Phys 132
Spring 2022

Electromagnetism and Optics: Class Exam III

21 April 2022

Name: Solution **Total:** /70

Instructions

- There are 10 questions on 7 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

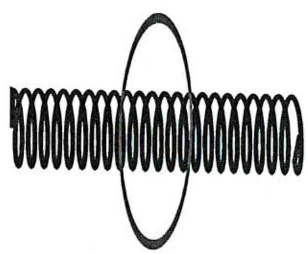
$$e = 1.61 \times 10^{-19}\,\mathrm{C} \qquad q_{\text{electron}} = -e \qquad q_{\text{proton}} = +e$$

$$m_{\text{electron}} = 9.11 \times 10^{-31}\,\mathrm{kg} \qquad m_{\text{proton}} = 1.67 \times 10^{-27}\,\mathrm{kg} \qquad c = 3.0 \times 10^{8}\,\mathrm{m/s}$$

$$k = 9.0 \times 10^{9}\,\mathrm{Nm^2/C^2} \qquad \epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/Nm^2} \qquad \mu_0 = 4\pi \times 10^{-7}\,\mathrm{Tm/A}$$

Question 1

An infinitely long solenoid and a separate single circular loop of wire are arranged so that their axes coincide. The loop has radius 0.50 m and carries a 4.0 A current. The solenoid has 400 turns per meter. Determine the current in the solenoid such that the net magnetic field in the center of the loop is zero.



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The fields must be opposite and equal in magnitude

$$B_{\text{solenoid}} = B_{\text{loop}}$$

$$\cancel{\mu_0} n I_{sol} = \frac{\cancel{\mu_0} I_{loop}}{2R} \Rightarrow I_{sol} = I_{loop} \frac{1}{2nR}$$

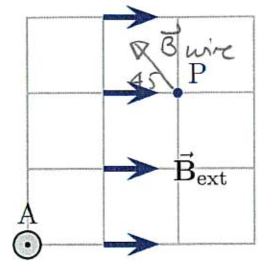
$$= I_{loop} \frac{1}{2 \times 400/\mathrm{m} \times 0.50\,\mathrm{m}}$$

$$= \frac{1}{400}\, 4.0\,\mathrm{A} = 0.010\,\mathrm{A}$$

Question 2

An infinitely long wire is perpendicular to the page and carries a 6.0 A current in the indicated direction. Separately a hidden external source produces the illustrated uniform magnetic field $\vec{\mathbf{B}}_{\mathrm{ext}}$ at all points; the magnitude of this field is $2.0 \times 10^{-\cancel{4}\,5}$ T. Determine the *magnitude* of the *net* magnetic field at the point labeled P. The grid units are each 0.010 m.

$\rightarrow 2.0 \times 10^{-5}\,\mathrm{T}$



There are two fields

$\vec{B}_{wire}$ and $\vec{B}_{ext}$

$\cancel{+1}$ +2

Then $\underbrace{\vec{B}_{net} = \vec{B}_{wire} + \vec{B}_{ext}}_{+1}$

The magnitude of the field produced by the wire is

$$B_{wire} = \frac{\mu_0 I}{\underbrace{2\pi r}_{+1}} = \frac{4\pi \times 10^{-7}\,\mathrm{Tm/A} \times 6.0\,\mathrm{A}}{2\pi \times 0.028\,\mathrm{m}}$$

$$r = \sqrt{2 \times 0.02^2} = \underbrace{0.028\,\mathrm{m}}_{+1}$$

$$= 4.3 \times 10^{-5}\,\mathrm{T} \quad +1$$

Then the components are

$$B_{wire\,x} = -B_{wire} \cos 45^\circ = -4.3 \times 10^{-5}\,\mathrm{T} \cos 45^\circ = -3.0 \times 10^{-5}\,\mathrm{T}$$

$$B_{wire\,y} = B_{wire} \sin 45^\circ = \ldots = 3.0 \times 10^{-5}\,\mathrm{T} \quad +2$$

So
$$\vec{B}_{net} = (-3.0 \times 10^{-5}\,\mathrm{T} + 2.0 \times 10^{-5}\,\mathrm{T})\,\hat{\imath} + 3.0 \times 10^{-5}\,\mathrm{T}\,\hat{\jmath}$$

$$= -1.0 \times 10^{-5}\,\mathrm{T}\,\hat{\imath} + 3.0 \times 10^{-5}\,\mathrm{T}\,\hat{\jmath} \quad +2$$

$$B_{net} = \sqrt{(1.0 \times 10^{-5}\,\mathrm{T})^2 + (3.0 \times 10^{-5}\,\mathrm{T})^2}$$

$$= 3.2 \times 10^{-5}\,\mathrm{T} \quad +2$$

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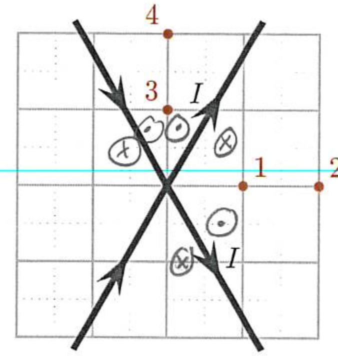
Question 3

