Fri: Exam III in class

Covers Ch 6, Ch 7

Lectures 28-36

HW 10-13

Bring: Prev two ½ letter sheets single side plus one new

Calculator

Given: Integral sheet, front + back.

Prev exams: 2020 Ex II Q 8

2020 Final Ex Q 10, 12, 13,

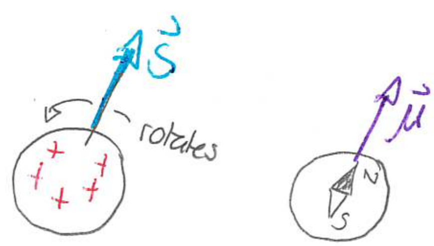
After break: * Classes via Zoom - see email.

* One more HW assignment

Spin angular momentum

For charged particles the spin angular momentum is accessible via the particle's magnetic dipole moment

$$\vec{\mu} = \frac{g}{2}\frac{Q}{M}\vec{S}$$



where Q = charge of particle

M = mass of particle

g = g-factor.

Thus if we can measure the components of $\vec{\mu}$, we can infer the components of the spin angular momentum

The theory of electromagnetism then provides a rule for the energy of the dipole in an external magnetic field. Thus

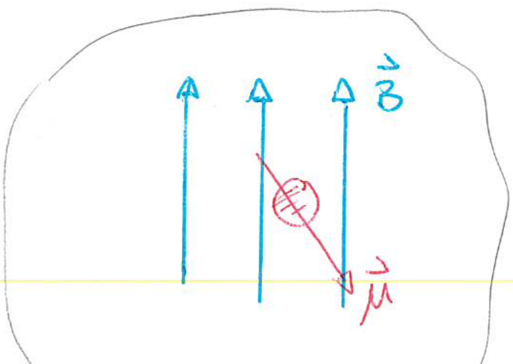
$$U = -\vec{\mu}\cdot\vec{B}$$

where $\vec{B}$ is the external field. ~~The force~~ on the dipole is

$$\vec{F} = -\vec{\nabla} U$$

$$= \vec{\nabla}(\vec{\mu}\cdot\vec{B}) = \frac{\partial(\vec{\mu}\cdot\vec{B})}{\partial x}\hat{x} + \frac{\partial(\vec{\mu}\cdot\vec{B})}{\partial y}\hat{y} + \frac{\partial(\vec{\mu}\cdot\vec{B})}{\partial z}\hat{z}$$

So



Particle in field

$\longrightarrow$

$$F_x = \frac{\partial}{\partial x}(\vec{\mu}\cdot\vec{B})$$

$$F_y = \frac{\partial}{\partial y}(\vec{\mu}\cdot\vec{B})$$

$$F_z = \frac{\partial}{\partial z}(\vec{\mu}\cdot\vec{B})$$

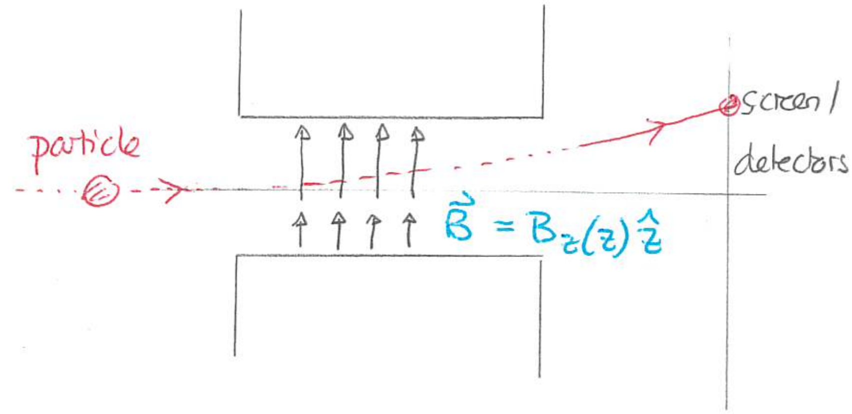
Quiz 1 80%

Stern- Gerlach experiment

We see that if we place a particle in an inhomogeneous external magnetic field then there will be a net force on the particle. This is the idea behind the Stern-Gerlach experiment (1920s).

