

Electromagnetism and Optics: Final Exam

12 December 2018

Name: Solution

Total: /150

Instructions

- There are 16 questions on 10 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$e = 1.61 \times 10^{-19} \text{ C} \quad q_{\text{electron}} = -e \quad q_{\text{proton}} = +e \quad \text{area sphere} = 4\pi r^2$$

$$m_{\text{electron}} = 9.11 \times 10^{-31} \text{ kg} \quad m_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg} \quad c = 3.0 \times 10^8 \text{ m/s}$$

$$k = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2 \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 \quad \mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$$

Question 1 ✓

Two oppositely charged particles, labeled A and B, are placed near to each other and held fixed. A has charge $+8 \text{ C}$ and B has charge -4 C . How does the *magnitude* of the force exerted by A on B, $F_{A \text{ on } B}$, compare to the magnitude of the force exerted by B on A, $F_{B \text{ on } A}$?



- i) $F_{A \text{ on } B} = \frac{1}{2} F_{B \text{ on } A}$
 ii) $F_{A \text{ on } B} = F_{B \text{ on } A}$
 iii) $F_{A \text{ on } B} = 2 F_{B \text{ on } A}$
 iv) $F_{A \text{ on } B} = 4 F_{B \text{ on } A}$
 v) $F_{A \text{ on } B} = 8 F_{B \text{ on } A}$

$$F_{A \text{ on } B} = k \frac{q_A q_B}{r^2}$$

$$F_{B \text{ on } A} = k \frac{q_B q_A}{r^2}$$

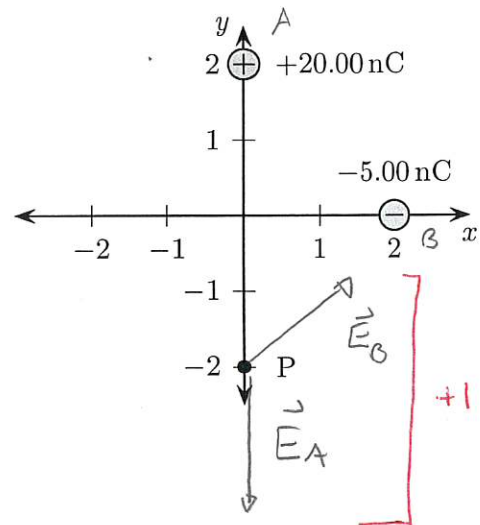
$$\underline{\hspace{2cm}} \\ = F_{A \text{ on } B}$$

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Question 2

Two charged particles are arranged as illustrated where distances are measured in meters.

- a) Determine the net electric field produced by the charge arrangement at the point labeled P.



$$E_A = k \frac{q_A}{r_A^2}$$

$$= 9.0 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \times \frac{20 \times 10^{-9} \text{C}}{(4.0 \text{m})^2}$$

$$= 11 \text{ N/C}$$

$$E_B = k \frac{q_B}{r_B^2} = 9.0 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \times \frac{5.00 \times 10^{-9} \text{C}}{8 \text{m}^2}$$

$$r_B^2 = (2 \text{m})^2 + (2 \text{m})^2 = 8 \text{m}^2$$

$$\vec{E}_A = -11 \text{ N/C} \hat{j}$$

$$\vec{E}_B = 5.6 \cos 45^\circ \hat{i} + 5.6 \sin 45^\circ \hat{j} = 4.0 \hat{i} + 4.0 \hat{j}$$

$$\vec{E} = \vec{E}_A + \vec{E}_B = 4.0 \hat{i} - 7.0 \hat{j} \text{ in N/C}$$

- b) Determine the magnitude of the net electric field at the point labeled P.

