{y}\int_{\theta_1}^{\theta_2}\frac{1}{(1/\cos^2\theta)^{3/2}}\frac{1}{\cos^2\theta}\,d\theta\ \hat{k}$$

$$= \frac{\mu_0}{4\pi}\frac{I}{y}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta\ \hat{k}$$

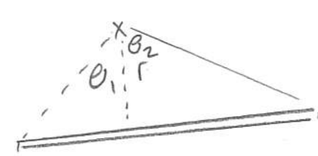
$$= \frac{\mu_0}{4\pi}\frac{I}{y}\sin\theta\Big|_{\theta_1}^{\theta_2}\ \hat{k}$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{I}{y}(\sin\theta_2-\sin\theta_1)\,\hat{k}$$

This gives:

The field above a finite straight segment is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r}\left[\sin\theta_2-\sin\theta_1\right]$$



with direction given by r.h. rule.

For an infinite wire $\quad \theta_1 = -\pi/2 \quad \theta_2 = \pi/2$ and this gives

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{r}\ (+\ \text{r.h. rule direction})$$

Warm Up 2