## **Electromagnetic Theory II: Homework 11**

Due: 16 March 2021

## 1 General plane wave solution to the three dimensional wave equation

a) Show that

$$f(\mathbf{r},t) = g(\mathbf{k} \cdot \mathbf{r} \pm \omega t),$$

where g(u) is any twice differentiable function of a single variable, is a solution to the three dimensional wave equation.

b) Suppose that

$$g(u) = \frac{A}{u^2 + b^2}$$

where A and b are constants. If  $\mathbf{k} = k \left( \hat{\mathbf{x}} + 2\hat{\mathbf{y}} \right) / \sqrt{5}$  and b = 2, sketch (in the xy plane) the location of the maximum displacement and the locations along which the displacement is half of the maximum value at t = 0. In which direction does the wave propagate?

## 2 Three dimensional waves

Consider the following candidates for three dimensional functions:

$$f_{1}(\mathbf{r},t) := A(x+2y-vt)e^{-(x+2y-vt)^{2}/a^{2}}$$

$$f_{2}(\mathbf{r},t) := A(x-vt)e^{-(x+2y-vt)^{2}/a^{2}}$$

$$f_{3}(\mathbf{r},t) := A(x^{2}+y^{2}-vt)e^{-(x^{2}+y^{2}-vt)^{2}/a^{2}}$$

$$f_{4}(\mathbf{r},t) := A\cos(x+y-vt)$$

$$f_{5}(\mathbf{r},t) := A\cos(x^{2}+y^{2}-vt)$$

where a > 0. Explain which of these might represent plane waves and which do not represent plane waves.

## 3 Spherical waves

Consider the three dimensional wave equation

$$\nabla^2 f = \frac{1}{v^2} \; \frac{\partial^2 f}{\partial t^2}.$$

In spherical coordinates, this gives

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

Assume a spherically symmetrical solution of the form

$$f(\mathbf{r},t) = \frac{h(r)}{r}e^{-i\omega t}$$

By substituting, determine a differential equation for h(r) and solve this for h(r) (use the complex representation when solving this).