Phys 231 2021

Lecture 12

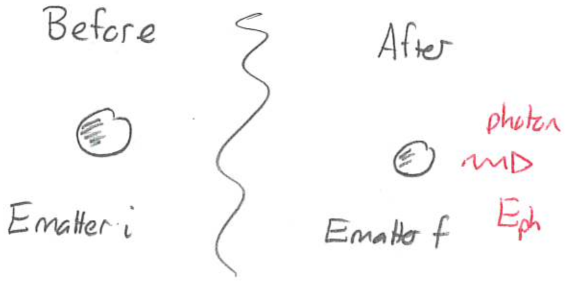
Mon: Read

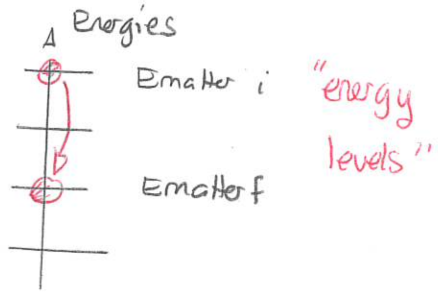
HW 4 by 5pm

Interaction between radiation and quantized matter

In many situations the possible energies of matter will be quantized. When such matter interacts with electromagnetic radiation, energy conservation will then imply that the spectrum of radiation emitted or absorbed will be discrete.

Emission

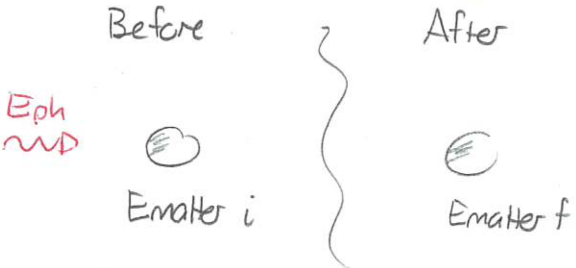




Then $E_{ph} = |\Delta E_{matter}| = E_{matter\ i} - E_{matter\ f}$

$$\Rightarrow \boxed{hf = \frac{hc}{\lambda} = E_{matter\ i} - E_{matter\ f}}$$

Absorption



$$\boxed{\Delta E_{ph} = |\Delta E_{matter}|}$$

$$\boxed{hf = \frac{hc}{\lambda} = E_{matter\ f} - E_{matter\ i}}$$

Given this, the remaining task is to determine the possible energy levels for matter; e.g.

1) hydrogen atom
2) helium atom
3) water molecule
4) copper crystal

Hydrogen atom models

Precise spectroscopy measurements give the wavelengths of electromagnetic radiation emitted or absorbed by a hydrogen atom. We find.

410.17 nm

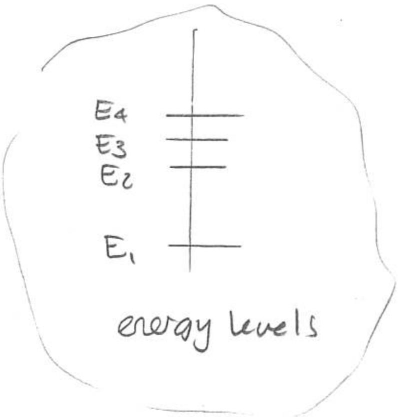
434.05 nm

486.13 nm

656.28 nm

Demo: Show NIST ASD database.

This information was known well before quantum theory. We then want to determine energy levels for the hydrogen atom so that



→

Various ΔE_{atom} give various photon energies

$E_{ph} = |\Delta E_{atom}|$

→

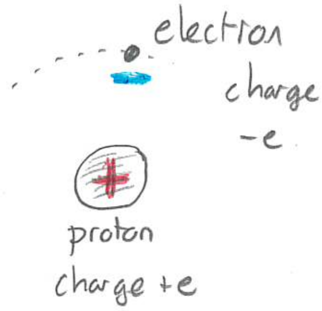
Predict wavelengths

$\lambda = \dfrac{hc}{|\Delta E_{atom}|}$

↓

Match observed wavelengths

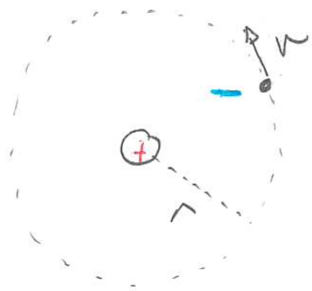
Any model for this will assume a basic structure for the hydrogen atom:



