

## Electromagnetism and Optics: Class Exam I

24 February 2022

Name: \_\_\_\_\_

Total: /70

### Instructions

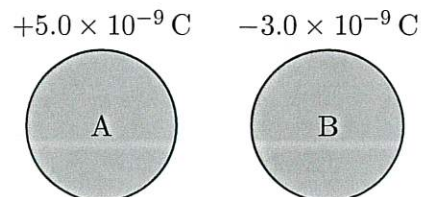
- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

### Physical constants and useful formulae

$$\begin{aligned}
 e &= 1.61 \times 10^{-19} \text{ C} & q_{\text{electron}} &= -e & q_{\text{proton}} &= +e \\
 m_{\text{electron}} &= 9.11 \times 10^{-31} \text{ kg} & m_{\text{proton}} &= 1.67 \times 10^{-27} \text{ kg} \\
 k &= 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2 & \epsilon_0 &= 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2
 \end{aligned}$$

### Question 1

Two identical metal spheres, A and B, initially hold the indicated charges. The spheres are brought into contact and charge flows from one to the other. Determine the charge on each sphere after this. Determine the number of electrons that flowed from one sphere to the other *and* describe the direction in which the electrons flowed.



+2 [ The total charged  $+5.0 \times 10^{-9} \text{ C} + (-3.0 \times 10^{-9} \text{ C}) = 2.0 \times 10^{-9} \text{ C}$  distributes evenly.  
 Thus each sphere has charge  $1.0 \times 10^{-9} \text{ C}$ .

+3 [ Sphere A loses  $4.0 \times 10^{-9} \text{ C}$  and thus gains electrons.  
 number  $e = \frac{Q}{e} = \frac{4.0 \times 10^{-9}}{1.6 \times 10^{-19}} = 2.5 \times 10^{10}$

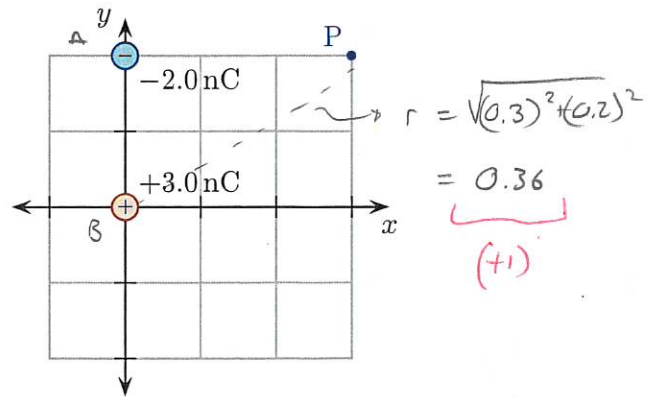
So  $2.5 \times 10^{10}$  electrons flow from B to A.

+1

/6

### Question 2

Two charged particles held fixed as illustrated. The grid units are 0.10 m. A third particle with charge  $3.0 \times 10^{-4} \text{ C}$  is placed at point P. Determine the net force exerted by the other two charges on the charge at P.



$$\vec{F} = \vec{F}_A + \vec{F}_B$$

(+1) [



Here  $\tan \theta = \frac{2}{3} \Rightarrow \theta = \arctan\left(\frac{2}{3}\right) = 33.7^\circ$  (+1)

Magnitudes:  $F_A = k \frac{|q_A q_P|}{r^2} = 9.0 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{2.0 \times 10^{-9} \text{ C} \times 3.0 \times 10^{-4} \text{ C}}{(0.30 \text{ m})^2} = 0.060 \text{ N}$  (+2)

$F_B = k \frac{|q_B q_P|}{r^2} = 9.0 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{3.0 \times 10^{-9} \text{ C} \times 3.0 \times 10^{-4} \text{ C}}{(0.36 \text{ m})^2} = 0.062 \text{ N}$  (+2)

Components  $F_{Ax} = -0.060 \text{ N}$   
 $F_{Ay} = 0 \text{ N}$  (+1)

$F_{Bx} = 0.060 \text{ N} \cos 33.7^\circ = 0.050 \text{ N}$   
 $F_{By} = 0.060 \text{ N} \sin 33.7^\circ = 0.033 \text{ N}$  (+3)