$$E = \sqrt{(4.0)^2 + (7.0)^2} = 8.0 \text{ N/C}$$

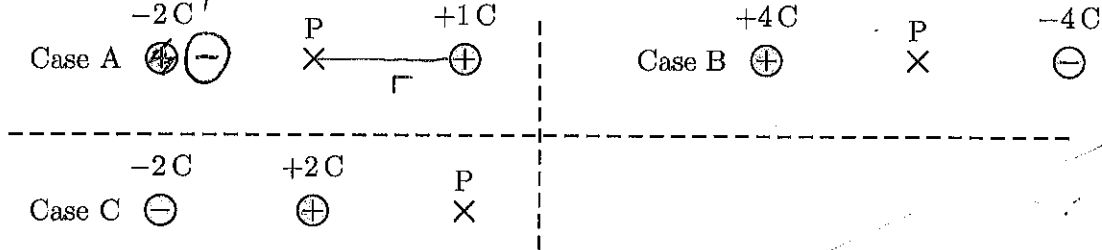
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$$V = V_1 + V_2$$

$$= k \frac{q_1}{r_1} + k \frac{q_2}{r_2} = \frac{k}{r} (q_1 + q_2)$$

Question 3 ✓

Various arrangements of fixed charges are as illustrated.



Which of the following (choose one) represents the rank of the electric potential at the points labeled P?

- i) $V_{\text{Case B}} = V_{\text{Case C}} > V_{\text{Case A}}$
- ii) $V_{\text{Case B}} > V_{\text{Case C}} > V_{\text{Case A}}$
- iii) $V_{\text{Case A}} > V_{\text{Case C}} > V_{\text{Case B}}$
- iv) $V_{\text{Case C}} > V_{\text{Case B}} > V_{\text{Case A}}$
- v) $V_{\text{Case A}} > V_{\text{Case B}} > V_{\text{Case C}}$

$$V_A = -\frac{k}{r}$$

$$V_B = 0 \frac{k}{r}$$

$$V_C = \frac{k}{r} \left(2 - \frac{2}{2} \right) = \frac{k}{r} \quad /5$$

Question 4 ✓

Three resistors, of various resistances are connected to batteries of different voltages. These satisfy $\Delta V_B = 2\Delta V_A$ and $\Delta V_C = 2\Delta V_A$. Separately $R_B = 4R_A$ and $R_C = 2R_A$. Rank the powers produced by the resistors from largest to smallest, indicating equality whenever this occurs. Explain your answer.

$$P = \Delta V I \Rightarrow P = \frac{(\Delta V)^2}{R}$$

$$P_A = \frac{(\Delta V_A)^2}{R_A}$$

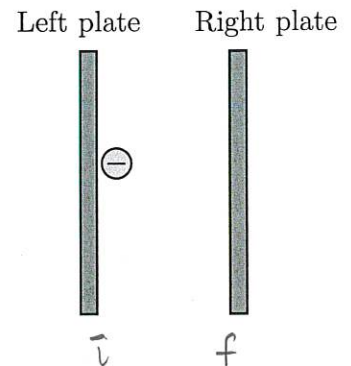
$$P_B = \frac{(\Delta V_B)^2}{R_B} = \frac{(2\Delta V_A)^2}{4R_A} = \frac{(\Delta V_A)^2}{R_A} \Rightarrow P_B = P_A$$

$$P_C = \frac{(\Delta V_C)^2}{R_C} = \frac{(2\Delta V_A)^2}{2R_A} = 2 \frac{(\Delta V_A)^2}{R_A} \Rightarrow P_C = 2P_A \quad /8$$

$$P_B = P_A < P_C$$

Question 5

Electrons are accelerated via an electric potential difference between two metal plates. Each electron is initially at rest on the left plate, at which the electric potential is 0 V. The potential on the right plate is such that electron moves right and attains speed 2.0×10^5 m/s at the instant just before it reaches the right plate. Determine the electric potential on the right plate which results in this motion. (Ignore any air resistance in this problem.)



(+1) $\Delta U_{elec} + \Delta K = 0$

(+2) $q \Delta V + \Delta K = 0$

(+1) $q(V_f - V_i) = -\Delta K \Rightarrow -e(V_f - 0) = -\left[\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right]$

(+1) $\Rightarrow -eV_f = -\frac{1}{2}mv_f^2$

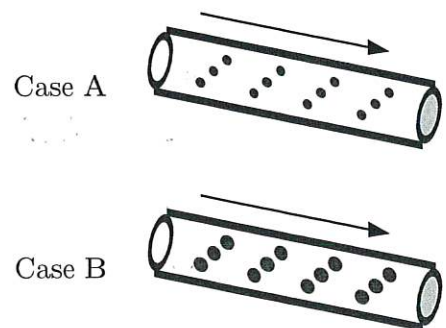
$\Rightarrow V_f = \frac{mv_f^2}{2e} = 0.11 \text{ V}$

