Tues: HW by 5pm Weds: Read

Fri: Exam III Covers Ch 6, Ch 7 Lectures 28 - 36
HW 10-13 2021 Ex II Q8

Spin angular momentum

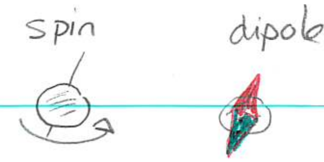
So far we have analyzed the hydrogen atom by considering the interaction between the charge of the electron and the charge of the nucleus.

It turns out that the electron has an additional property that helps describe the structure of any atom. This is spin angular momentum and facts about this can be deduced from the connection between spin angular momentum and magnetic dipole moment.

$$U = \frac{-ee}{4\pi\epsilon_0 r}$$

not just charge

electron

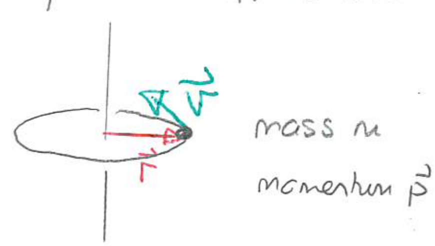


Either of these quantities when measured in a quantum system yield discrete outcomes.

We initially describe spin angular momentum for classical rigid objects. This is rooted in orbital angular momentum of a point particle. Suppose that a particle orbits in a circle as illustrated.

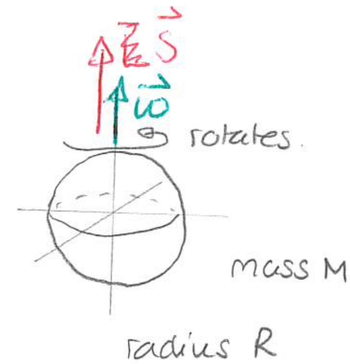
Then it has orbital angular momentum

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = mvr\,\hat{z}$$



This can be extended to a solid object by breaking it into imaginary infinitesimal pieces and adding the angular momentum of each piece. In the case of an object that spins around its center of mass we call the angular momentum <u>spin angular momentum.</u>

For example for a solid sphere the spin angular momentum is

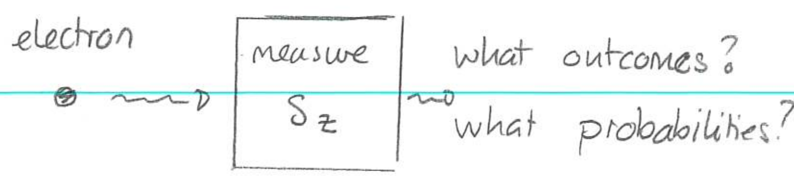


$$\vec{S} = \frac{2}{5} M R^2 \vec{\omega}$$

where $\vec{\omega}$ is the angular velocity vector. We might expect that an electron constitutes such a spinning solid object and therefore has a spin angular momentum. It emerges that it does although it cannot be associated with the rotation of a solid object. Regardless of this the spin angular momentum is a vector

$$\vec{S} = S_x \hat{x} + S_y \hat{y} + S_z \hat{z}$$

and we can measure any of the components.

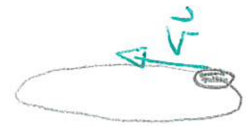


We now ask how one could measure any or all of the components of spin angular momentum.

Magnetic dipole moment

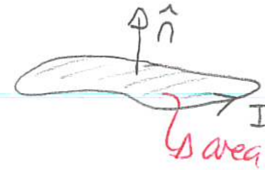
Consider an orbiting charged particle. This will

1) produce a magnetic field
2) respond to external magnetic fields.