So  $F_{\text{net}x} = F_{Ax} + F_{Bx} = -0.060 \text{ N} + 0.050 \text{ N} = -0.010 \text{ N}$   
 $F_{\text{net}y} = F_{Ay} + F_{By} = 0 \text{ N} + 0.033 \text{ N} = 0.033 \text{ N}$  (+1)

$$\vec{F}_{\text{net}} = -0.010 \text{ N} \hat{i} + 0.033 \text{ N} \hat{j}$$

/12

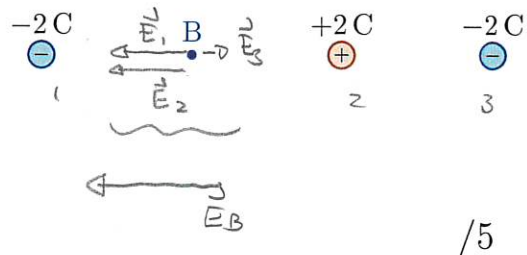
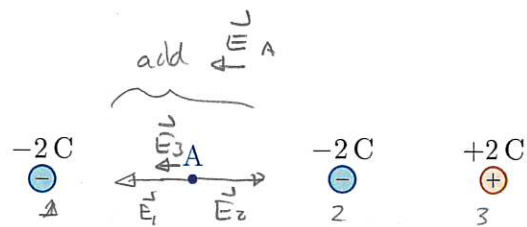
### Question 3

Two arrangements of fixed charges are as illustrated. One arrangement produces field  $\vec{E}_A$  at point A. The other produces field  $\vec{E}_B$  at point B. Which of the following (choose one) best ranks the *magnitude* of the total electric field at the indicated points?

i)  $E_A = E_B$

ii)  $E_A < E_B$

iii)  $E_B < E_A$



/5

### Question 4

A positively charged particle, Zog, is held fixed. Another charged particle, X, is given a brief kick and moves toward Zog. Consider the motion of X over an interval as it approaches Zog.



$V = k \frac{q}{r}$  decreases  $\Rightarrow \Delta V > 0$

a) Which of the following (choose one) is true regarding the change in electric potential experienced by X over the interval?

i)  $\Delta V > 0$  if X is positive,  $\Delta V < 0$  if X is negative,

ii)  $\Delta V < 0$  if X is positive,  $\Delta V > 0$  if X is negative,

iii)  $\Delta V > 0$  regardless of X's charge.

iv)  $\Delta V < 0$  regardless of X's charge.

b) Which of the following (choose one) is true regarding the change in electric potential energy over the interval?

i)  $\Delta U_{elec} > 0$  if X is positive,  $\Delta U_{elec} < 0$  if X is negative,

ii)  $\Delta U_{elec} < 0$  if X is positive,  $\Delta U_{elec} > 0$  if X is negative,

iii)  $\Delta U_{elec} > 0$  regardless of X's charge.

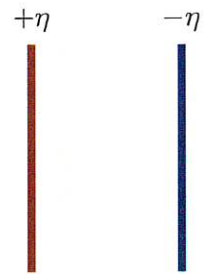
iv)  $\Delta U_{elec} < 0$  regardless of X's charge.

$\Delta U_{elec} = q \Delta V$   
+1

/8

### Question 5

Two parallel infinite plates are separated 0.0040 m. Each plate is uniformly charged and the charge density on the left plate is exactly the opposite of that on the right plate. A proton is released from rest at the left (positively charged plate). The speed of the proton when it reaches the plate on the right is  $5.0 \times 10^4$  m/s.



a) Determine the acceleration of the particle.

$$(1) \quad \left[ v_f^2 = v_i^2 + 2a \Delta x \right]$$

$$v_i = 0 \text{ m/s} \quad v_f = 5.0 \times 10^4 \text{ m/s}$$

$$\Delta x = 0.0040 \text{ m}$$

$$(2) \quad \left[ \frac{v_f^2 - v_i^2}{2 \Delta x} = a \quad \Rightarrow \quad a = \frac{(5.0 \times 10^4 \text{ m/s})^2}{2 \times 0.0040 \text{ m}} \right]$$

$$= 3.1 \times 10^{11} \text{ m/s}^2 \quad \rightarrow \quad \vec{a} = 3.1 \times 10^{11} \text{ m/s}^2 \hat{c}$$

b) Determine the electric field produced by the plates.

$$(2) \quad \left[ \vec{F}_{\text{net}} = m \vec{a} \quad \vec{F}_{\text{net}} = \vec{F}_{\text{elec}} = q_e \vec{E} \quad \Rightarrow \quad q_e \vec{E} = m \vec{a} \right]$$

$$\Rightarrow \quad \vec{E} = \frac{m}{q_e} \vec{a}$$

$$(3) \quad \left[ \vec{E} = \frac{1.67 \times 10^{-27} \text{ kg}}{1.6 \times 10^{-19} \text{ C}} 3.1 \times 10^{11} \text{ m/s}^2 \hat{c} \right]$$

$$= 3.3 \times 10^3 \text{ N/C} \hat{c}$$

c) Determine the charge density  $\eta$ .