Given

$$\vec{B} = B_z(z)\,\hat{z}$$

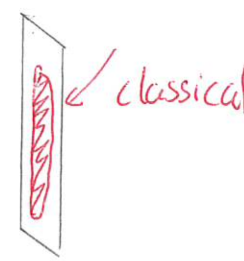


we get

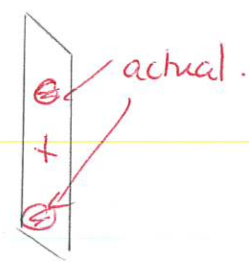
$$F_z = \mu_z \frac{\partial B_z}{\partial z}$$

There will be a force that deflects the particle and the deflection will be proportional to μ_z. This can be registered on a distant screen.

Classically we expect a continuous range of deflections since μ_z can take on a continuous range of values



The actual experiment, done with electrons, reveals one of two deflections.



Demo: Toutestquantique video

The experiment was done using silver atoms, In their ground state these have zero orbital angular momentum. Thus the experiment reveals the spin angular momentum of the electron. The conclusions are:

1) Atoms / subatomic particles have an intrinsic spin angular momentum. Calculations reveal that this cannot arise from the particle's motion.

2) Any component of the spin angular momentum can only assume discrete values when measured. Thus spin angular momentum is quantized.

Spin-1/2 particle

An electron is an example of a particle for which the z-component can take one of two values

Possible outcomes

Measure S_z

$S_z = +\hbar/2 \rightsquigarrow$ spin up $\rightsquigarrow$

$S_z = -\hbar/2 \rightsquigarrow$ spin down $\rightsquigarrow$

This is a spin-1/2 particle:

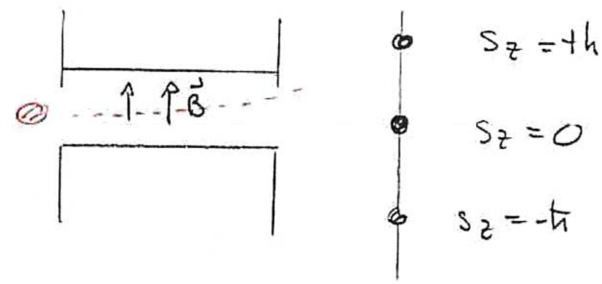
> A spin-1/2 particle is one such that if any single component of spin is measured the outcome will be <u>one of two possibilities</u>

The component of the spin can be described by

$$\boxed{S_z = m_s \hbar \qquad \text{where} \quad m_s = +1/2 \text{ or } m_s = -1/2}$$

General spin measurements

There are other possibilities for spin measurements for subatomic particles. For example, a spin-1 particle would give one of three outcomes if μ_z and S_z were measured. These correspond to three possible values of S_z:



$S_z = +\hbar$

$S_z = 0$

$S_z = -\hbar$

or in short $\quad S_z = m_s \hbar \quad$ where $\quad m_s = -1, 0, 1$

The most general case of spin for subatomic particles and quantum systems is described by:

> A spin-s particle is described by a number
>
> $s = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$
>
> and the possible outcomes of a measurement of S_z are:
>
> $S_z = m_s \hbar$
>
> where $\quad m_s = -s, -s+1, -s+2, \ldots, s-2, s-1, s$

Quiz 2 100%

The number s is called the spin quantum number.

Electron spin

Direct experiments and much indirect evidence indicates that an electron has spin-$\frac{1}{2}$. Again this is not associated with any motion of the electron - it is an inherent property of the electron. Thus the z-component of the electron can be in one of two states.

This needs to be incorporated into the description of the state of an electron, including when it appears in a hydrogen atom.

spin up	spin down
$m_s = +\frac{1}{2}$	$m_s = -\frac{1}{2}$
$S_z = +\frac{\hbar}{2}$	$S_z = -\frac{\hbar}{2}$

It may appear that this requires an additional wavefunction or factor of a wavefunction.

But the spin does not refer to a location and so there is no wavefunction that depends on position. In quantum theory this is resolved by extending the formalism to include vectors that describe the state of such systems. In brief this would involve two vectors such as

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

that correspond to spin up and spin down respectively.

A comprehensive formalism for quantum theory describes how to use such vectors to predict the behavior and measurement outcomes for such particles.

For an electron in a hydrogen atom the state must be represented by

- numbers that describe its spatial state n, l, m_l
- numbers " " its spin state

So we need

$n = 1, 2, 3, \ldots$ $\longrightarrow$ $l = 0, 1, 2, \ldots, n-1$

$\longrightarrow$ $m_l = -l, -l+1, \ldots, l-1, l$

$m_s = -\hbar/2, +\hbar/2$

So possible states for an electron in a hydrogen atom are: