Phys 231 2021

Lecture 11

Thurs: Seminar via Zoom??

Fri: Read 4.6

Mon: HW

deBroglie relation

The method for describing certain phenomena associated with material particles is simplest for the case of free particles:

Physical Situation

free particle with momentum $\vec{p}$ in x direction

Mathematical description

Associate complex wave with particle

$$\Psi = Ae^{i(kx-\omega t)}$$

where $k = 2\pi/\lambda$ and

$$\lambda = \frac{h}{p}$$ de Broglie

↓

Track /propagate this wave according to usual wave propagation /optics.

↓

Probability $\sim |\Psi|^2$ →

Physical Outcomes

Describes what happens when measurements are performed

The deBroglie relation is crucial since it provides wavelengths that are important for interference situations. Note that

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p$$

We define "h-bar" as

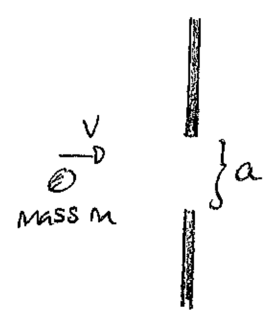
$$\hbar = \frac{h}{2\pi}$$

Then $$\boxed{k = \frac{p}{\hbar}}$$

Evidence for the wave picture

Particle diffraction and interference experiments were only done decades after the proposal of the wave picture and the deBroglie relation. The reason is that for any diffraction or interference effects to be observed the wavelength must be larger than or comparable to the slit width. We can consider a particle with mass m and velocity v moving toward a slit with width a. We need

$$a < \lambda = \frac{h}{mv}$$



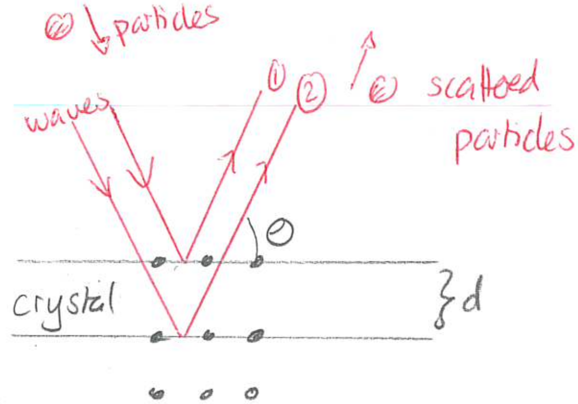
$$\Rightarrow \quad a < \frac{h}{mv}$$

Consider a particle with mass $0.00050\,kg$ and speed $0.20\,m/s$

Then
$$a < \frac{6.63 \times 10^{-34}\,Js}{0.00050\,kg \times 0.20\,m/s} = 6.63 \times 10^{-30}\,m$$

We cannot provide material gaps of this size.

However, the combination of subatomic particles and crystals that act as gratings can provide circumstances where such interference effects are noticeable. Then the Bragg condition determines the angles at which electrons are strongly scattered



$$\boxed{2d\sin\theta = m\lambda}$$

where $m = 0, 1, 2, \ldots$ For crystals $d \sim 10^{-9}$ m and for electrons $\lambda = \frac{h}{mv} \sim \frac{10^{-33}}{10^{-30} \times V} \approx \frac{10^{-3}}{V}$ can give plausible wavelengths so that scattering is observed. Such effects were first observed by Davison + Germer in 1927

Demo: PhET Electron Scattering (Java only)

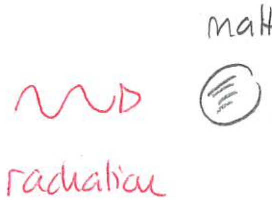
Demo: Google electron diffraction images

Quiz 1 80%

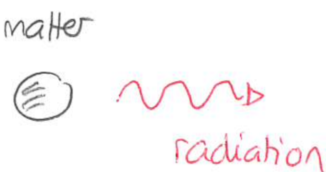
Demo: Electron microscopy images. (Wikipedia)

