# Electromagnetic Theory II: Homework 14

Due: 30 March 2021

## 1 Potentials for a spherical situation

Suppose that, in spherical coordinates,

$$V = 0$$

and

$$\mathbf{A} = \begin{cases} 0 & \text{if } r < R \\ \dfrac{\alpha}{r}\cos(\omega t)\hat{\boldsymbol{\phi}} & \text{if } r > R \end{cases}$$

where $\alpha, \omega$ and $R$ are positive constants.

a) Determine the electric and magnetic fields associated with these potentials.

b) Determine the charge and current distributions that give rise to these potentials.

c) Determine the Poynting vector associated with these fields. Which way is energy transported by the fields?

## 2 Time-varying electric fields

The potential formulation allows one to construct time-varying electric fields that are not accompanied by induced magnetic fields. Suppose that $\mathbf{A} = 0$ and that

$$V = \begin{cases} 0 & \text{if } r < R \\ \dfrac{q}{4\pi\epsilon_0 r}\cos(\omega t) & \text{if } r > R \end{cases}$$

where $q$ and $\omega > 0$ are constants.

a) Determine the electric and magnetic fields produced by these potentials. Sketch the fields.

b) Do these fields radiate electromagnetic energy? Explain your answer.

c) Determine the source charge and current densities that produce these fields.

d) Verify that these source charges and currents satisfy the continuity equation for $r > R$.

## 3 Localized charge and current distribution and time varying fields

A charge and current distribution produces a potential, given in cylindrical coordinates by

$$V = \begin{cases} 0 & \text{if } s < R \\ \dfrac{\sigma(t)R}{\epsilon_0}\ln s & \text{if } s > R \end{cases}$$

where $\sigma(t)$ has units of $\mathrm{C/m^2}$ and varies with time. Is it possible that this can be produced by a charge and current density which is strictly zero beyond $R' > R$? Explain your answer.