- a single proton (nucleus)
- a single electron (periphery)

Classical models would assume a specific orbit for the electron. In the simplest such models, the proton is stationary and the electron orbits in a circular path. Thus a model might have:

1) electron orbits in a circular path with radius r and speed v. The electron has mass m_e



2) the laws of classical mechanics and electromagnetism apply to the situation. These include:

Newton's 2^{nd} Law : $\vec{F} = m_e \vec{a}$

Centripetal acceleration : $a = \frac{v^2}{r}$

Coulombs law for electrostatic forces : $F_{elec} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

Electrostatic potential energy : $U_{elec} = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$

Kinetic energy $K = \frac{1}{2} m_e v^2$

Orbital angular momentum : $L = m_e v r$

We now apply classical mechanics to determine the total energy $E = K + U_{elec}$ for an orbit.

Quiz 1 10%-30%

The energy is:

$$E = \frac{1}{2} m_e v^2 + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

But $q_1 = e$, $q_2 = -e$. Thus

$$E = \frac{1}{2} m_e v^2 - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

In order to determine the velocity consider forces:

Quiz 2

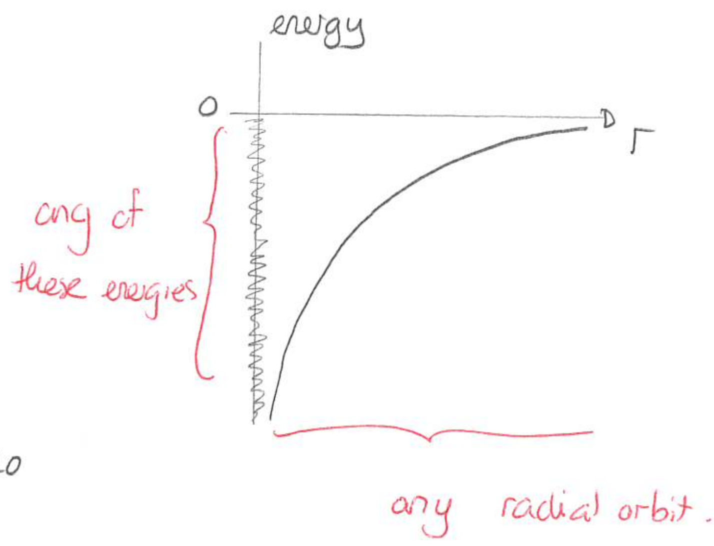
$$|F| = m_e a \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{m_e v^2}{r}$$

$$\Rightarrow m_e v^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

Thus we obtain

For a classical circular orbit with radius r, the energy of the electron is $E = -\frac{1}{8\pi\epsilon_0} \frac{e^2}{r}$

So classically we predict a continuous range of energies and therefore a continuous emission spectrum.



The first attempt to quantize this was produced by Bohr who added one more requirement:

The orbital angular momentum can only take on values given by

$$L = n\hbar$$

where $n = 1, 2, 3, \ldots$

This is really an assumption but we can see how one might arrive at this:

$$L = m_e v r = p r$$

But deBroglie gives $\lambda = h/p \Rightarrow p = h/\lambda$

$$\Rightarrow L = \frac{hr}{\lambda}$$

Now if one considered a wave encircling the nucleus we would need that the circumference is an integral number of wavelengths.

