

## Electromagnetism and Optics: Class Exam III

7 November 2018

Name: \_\_\_\_\_

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### Instructions

- There are 9 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

### Physical constants and useful formulae

$$e = 1.61 \times 10^{-19} \text{ C} \quad q_{\text{electron}} = -e \quad q_{\text{proton}} = +e$$

$$m_{\text{electron}} = 9.11 \times 10^{-31} \text{ kg} \quad m_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg} \quad c = 3.0 \times 10^8 \text{ m/s}$$

$$k = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2 \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \quad \mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$$

### Question 1

An experimenter aims to create a magnetic field by running a current around a circular loop.

- 5 a) The loop has radius 0.080 m. Determine the current in the loop needed to provide a magnetic field with magnitude  $5.0 \times 10^{-2} \text{ T}$  at the center of the loop?

$$B = \frac{\mu_0 I}{2R} \Rightarrow I = \frac{B2R}{\mu_0} = \frac{5.0 \times 10^{-2} \text{ T} \times 2 \times 0.080 \text{ m}}{4\pi \times 10^{-7} \text{ T m/A}}$$

$$= 6400 \text{ A}$$

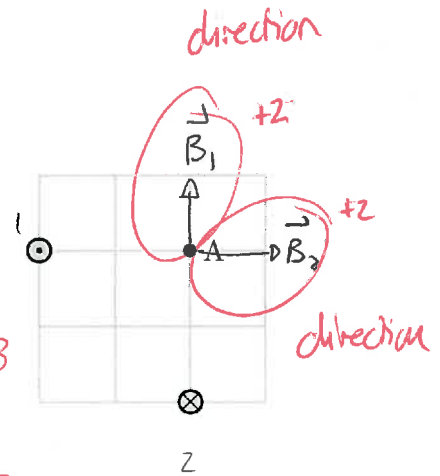
- 3 b) Suppose that the radius of the loop is then halved, while the current remains <sup>constant</sup>. By what factor does this increase (multiply) the magnetic field at the center of the loop? Explain your answer.

$$B = \frac{\mu_0 I}{2R} \quad \text{halving } R \text{ means } B_{\text{new}} = 2B_{\text{old}}$$

Factor of 2 /8

### Question 2

Two infinitely long wires are perpendicular to the page and carry identical currents with magnitudes 10 A in the indicated directions. Determine the net magnetic field produced by the two currents at the point labeled A. The grid units are each 0.010 m.



$$\vec{B} = \vec{B}_1 + \vec{B}_2 \quad ] \quad \text{or add vectors } 2 + 3$$

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \text{ Tm/A} \times 10 \text{ A}}{2\pi \times 0.020 \text{ m}} \quad ] \quad +1 \quad \text{or}$$

$$= 1.0 \times 10^{-4} \text{ T}$$

$$\vec{B}_1 = 1.0 \times 10^{-4} \text{ T } \hat{j}$$

Likewise  $\vec{B}_2 = 1.0 \times 10^{-4} \text{ T } \hat{i}$

$$\Rightarrow \vec{B} = 1.0 \times 10^{-4} \text{ T } \hat{i} + 1.0 \times 10^{-4} \text{ T } \hat{j} \quad ] \quad +1$$

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### Question 3

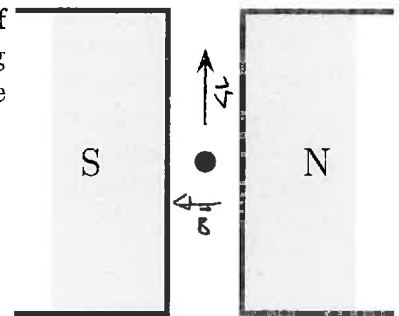
At one instant, a proton moves as illustrated between the poles of two closely spaced magnets as illustrated. Which of the following (choose one) represents the direction of the force exerted by the magnetic field produced by the magnets on the proton?