Two infinitely long wires are oriented as illustrated and carry currents with the same magnitude. Let B_1 be the magnitude of the net magnetic field at the location 1, B_2 be the magnitude of the net magnetic field at the location 2, ... Which of the following (choose one) ranks the *magnitudes* of the net fields?



i) $B_1 = B_2 < B_3 = B_4$
ii) $B_1 = B_2 < B_4 < B_3$
iii) $B_1 = B_2 < B_3 < B_4$
iv) $B_2 = B_4 < B_1 = B_3$
v) $B_2 < B_4 < B_1 < B_3$

At 1,2 the fields cancel
$\Rightarrow B_1 = B_2 = 0$

At 3,4 the fields add

3 is closer $\Rightarrow B_3$ larger

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Question 4

A positively charged particle is fired directly toward the south pole of a bar magnet. The particle initially approaches the magnet along a line that runs midway through the magnet and parallel to the length of the magnet.

The arrow indicates the initial direction of motion. Which of the following (choose one) is true regarding the direction of the force exerted on the particle?

i) There is no force.
ii) The magnet attracts the particle.
iii) The magnet repels the particle.
iv) The magnet exerts a force up.

+2

Briefly explain your answer.

$\vec{B} \leftarrow$

$\vec{v} \leftarrow$

+2

$\vec{F} = q\vec{v} \times \vec{B}$ +2

parallel $\Rightarrow \vec{F} = 0$

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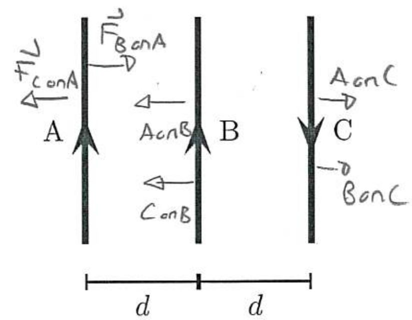
Question 5

Three wires, labeled A, B and C each carry currents in the indicated directions. The separation between adjacent wires is d. The current in wire A has magnitude I, that in wire B has magnitude I and that in wire C has magnitude $2I$. The length of each wire is L.



a) Assuming that the wires are close enough to approximate them as infinitely long, determine an expression for the *net force on each wire* in terms of I, d, L and constants.

Between two wires $F = \dfrac{\mu_0 I_1 I_2 L}{2\pi d}$ (+1) or comparable B then $\vec F = I\vec l \times \vec B$

Note directions

adding including directions... (+3)

$\vec F_{net\ on\ A} = \vec F_{B\ on\ A} + \vec F_{C\ on\ A}$

$$F_{net\,A} = \frac{\mu_0}{2\pi} L\left(\frac{II}{d} - \frac{2II}{2d}\right) = 0$$ (+2) (+1)

$\vec F_{net\ on\ B} = \vec F_{A\ on\ B} + \vec F_{C\ on\ B}$

$$F_{net\,B} = -\frac{\mu_0}{2\pi} L\left(\frac{II}{d} + \frac{2II}{d}\right)\hat{\imath}$$ (+1)

$$= -\frac{\mu_0}{2\pi}\frac{LI^2}{d}3\hat{\imath}$$ (+1)

(+1) (+1)

$\vec F_{net\ on\ C} = \vec F_{A\ on\ C} + \vec F_{B\ on\ C} \Rightarrow$

$$F_{net\ C} = \frac{\mu_0}{2\pi} L\left(\frac{I\cancel{2}I}{\cancel{2}d} + \frac{I2I}{d}\right)\hat{\imath}$$

$$= \frac{\mu_0}{2\pi}\frac{LI^2}{d}(3)\hat{\imath}$$

b) Rank the wires in order of increasing *magnitude of the net force* on the wire.

$F_{net\,A} < F_{net\,B} = F_{net\,C}$ (+1)

see above

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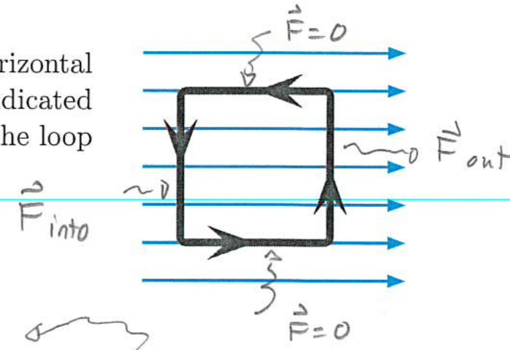
$\vec{F} = I\vec{l} \times \vec{B}$

on each edge

Question 6

A square loop is initially held at rest in the indicated horizontal magnetic field. A current flows around the loop in the indicated direction. The loop is then released. Immediately after the loop is released, which way (choose one) does it move?



i) The entire loop moves to the right.

ii) The entire loop moves out of the page.

iii) The loop rotates about an axis pointing vertically.

iv) The loop rotates clockwise as viewed looking at the page.