$$E = \frac{\eta}{\epsilon_0} \quad \eta = \epsilon_0 E$$

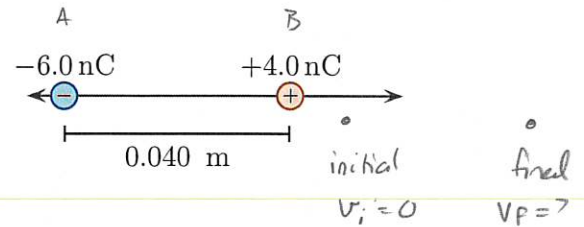
$$= 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \times 3.3 \times 10^3 \text{ N/C}$$

$$= 2.9 \times 10^{-8} \text{ C/m}^2$$

/12

### Question 6

Two point charges, separated by 0.040 m, are held fixed. Another particle with charge  $+3.0 \mu\text{C}$  and mass  $0.0010 \text{ kg}$  is released from rest a distance of 0.010 m to the right of the  $+4.0 \text{ nC}$  particle. Determine the speed of the  $+3.0 \mu\text{C}$  particle when it is 0.030 m to the right of the  $+4.0 \text{ nC}$  particle.



$$+1 \quad \Delta K + q \Delta V = 0$$

$$+4 \quad \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + q (V_f - V_i) = 0$$

$$v_f^2 = - \frac{2q}{m} (V_f - V_i) \quad \Rightarrow \quad v_f = \sqrt{- \frac{2q}{m} (V_f - V_i)}$$

$$\text{Then } V_f = k \frac{q_A}{r_{AF}} + k \frac{q_B}{r_{BF}}$$

$$= 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2 \left[ \frac{-6.0 \times 10^{-9} \text{ C}}{0.070 \text{ m}} + \frac{4.0 \times 10^{-9} \text{ C}}{0.030 \text{ m}} \right] = 429 \text{ V}$$

$$V_i = k \frac{q_A}{r_{Ai}} + k \frac{q_B}{r_{Bi}}$$

$$= 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2 \left[ \frac{-6.0 \times 10^{-9} \text{ C}}{0.050 \text{ m}} + \frac{4.0 \times 10^{-9} \text{ C}}{0.010 \text{ m}} \right] = 2520 \text{ V}$$

$$+2 \quad v_f = \sqrt{\frac{-2 \times 3.0 \times 10^{-6} \text{ C} \times (429 \text{ V} - 2520 \text{ V})}{0.0010 \text{ kg}}} = 3.5 \text{ m/s}$$



### Question 7

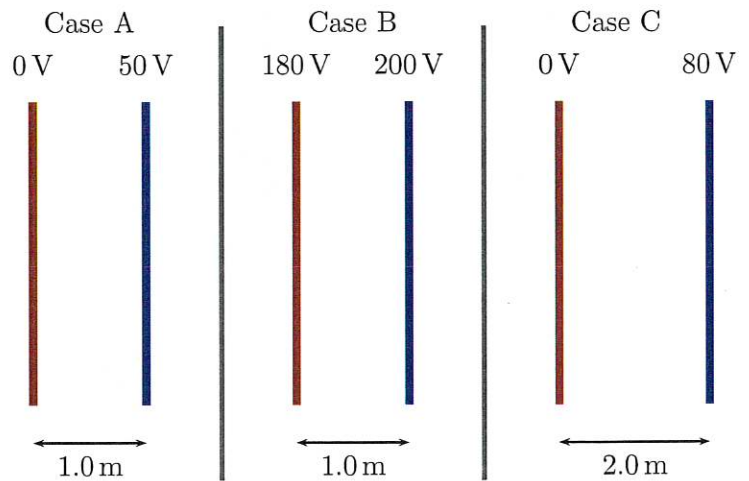
Three arrangements of infinitely large parallel conducting plates are as illustrated. Each plate is uniformly charged and held at a fixed electrostatic potential. Rank the electric fields in order of increasing magnitude. Briefly explain your answer.

$$E = -\frac{dV}{dx} = -\frac{\Delta V}{\Delta x}$$

$$E_A = -\frac{50}{1} = -50 \text{ N/C}$$

$$E_B = -\frac{20}{1} = -20 \text{ N/C}$$

$$E_C = -\frac{80}{2.0} = -40 \text{ N/C}$$



$$|E_B| < |E_C| < |E_A|$$

/6

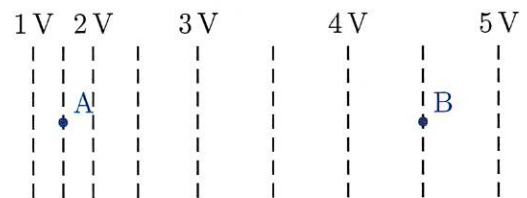
### Question 8

Hidden source charges produce the illustrated equipotentials. Let  $E_A$  be the magnitude of the electric field at point A and  $E_B$  be the magnitude of the field at point B. Which of the following is true?

i)  $E_A < E_B$

ii)  $E_A = E_B$

iii)  $E_A > E_B$



Briefly explain your answer.

$$E = -\frac{\Delta V}{\Delta x} \quad \text{steeper at A} \quad \Rightarrow \quad E_A > E_B$$

/6