- i)  $\uparrow$
- ii)  $\downarrow$
- iii)  $\leftarrow$
- iv)  $\rightarrow$
- v) Into the page
- vi) Out of the page

$$\vec{F} = q \vec{v} \times \vec{B}$$

$\vec{v} \times \vec{B}$  is out

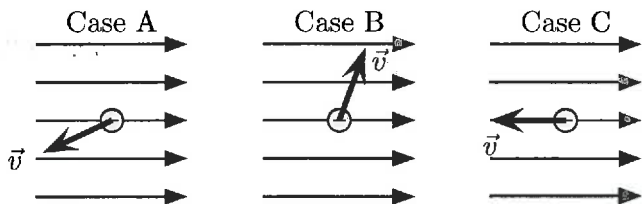
$q$  positive  $\Rightarrow \vec{v} \times \vec{B}$  out



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### Question 4

Identical negatively charged particles move through identical horizontally oriented magnetic fields with identical speeds in different directions as illustrated. The velocities of the particles are as indicated. Rank the situations in order of **increasing** magnitude of the magnetic force exerted on each particle. Explain your answers.



$$\vec{F} = q \vec{v} \times \vec{B}$$

magnitude  $F = |q|vB\sin\theta$

where  $\theta$  is angle between  $\vec{v}$  and  $\vec{B}$

For C  $\theta = 180^\circ \Rightarrow \sin\theta = 0 \Rightarrow F_C = 0$

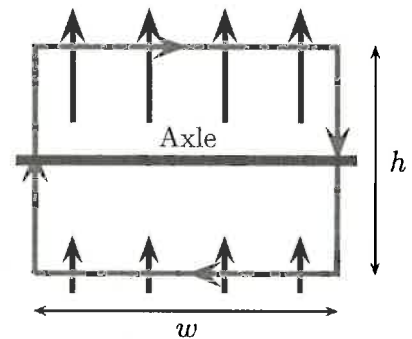
For A  $|\sin\theta| < |\sin\theta|$  for B  $\Rightarrow F_B > F_A$

$$F_C < F_A < F_B$$

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### Question 5

A rectangular loop with illustrated dimensions is placed in a vertical magnetic field. The field strength varies as indicated, with the magnitude of the field along the upper wire equal to  $B_1$  and the magnitude of the field along the lower wire equal to  $B_2$ . Determine an expression for the net force on the loop.



The current in the loop is  $I$ .

On the top  $\vec{I}$  is  $\rightarrow$   $B$  is  $\uparrow$  so  $\vec{F} = I\vec{\ell} \times \vec{B} \Rightarrow \vec{F}$  is out  $+2$

On the bottom  $\vec{I}$  is  $\leftarrow$  so  $\vec{F} = I\vec{\ell} \times \vec{B} \Rightarrow \vec{F}$  into  $+2$

On sides  $\vec{\ell} \times \vec{B} = 0$  so  $\vec{F} = 0$  on sides  $+1$

On top  $F = I\ell B \sin\theta$   
 $= IwB_1 \sin 90^\circ = IwB_1$   $+1$

Likewise on bottom  $F = IwB_2 \sin\theta = IwB_2$   $+1$

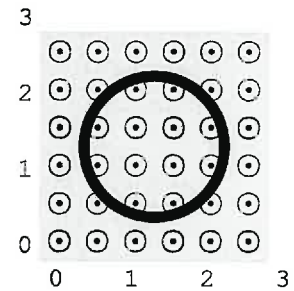
So  $\vec{F}_{\text{net}} = IwB_1 - IwB_2$  subtract out of page  $+2$

$\vec{F}_{\text{net}} = Iw(B_1 - B_2)$  out of page  $+1$

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### Question 6

A loop is held stationary in a uniform magnetic field as illustrated. The magnetic field strength increases over time. Determine whether or not any current flows through the loop and, if there is, determine its direction. Explain your answer.