right side moves out, left side in

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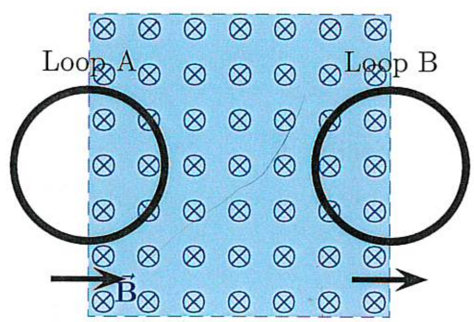
Question 7

There is a uniform constant magnetic field throughout the shaded region. Two loops identical loops are dragged at the at the same rate to the right. Which of the following (choose one) is true regarding the currents in the loops at the indicated moment?



i) $I_A = I_B = 0$.

ii) I_A is clockwise, I_B is clockwise.

iii) I_A is clockwise, I_B is counterclockwise.

iv) I_A is counterclockwise, I_B is clockwise.

v) I_A is counterclockwise, I_B is counterclockwise.

Briefly explain your answer.

Loop A

Flux increasing into

$\Rightarrow$ Induced field out

$\Rightarrow$ r.h.r. gives current is C.C.W

Loop B

Flux decreasing into

$\Rightarrow$ Induced field in

$\Rightarrow$ r.h.r. give c.w.

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Question 8

A circular loop is placed in a uniform magnetic field. The radius of the loop can be made to decrease as time, t, passes and it does so in such a way that the area of the loop is $A = 0.50\,t^3$ (the constant has units of $\mathrm{m^2/s^3}$). The field is always perpendicular to the loop and has a constant magnitude of 4.0 T. Determine the magnitude of the EMF induced in the loop at $t = 2.0\,\mathrm{s}$.

$\mathcal{E} = \left|\frac{d\Phi}{dt}\right|$ and $\Phi = AB\cos\theta$ so 0°

$= 0.50\,t^3\,4.0\,\mathrm{T}\cos 0^\circ$

$= 2.0\,\mathrm{m^2/s^3}\,t^3$

$\frac{d\Phi}{dt} = 2.0 \times 3t^2 = 6.0\,\mathrm{m^2/s^4}\,t^2$

At $t = 2.0\,\mathrm{s}$ $\frac{d\Phi}{dt} = 24 \Rightarrow \mathcal{E} = 24\,\mathrm{V}$

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Question 9

A wave on a string is described by displacement

$$y(x,t) = A\sin\left(6.28\,\mathrm{m}^{-1}\,x - 20\,\mathrm{s}^{-1}\,t - \pi/2\right).$$

a) Consider a snapshot of the wave at $t = 0\,\mathrm{s}$. Determine the locations of the two crests that are nearest to $x = 0\,\mathrm{m}$ and to its right (i.e. for $x > 0$) at this instant.

At $t=0$ $y(x,t) = A\sin(6.28\,\mathrm{m}^{-1}x - \pi/2)$

For a crest the argument of sin must be $\pi/2, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

For $\pi/2$ $6.28\,\mathrm{m}^{-1}x - \pi/2 = \pi/2 \Rightarrow 6.28\,\mathrm{m}^{-1}x = \pi \Rightarrow x = \frac{\pi}{6.28\,\mathrm{m}^{-1}} = 0.50\,\mathrm{m}$

For $\frac{5\pi}{2}$ $6.28\,\mathrm{m}^{-1}x - \pi/2 = \frac{5\pi}{2} \Rightarrow 6.28\,\mathrm{m}^{-1}x = 3\pi \Rightarrow x = \frac{3\pi}{6.28\,\mathrm{m}^{-1}} = 1.50\,\mathrm{m}$

b) Determine the distance between successive crests at $t = 0\,\mathrm{s}$.

(+2) From the previous part 1.0 m.

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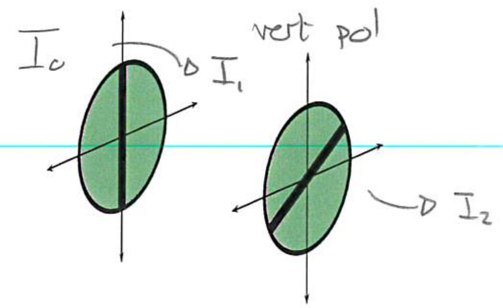
Question 10

Unpolarized light with intensity I_0 is incident from the left on a polarization filter, whose transmission axis is oriented vertically. This transmitted light is later incident on a second polarization filter, whose transmission axis 30° from the horizontal. Determine an expression for the intensity of the light transmitted by the second polarization filter (in terms of I_0).



$I_1 = I_0 \frac{1}{2}$ ↑ vert

$I_2 = I_1 \cos^2 60^\circ$ 60° 30°

$\theta = 30^\circ$ (-2 pts)

$\Rightarrow I_2 = \frac{1}{2} I_0 \cos^2 60^\circ$

$\frac{1}{2} I_0$ missing (-2 pts)

$= \frac{1}{2} I_0 \left(\frac{1}{2}\right)^2$

$= \frac{1}{8} I_0$

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