So $2\pi r = n\lambda \Rightarrow \lambda = \frac{2\pi r}{n}$

$$\Rightarrow L = \frac{h}{2\pi} n = \hbar n.$$

Then this rule gives

$$m_e v r = n\hbar \qquad n=1,2,3\ldots$$

and this implies that only certain orbital radii exist. Thus

The Bohr model predicts that possible orbital radii are

$$r_n = \frac{4\pi\epsilon_0 \hbar^2}{m e^2}\, n^2$$

where $n=1,2,3,\ldots$

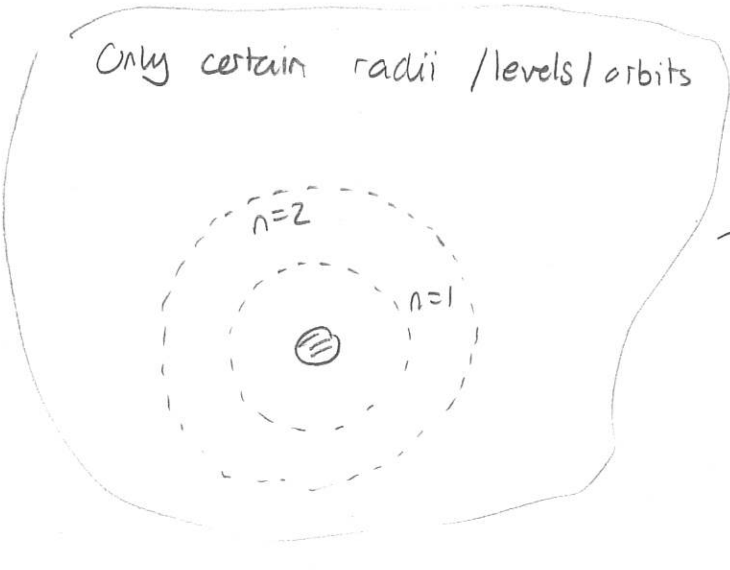
Then the prediction becomes:

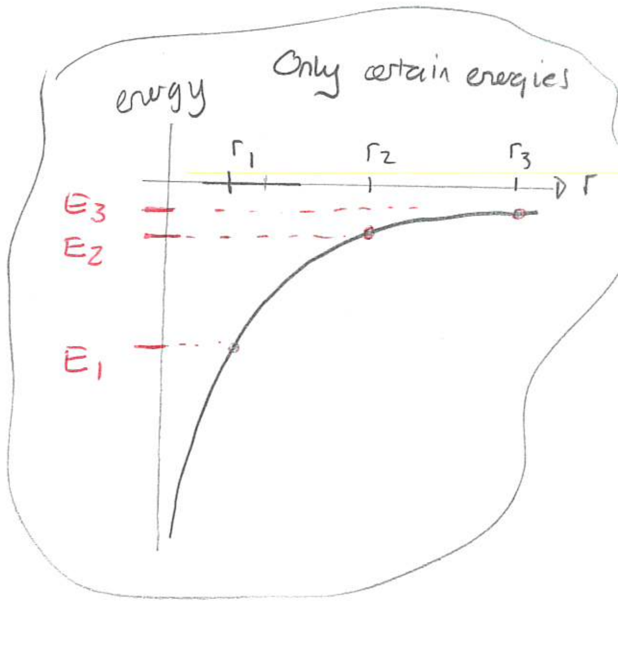
The Bohr model predicts that only certain possible energies exist:

$$E_n = -\frac{m_e e^4}{2(4\pi\epsilon_0)^2\hbar^2}\,\frac{1}{n^2}$$

where $n=1,2,3\ldots$

Thus we have





The smallest value of n corresponds to the smallest orbital radius and the lowest energy. Here $n=1$ and

$$r_1 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} \equiv a_0 \quad \text{(Bohr radius)}$$

Calculations give

$$a_0 = 0.0529\,\text{nm}$$

Then

$$E_1 = \frac{-m_e e^4}{2(4\pi\epsilon_0)^2\hbar^2} = -13.6\,\text{eV}$$

is the lowest (ground state) energy.

Quizz 100%

The predicted energy transitions predict:

1) a discrete spectrum
2) actual observed spectral wavelengths