(+5)

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Question 6

Charged particles pass through two cylindrical sections of wire. The number of particles that emerge through the shaded end of each wire, N , is observed for a period of 10 s. In case A, each particle has charge $+4.0 \text{ C}$ and $N = 1600$. In case B, each particle has charge $+12 \text{ C}$ and $N = 800$. Which of the following (choose one) is correct regarding the current in case A, I_A , and the current in case B, I_B ?



i) $I_A = 2I_B$

ii) $I_A = \frac{1}{3}I_B$

iii) $I_A = \frac{2}{3}I_B$

iv) $I_A = \frac{3}{2}I_B$

$I = \frac{\Delta Q}{\Delta t}$

$= \frac{N q_{\text{single charge}}}{\Delta t \leftarrow \text{same}}$

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4 For B $N_B = \frac{1}{2}N_A$

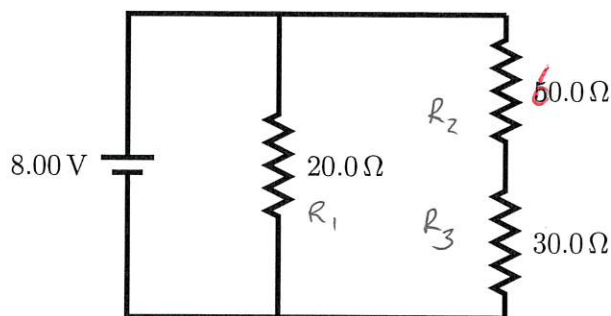
$q_B = 3q_A$

$N_B q_B = \frac{3}{2}N_A q_A \Rightarrow I_B = \frac{3}{2}I_A$

$I_A = \frac{2}{3}I_B$

Question 7 ✓

Consider the illustrated circuit.



- a) Determine the current provided by the battery.

(+2) [$\Delta V = I R_{\text{equiv}}$]

R_2, R_3 are in series and have an equivalent resistance of $R_2 + R_3 = 90 \Omega$

This is in parallel with R_1 , so

(+2) [$\frac{1}{R_{\text{equiv}}} = \frac{1}{R_1} + \frac{1}{90 \Omega}$] (+1)

$= \frac{1}{20} + \frac{1}{90} = \frac{11}{180 \Omega}$

$\Rightarrow R_{\text{equiv}} = \frac{180 \Omega}{11} = 16.4 \Omega$

Thus $8.00 \text{ V} = I 16.4 \Omega$

$\Rightarrow I = \frac{8.00 \text{ V}}{16.4 \Omega} = 0.49 \text{ A}$

- b) Suppose that 20.0Ω resistor is replaced by a 10.0Ω resistor. Explain whether the current through the 60.0Ω resistor increases, decreases or stays the same as a result of this change.

(+4) [Same. ΔV across 60Ω and 30Ω series combination stays the same]

Let out be +ve
in be -ve

$$\vec{B} = \vec{B}_{\text{horiz}} + \vec{B}_{\text{vert}}$$

Question 8

Two infinitely long wires are oriented as illustrated and each carry the same current. Which of the following (choose one) ranks the magnitudes of the net magnetic fields at the indicated points?

- i) $B_3 < B_4 = B_2 < B_1$
- ii) $B_3 < B_4 = B_1 < B_2$
- iii) $B_2 < B_1 < B_4 < B_3$
- iv) $B_2 < B_1 < B_3 < B_4$

$$B_1 = \frac{\mu_0 I}{2\pi} \left(\frac{1}{2} - 1 \right) = - \frac{\mu_0 I}{2\pi} \left(\frac{1}{2} \right)$$