The flux increases over time  $\Rightarrow \mathcal{E} \neq 0 \Rightarrow I \neq 0$

Now initially  $\odot \vec{B}$  smaller  
later  $\odot \vec{B}$  larger

$\Rightarrow$  induced  $\vec{B} (\otimes)$  into.  $\Rightarrow$  current clockwise by r.h. rule

$\uparrow$  less space

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### Question 7

A circular loop has a radius 0.050 m and is placed in a magnetic field that is perpendicular to it and has magnitude  $B = 6.0 \times 10^{-5} \sin(\omega t)$  where  $\omega = 800 \text{ s}^{-1}$ . Determine the EMF produced in the loop at  $t = 0 \text{ s}$ .

$$\mathcal{E} = \left| \frac{d\Phi_m}{dt} \right| \quad +1$$

$$\Phi_m = AB \sin \theta \quad \xrightarrow{+1} \quad 90^\circ \quad = AB = \pi r^2 B$$

$$\Rightarrow \Phi_m = \pi \times (0.050 \text{ m})^2 \times 6.0 \times 10^{-5} \sin(\omega t) \quad \xrightarrow{+3} \\ = 4.7 \times 10^{-7} \sin(\omega t)$$

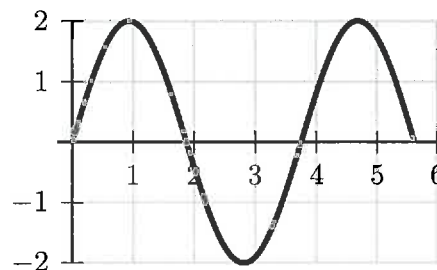
$$\text{Then } \frac{d\Phi_m}{dt} = \omega 4.7 \times 10^{-7} \cos(\omega t) \quad \xrightarrow{+4}$$

$$\text{At } t=0 \quad \frac{d\Phi_m}{dt} = 800 \text{ s}^{-1} \times 4.7 \times 10^{-7} \cos(0) \quad \xrightarrow{+1} \\ \Rightarrow \mathcal{E} = 3.8 \times 10^{-4} \text{ V}$$

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### Question 8

A snapshot of a wave at one instant ( $t = 0$ ) is illustrated. The axis units are in meters. The frequency of the wave is 50 Hz.



- a) Determine an expression for the wave in the form  $y = A \sin(kx)$  at this instant (i.e. find  $A$  and  $k$ ).

$$A = 2.0 \text{ m}$$

+1

=0

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.75} \quad +1 \quad \lambda = 3.75 \text{ m} \quad +1$$

need this

$$y = 2.0 \text{ m} \sin\left(\frac{2\pi}{3.75}x\right) \quad +1$$

- b) Determine the amount of time that it takes for the wave to travel 600 m to the right.

$$v = \lambda f = 3.75 \text{ m} \times 50 \text{ Hz} = 187.5 \text{ m/s} \quad +1$$

$$+1 \left[ \frac{\Delta x}{\Delta t} = v \Rightarrow \frac{\Delta x}{v} = \Delta t = \frac{600 \text{ m}}{187.5 \text{ m/s}} = 3.2 \text{ s} \right] \quad +2$$

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### Question 9

Unpolarized light with intensity  $I_0$  is incident from the left on a polarizer whose transmission axis is  $45^\circ$  from the horizontal. This transmitted light is later incident on a second polarizer, whose transmission axis is horizontal. Which of the following (choose one) is the intensity of the light transmitted by the second polarizer?

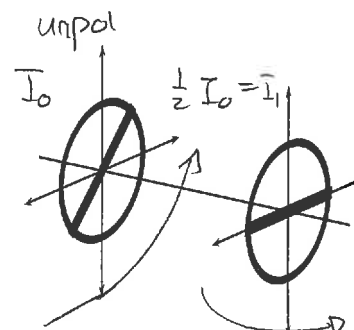
i)  $I_{\text{trans}} = 0.$

ii)  $I_{\text{trans}} = \frac{1}{8} I_0.$

iii)  $I_{\text{trans}} = \frac{1}{4} I_0.$

iv)  $I_{\text{trans}} = \frac{1}{2} I_0.$

v)  $I_{\text{trans}} = I_0.$



At this stage  
polarized along  $45^\circ$

$$I_2 = I_1 \frac{1}{2}$$

$$\text{So } I_2 = \frac{1}{2} \left( \frac{1}{2} I_0 \right) = \frac{1}{4} I_0$$

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