Electromagnetic Theory II: Homework 15

Due: 2 April 2021

1 Lorentz and Coulomb gauge

Suppose that the vector potential is

$$\mathbf{A} = A_0 \cos(kx - \omega t)\,\hat{\mathbf{y}}$$

and the scalar potential is

$$V = V_0 \cos(kx - \omega t).$$

a) Is this potential in the Coulomb gauge?

b) Determine conditions such that this potential is in the Lorentz gauge.

c) Determine the fields produced by these potentials.

2 Griffiths, *Introduction to Electrodynamics, 4ed*, 10.3, page 440.

3 Gauge choices for potentials

Consider the potentials in spherical coordinates:

$$V = 0$$

and

$$\mathbf{A} = \begin{cases} 0 & \text{if } r < R \\ \dfrac{\alpha}{r}\cos(\omega t)\hat{\boldsymbol{\phi}} & \text{if } r > R \end{cases}$$

where α and R are positive constants.

a) Are these in the Coulomb gauge?

b) Show that a spherically symmetrical gauge function, λ, that transforms potentials to the Lorentz gauge satisfies

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \lambda}{\partial r}\right) = \frac{1}{c^2}\frac{\partial^2 \lambda}{\partial t^2}.$$

c) Show that a solution to this is

$$\lambda(r,t) = \frac{\beta}{r}\cos(kr - \omega_0 t)$$

provided that ω_0 and k satisfy a particular relationship.

d) Express the potentials in the new gauge.