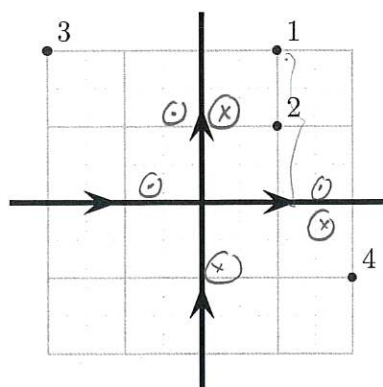
$$B_2 = \frac{\mu_0 I}{2\pi} (1 - 1) = 0$$

$$B_3 = \frac{\mu_0 I}{2\pi} \left(\frac{1}{2} + \frac{1}{2} \right)$$

$$= \frac{\mu_0 I}{2\pi} (1)$$

$$B_4 = \frac{\mu_0 I}{2\pi} \left(-\frac{1}{2} - 1 \right)$$

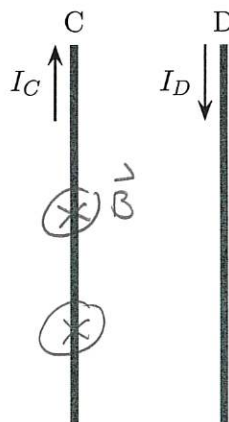
$$B_4 = \frac{\mu_0 I}{2\pi} \left(-\frac{3}{2} \right)$$



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Question 9

Two parallel wires carry currents as illustrated. Wire D is infinitely long and wire C has length 2 m and is 0.25 m from wire D. Both wires carry currents of 3 A but do so in opposite directions. Determine the direction and magnitude of the magnetic field produced by wire D at the location of wire C. Use this to determine the force exerted by D on C.



$$B_{\text{from D}} = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \text{ Tm/A} \times 3 \text{ A}}{2\pi \times 0.25 \text{ m}}$$

$$= 2.4 \times 10^{-6} \text{ T}$$

into page

$$F = I \vec{l} \times \vec{B}$$



$\vec{l} \times \vec{B}$ is ←

$$F = I l B \sin \theta$$

$$= 3 \text{ A} \times 2 \text{ m} \times 2.4 \times 10^{-6} \text{ T} \sin 90^\circ$$

$$= 1.4 \times 10^{-5} \text{ N}$$

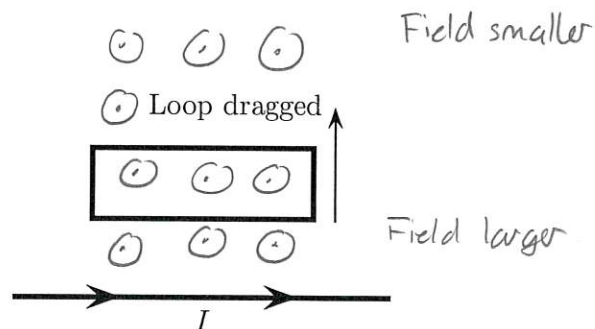
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$$Use F = \frac{\mu_0 I_1 I_2 l}{2\pi d^2} \text{ only}$$

Question 10

An infinitely long wire carries a constant current, I , in the indicated direction. A rectangular loop lies in the plane of the wire and initially has no current. It is then dragged in the indicated direction. Describe whether there is a current in the loop while it is dragged and, if so, what the direction of the current is. Explain your answer.



The long straight wire produces a magnetic field in the indicated direction. The loop is dragged into a region where the field

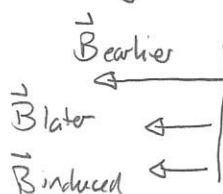
+2 decreases $\Rightarrow$ flux changes $\Rightarrow$ current is induced. [Current is present]

Right side view By Lenz's law

The induced field points out

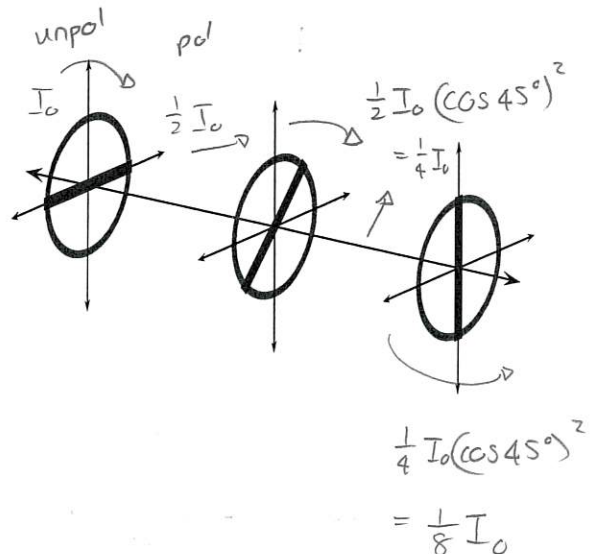
$\Rightarrow$ induced current is counterclockwise

