Electromagnetic Theory II: Homework 3

Due: 5 February 2021

1 Axial polarization

In situations where there is no free charge,

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho_{\text{free}} = 0$$

and it would appear that $\mathbf{D} = 0$. The aim of this exercise is to check whether this is always true.

a) Suppose that $\mathbf{D} = 0$. Determine an expression for the electric field in terms of the polarization in this case. This will be true for *any* situation where $\mathbf{D} = 0$.

As a specific example consider a sphere of radius R which contains no free charge and which has polarization (in spherical coordinates)

$$\mathbf{P} = P(r)\hat{\boldsymbol{\phi}}$$

where $P(r)$ is any function.

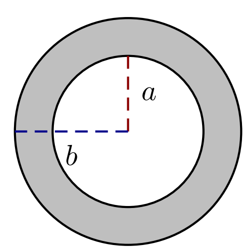
b) Assuming that $\mathbf{D} = 0$ determine the electric field within the sphere. Does this field satisy the requirement that $\boldsymbol{\nabla} \times \mathbf{E} = 0$ for electric fields?
c) One way of calculating the electric field is to use the usual rules of electrostatics to calculate the fields produced by the bound charge distributions. Determine the bound surface and volume charge densities and use these to determine the electric field within the sphere. Is the result consistent with the answer from the previous part?
d) This is partly resolved by considering $\boldsymbol{\nabla} \times \mathbf{D}$. If this were zero then the usual electrostatics *would guarantee* that $\mathbf{D} = 0$. Determine an expression for $\boldsymbol{\nabla} \times \mathbf{D}$. Is it zero?
e) In general, is it true that if there is no free charge then $\mathbf{D} = 0$?

2 Polarized cylindrical object

An infinitely long cylindrical pipe has inner radius a and outer radius b. The pipe consists of a material that has polarization, given in cylindrical coordinates by

$$\mathbf{P}(\mathbf{r}') = \frac{\alpha}{s'^2}\hat{\mathbf{s}}$$

where $\alpha > 0$. There is no free charge present. Determine the electric displacement and the electric field at all points.



3 Point charge within a dielectric sphere

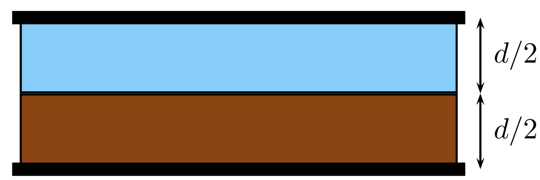
A point charge with charge q is located at the center of a sphere, with radius R, made of a linear dielectric material with permittivity ϵ.

a) Determine the electric displacement and electric field at all points.

b) Determine the electrostatic potential at all points, assuming the the potential at infinity is zero.

4 Parallel plate capacitor with two layers of dielectric

A parallel plate capacitor has plates of area A separated by distance d. The gap between the plates is filled with two dielectric materials, each of thickness $d/2$. The upper material has permittivity ϵ_1 and the other has permittivity ϵ_2. The free charge per unit area on the upper plate is $+\sigma_{\text{free}}$ and on the lower plate is $-\sigma_{\text{free}}$. Assume that the plates are so large that they can be approximated as infinite.



a) Determine the electric field at all points between the plates.

b) Determine the capacitance of this capacitor.

c) Determine expressions for the bound charge densities on the upper and lower surfaces of each dielectric.