Electromagnetic Theory II: Homework 5

Due: 12 February 2021

1 Torque exerted by one current loop on another

A small circular loop with radius a lies in the xz plane and is centered on the origin. It carries current I_1 counterclockwise (when viewed down the $+y$ axis toward the origin). Another loop with radius b lies in the xy plane on the $+y$ axis at a distance R from the origin. This loop carries current I_2 clockwise (when viewed down the $-\hat{\mathbf{z}}$ direction).

a) Assuming $R \gg a, b$ the loops can be regarded as dipoles. Determine the torque exerted by the loop of radius a on the other loop.

b) Determine the torque exerted by the loop of radius b on the other loop.

2 Stern-Gerlach experiment

Suppose that a beam of particles, each with mass M and velocity $\mathbf{v} = v_x\hat{\mathbf{x}}$, enters a region in which the magnetic field is

$$\mathbf{B} = (B_0 z)\,\hat{\mathbf{z}}$$

where $B_0 = 600\,\text{Tesla/m}$. This region extends in the x direction for distance L_B. A detector is placed L_D beyond the end of the magnetic field region. The setup is illustrated in Fig. 1.

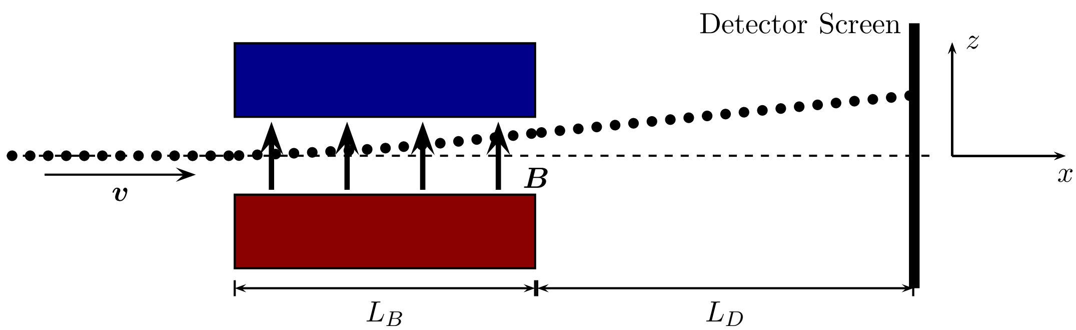


Figure 1: Question 2

a) Suppose that prior to entering the magnetic field, the z component of each particle's magnetic dipole moment has the same value, m_z. Find an expression, in terms of M and m_z, for the acceleration of the particles while they are in the region with non-zero magnetic field. Ignore all forces on any particle except that exerted by the magnetic field and assume that the orientation of the magnetic dipole moment vector remains constant as the particle passes through the field.

b) Assume that the particles follow trajectories governed by classical mechanics. Find an expression for the total deflection in the z direction in terms of M, m_z, v_x, L_B and L_D. Verify that it is proportional to m_z.

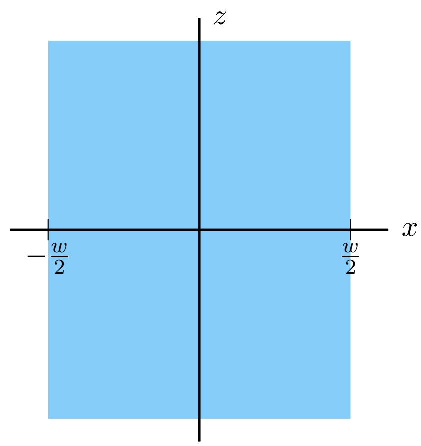
3 Bound current densities

Consider an infinite slab of material extending across the range $-w/2 \leqslant x \leqslant w/2$, $-\infty \leqslant y \leqslant \infty$, and $-\infty \leqslant z \leqslant \infty$. (this is parallel to the yz plane). The material has magnetization

$$\mathbf{M} = M(x)\hat{\mathbf{z}}$$

where $M_0 > 0$ and $a > 0$ are constants. The following questions apply to a section that extends across the material and beyond its edges, for which

$$\begin{aligned} -w \leqslant x \leqslant w, \\ y = 0, \\ 0 \leqslant z \leqslant h. \end{aligned}$$



a) For the special case $\mathbf{M} = M_0 \frac{x^2}{a^2}\hat{\mathbf{z}}$, determine the total bound current that flows across the section described above.

b) Determine the total bound current that flows across the section described above for any magnetization. *Hint: You should get the same result for any magnetization.*

4 Bound current densities: sphere

A sphere with radius R has magnetization

$$\mathbf{M} = \frac{\alpha}{r}\,\hat{\boldsymbol{\phi}}$$

where $\alpha > 0$ is a constant.

a) Determine the bound current densities inside and on the surface of the sphere.

b) Using a diagram describe how the bound current flows across the surface of and through the sphere.