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Question 11

Three linear polarizers are arranged with their transmission axes as illustrated. For the first and leftmost the axis is horizontal, for the middle the axis is midway between horizontal and vertical and for the last and rightmost it is vertical. Unpolarized light of intensity I_0 is incident from the left on the leftmost polarizer. Which of the following (choose one) best represents the intensity of the light after it emerges from the rightmost polarizer?



i) $I = 0$

ii) $I = I_0$

iii) $I = \frac{1}{2} I_0$

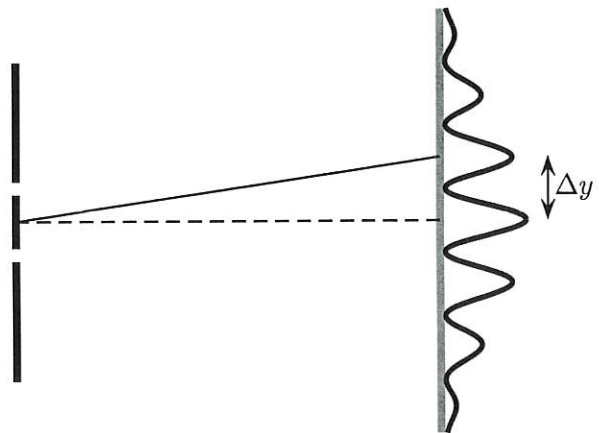
iv) $I = \frac{1}{4} I_0$

v) $I = \frac{1}{8} I_0$

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Question 12 ✓

Monochromatic light (i.e. light of one frequency) is incident on a double slit. The slit separation is 0.0500 mm. The interference pattern is projected onto a screen 2.00 m from the slits and the intensity profile is as illustrated. The distance between the first bright fringe and the central maximum is $\Delta y = 2.25$ cm. Determine the wavelength of the light.



$$y_m \approx \frac{mL\lambda}{d} \quad (+2) \quad \text{or } \sin \theta_m = m\lambda$$

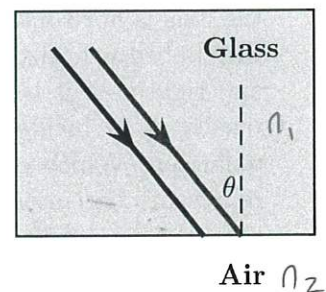
$$\lambda = \frac{y_m d}{mL} = \frac{2.25 \times 10^{-2} \text{ m} \times 0.0500 \times 10^{-3} \text{ m}}{2.00 \text{ m}} \quad (+3)$$

$$= 5.63 \times 10^{-7} \text{ m} \quad (+3)$$

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Question 13 ✓

Red and violet light rays travel parallel to each other through a glass slab toward the edge of the slab, where it meets air. While keeping the rays parallel to each other, the angle θ from the normal is increased starting from $\theta = 0^\circ$. Initially both colors emerge into the air. As θ is increased, which, if either, of the colors will be the first to disappear from the air (i.e. no longer emerge from the bottom of the slab)? The index of refraction of the glass is 1.54 for violet light and 1.52 for red light. The index of refraction for both in air is 1.00. Explain your answer.



For total internal reflection:

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$\Rightarrow n_1 \sin \theta_c = n_2$$

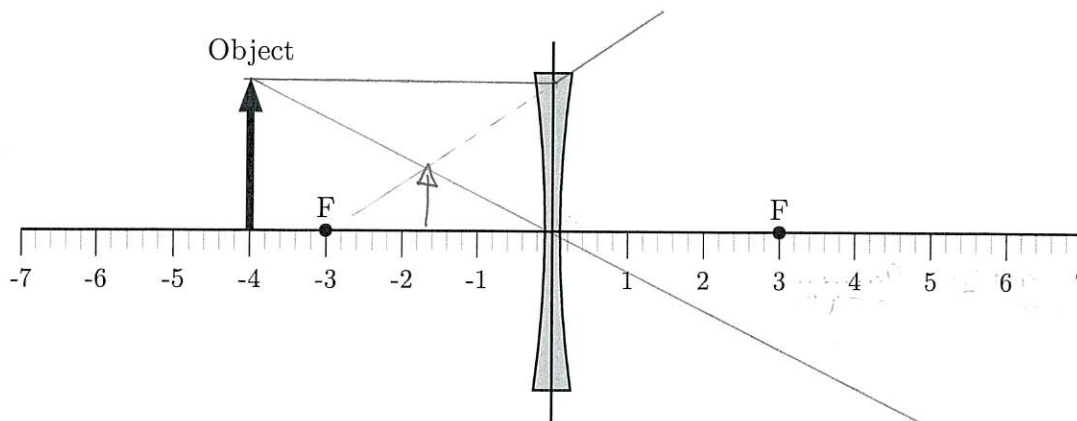
$$\Rightarrow \sin \theta_c = \frac{n_2}{n_1}$$