Matter + radiation interactions

An additional crucial aspect of early quantum theory is the set of rules which describe interactions between matter + radiation. Examples are



absorption of radiation



emission of radiation

In the case of emission, in many situations the light emitted has a spectrum consisting of discrete frequencies or wavelengths

Demo: McQuarrie images

Generically we can describe such situations via:

Only certain discrete energies are possible for matter

Energy associated with radiation is absorbed or emitted via single photons, each with energy $E_{ph} = hf = hc/\lambda$

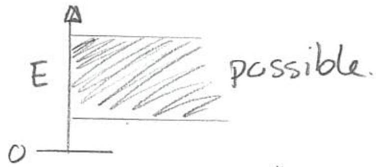
Energy is conserved on a per photon basis

$$|\Delta E_{matter}| = E_{ph}.$$

Now consider competing models for matter.

Classical

Any energy in some range is possible



$\Downarrow$

$|\Delta E_{matter}|$ has continuous range

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E_{ph} has continuous range

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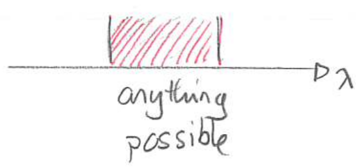
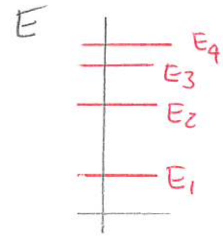
λ emitted/absorbed has continuous range



Quantum

Only some energies possible



$\Downarrow$

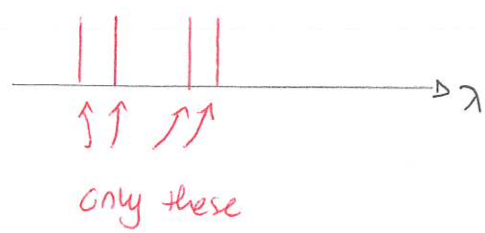
Only certain discrete $|\Delta E_{matter}|$

$\Downarrow$

Only certain discrete E_{ph}

$\Downarrow$

Only certain discrete λ

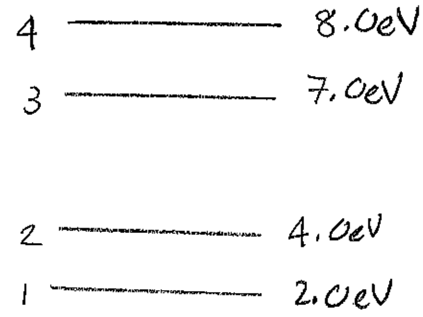


Additionally in quantum theory, transitions between any pair of matter energy levels are possible.

Quiz 2 50% – 70%

Quiz 3 100%

Example: Consider a hypothetical system whose energy levels are as illustrated



Determine:

a) smallest wavelength emitted

b) largest wavelength "

Answer: In all cases

$$|\Delta E_{atom}| = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{|\Delta E_{atom}|}$$

a) largest $\Delta E_{atom} \Rightarrow 4 \to 1$ and $|\Delta E_{atom}| = 6.0eV$

$$|\Delta E_{atom}| = 6.0eV \times 1.6\times10^{-19} J/eV$$

$$= 9.6\times10^{-19} J.$$

$$\lambda = \frac{hc}{9.6\times10^{-19}J} = \frac{6.63\times10^{-34} J\cdot s \times 3.0\times10^{8} m/s}{9.6\times10^{-19} J} = 207 nm.$$

b) smallest $|\Delta E_{atom}| \Rightarrow 4 \to 3$ and $|\Delta E_{atom}| = 1.0eV$

$$= 1.6\times10^{-19} J.$$

$$\lambda = \frac{hc}{1.6\times10^{-19}J} = \frac{6.63\times10^{-34} J\cdot s \times 3.0\times10^{8} m/s}{1.6\times10^{-19} J} = 1240 nm$$