These can be described to a good approximation via the magnetic dipole moment of the moving charge. For a loop of current the dipole moment is

$$\vec{\mu} = I A \hat{n}$$



where A is the area of the loop and $\hat{n}$ the normal vector perpendicular to the surface of the loop. There are general rules for determining such magnetic dipole moments for any arrangement of moving charges and currents. For a point charge moving with velocity $\vec{v}$ the rule becomes (see PHYS 311)

$$\vec{\mu} = \frac{1}{2}\vec{r} \times Q\vec{v} = \frac{1}{2} Q(\vec{r} \times \vec{v})$$

$\rightarrow \vec{\mu} = \frac{1}{2}\int \vec{r} \times \underbrace{\rho_c}_{\text{charge density}} \vec{v}\, dV$

We can then see that

$$\vec{\mu} = \frac{1}{2}\frac{Q}{M}\underbrace{(\vec{r} \times \vec{p})}_{\vec{L}}$$

Thus, for a moving point charge the magnetic dipole moment is related to the orbital angular momentum by

$$\boxed{\vec{\mu} = \frac{Q}{2M}\vec{L}}$$

We can extend this analysis to rigid massive charge objects that spin

We find that

> If the mass is distributed in the same way as the charge then
>
> $$\vec{\mu} = \frac{Q}{2M}\vec{S}$$
>
> where Q = total charge of the spinning object
> M = " mass " " " "

For example

uniform mass distribution / uniform charge distribution

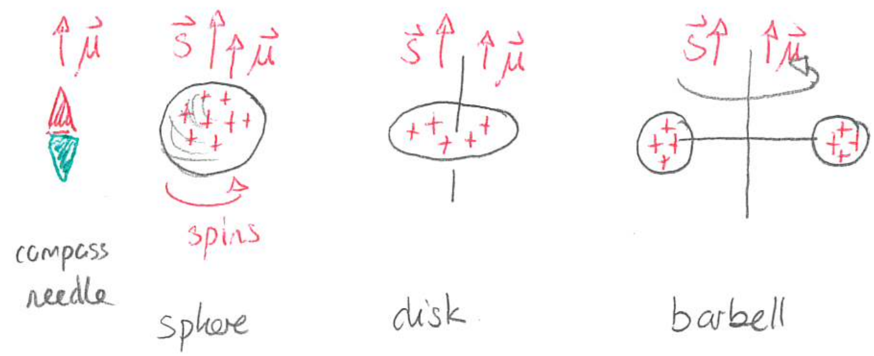
$$\vec{\mu} = \frac{Q}{2M}\vec{S}$$

Quiz!

The previous rule is only true if the charge is uniformly distributed. Even when it is not a general rule applies.

> For rigid rotating distributions of mass and charge
>
> $$\vec{\mu} = \frac{g}{2}\frac{Q}{M}\vec{S}$$
>
> where Q = total charge of distribution
> M = " mass " "
> g = g-factor (describes how the charge and mass distributions are related.

We can calculate these for all types of distributions:



→ if mass is distributed in the same way as charge then $g=1$

→ if mass + charge are distributed differently $g \neq 1$.

Again the magnetic dipole moment has components

$$\vec{\mu} = \mu_x \hat{x} + \mu_y \hat{y} + \mu_z \hat{z}$$

and

$$\mu_x = \frac{gQ}{2M} S_x$$

$$\mu_y = \frac{gQ}{2M} S_y$$

$$\mu_z = \frac{gQ}{2M} S_z$$

so

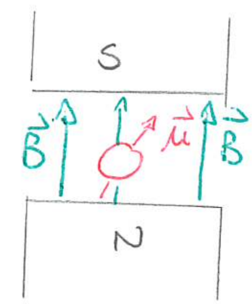
If one can measure the components of the magnetic dipole moment then one can obtain the components of spin

The question now becomes how to measure the magnetic dipole moment of any object, particularly subatomic particles.

Magnetic Dipoles in Magnetic Fields

The crucial step is to place the magnetic dipole in a magnetic field produced by external magnets.

Classical electromagnetic theory gives:



> The energy of a dipole in an external field is
>
> $$U = -\vec{\mu}\cdot\vec{B}$$

Then the force exerted on the dipole is:

$$\vec{F} = -\vec{\nabla} U$$
$$= -\vec{\nabla}(-\vec{\mu}\cdot\vec{B}) = \vec{\nabla}(\vec{\mu}\cdot\vec{B})$$

Thus we need to place the particle in a magnetic field and observe the forces on the particle

Quizz

Demo: Stern-Gerlach (toutestquantique)