So $\sin \theta_c$ is smaller for violet
 $\Rightarrow \theta_c$ is smaller " "
 $\Rightarrow$ violet disappears first

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Question 14

An object, whose height is 2 cm, is placed to the left of a concave lens as illustrated. The focal points of the lens are labeled, F. The units along the horizontal axis are centimeters.



- Trace rays as accurately as possible to indicate the image produced by the lens.
- Determine the location and height of the image using equations.

$$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \Rightarrow \frac{1}{s'} + \frac{1}{4\text{cm}} = \frac{1}{-3\text{cm}}$$

$$\Rightarrow \frac{1}{s'} = \frac{1}{-3\text{cm}} - \frac{1}{4\text{cm}} = -\frac{7}{12\text{cm}}$$

$$\Rightarrow s' = -\frac{12\text{cm}}{7} = -1.7\text{cm}$$

$$m = -\frac{s'}{s} \Rightarrow m = -\frac{-1.7\text{cm}}{4} = 0.43$$

$$\frac{h'}{h} = m \Rightarrow h' = h m = 2\text{cm} \times 0.43 = 0.86\text{cm}$$

- Zog claims that it is impossible to use this lens to produce an enlarged image of the object using just this lens. Is Zog's claim true or false? Explain your answer.

Yes the diagonal line through center always intersects beneath the line through focal pt.

Question 15

A convex lens is constructed from glass with an index of refraction 1.55. In air (index of refraction 1.00) the lens has a focal length of 48 mm. Suppose that the lens is immersed in water (index of refraction 1.33). Which of the following (choose one) is true?

- i) The focal length of the lens in water is exactly 48 mm.
- ii) The focal length of the lens in water is more than 48 mm.
- iii) The focal length of the lens in water is less than 48 mm.

Briefly explain your answer.

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1 \right) \underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2} \right)}_{\text{same}}$$

$\frac{n_2}{n_1} - 1$ is smaller in water
 $\Rightarrow f$ larger in water.

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Question 16

A string with length 1.20 m is stretched so that the speed of waves on the string is 480 m/s.

- a) Determine the frequency of the standing wave that has one antinode.



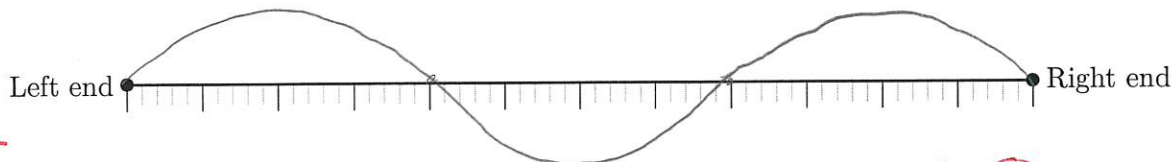
$$\lambda = 2L = 2.40 \text{ m}$$

$$v = \lambda f$$

$$\Rightarrow 480 \text{ m/s} = 2.40 \text{ m} f \Rightarrow f = 200 \text{ Hz}$$

- b) Sketch the standing wave with three antinodes using the axis below and determine its wavelength and frequency.

+3



$$3 \frac{\lambda}{2} = L$$

$$\Rightarrow \lambda = \frac{2}{3} L = \frac{2}{3} 1.20 \text{ m} = 0.80 \text{ m}$$

$$v = \lambda f$$

$$\Rightarrow 480 \text{ m/s} = 0.80 \text{ m} f$$

$$\Rightarrow f = 600 \text{ Hz}$$

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