Electromagnetic Theory II: Homework 21

Due: 28 April 2021

1 Force exerted by a point source charge

Consider a source point charge with charge q and a test charge with charge Q that moves with constant velocity $\mathbf{v} = v\hat{\mathbf{x}}$ with respect to the source charge. The aim of this exercise is to compare the fields produced by the source and force it exerts on the test when viewed from two frames: one in which the source is at rest and the other in which the test is at rest. Assume that Maxwell's equations and the Lorentz force law are valid in both frames.

a) Consider the situation as observed in a frame in which the source is at rest. Determine the electric and magnetic fields produced by the source and the force that they exert on the test particle when it is at location $\mathbf{r} = x\hat{\mathbf{x}}$.

b) Consider the situation as observed in a frame in which the test particle is at rest at $\mathbf{r} = x\hat{\mathbf{x}}$. Determine the electric and magnetic fields produced by the source as it passes the origin (use the results from Ch. 11 for a particle that moves with constant velocity). Use these to determine the force exerted on the test particle. Is it the same as when viewed from the other frame?

2 Force exerted by an infinite line of charges: including Lorentz contraction

Consider an infinite line of equally spaced identical charged particles that are stationary relative to each other. Suppose that a test particle with charge Q is at rest relative to these and a distance s from the line. Assume that the spacing between adjacent source charges is so small that the fields that they produce can be approximated by a continuous distribution of charge.

a) In the frame in which the source are at rest, determine the electric and magnetic fields at the test. Determine the force $\mathbf{F}'$ which these fields exert on a stationary test particle.

b) Suppose that the situation is observed from another inertial frame in which the particles move with velocity v along the line of charge. Using the length-contracted charge density, determine the electric and magnetic fields at the test. Determine the force $\mathbf{F}$ which these fields exert on the test particle.

c) How are $\mathbf{F}$ and $\mathbf{F}'$ related?