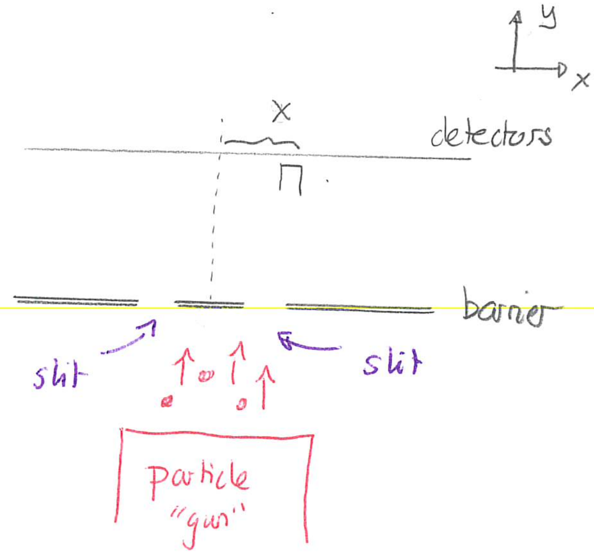
Weds: Read. 4.2

Diffraction and Interference for Matter

We now consider whether it is possible to observe wavelike phenomena with matter. This means that we aim to observe interference or diffraction of particle matter such as electrons, neutrons, atoms or molecules. A typical experiment of this type would be

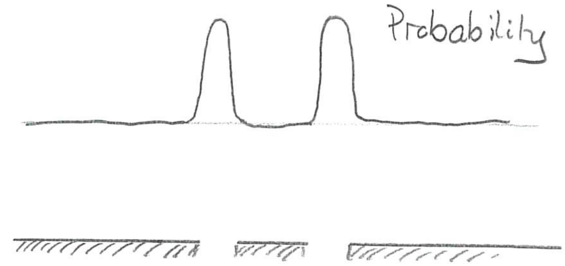


1) fire identical particles, each with same momentum and energy toward barrier
2) detect particle arrival at various locations beyond the barrier.
3) arrange for the experiment to be such that only one particle is present at any single time.
4) although the particles each have the same momentum their horizontal position can vary.
5) any particle that strikes the barrier is absorbed/removed

When such experiments are done one observes:

1) each particle only arrives at one location/detector (particles don't split)
2) one can count the number of particles that arrive at each detector location:

If the particles behaved as classical "pool balls" then the arrival probability would be as illustrated.



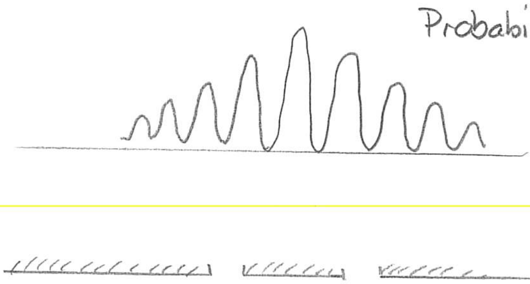
The actual experiments have been done with:

1) electrons
2) neutrons
3) larger molecules.

Demo: IMM movie → YouTube version 10:50 min

Demo: Show images from Zeilinger article

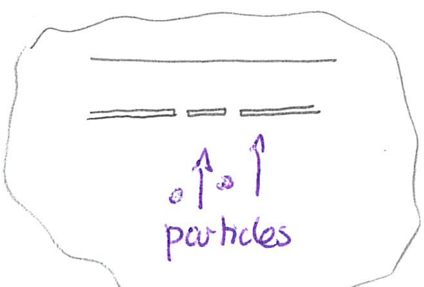
~~Demo: Show Cronin~~
~~440~~



Experiments like this reveal a probability distribution that is similar to the intensity profile for interference of light. Thus

> Interference experiments with matter reveal wavelike behavior of matter, including electrons, neutrons and various molecules. This appear to be a general property of matter.

So we have



→ Describe particles using waves Ψ → Interference / superposition of waves → Intensity $|\Psi|^2$ → Probability

Demo: Slides 1-3

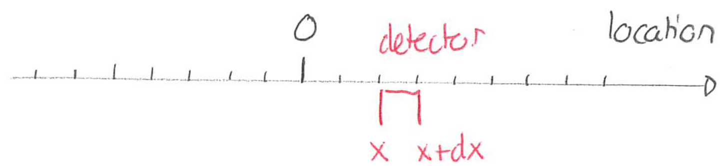
Quiz 1

Demo: Slide 4 Remove right barrier.

Demo: Zeilinger article, Cronin article pg 1056

Probabilities

Experiments indicate that one cannot predict the arrival location any single particle with certainty. On the other hand quantum theory can predict the probabilities with which particles will arrive at various locations. We need a language for such probabilities. For positions this is complicated by the fact that position is represented by a continuous variable. There are infinitely many possibilities.



However, any detector will have a finite resolution or width dx. With such a detector one can determine the fraction of incident particles that reach the detector. Let

x = left edge of detector

dx = width of detector

$N(x)$ = number of particles entering detector

N = total number of particles arriving at any location.

Then an intuitive idea of probability of arrive is

$$\frac{N(x)}{N} \Big\} \text{ measured}$$

and this can be measured. Then theory will predict what this should be. This will give a theoretical quantity: the probability density or probability distribution, $P(x)$.

We will define $P(x)$ correctly later. However roughly

$$\boxed{P(x)\,dx = \text{prob particle arrives in region } x \to x+dx}$$

and we expect that

$$\underbrace{P(x)dx}_{\text{calculated}} \approx \left.\frac{N(x)}{N}\right\} \text{measured}$$

For the moment we will use probability densities in conceptual ways

Matter Waves

The following process can describe particle interference experiments.

Particles with known mass, momentum, ...

↓

Associate complex wave with particles

$\Psi(x,y,t)$

↓

Barrier / slit arrangement

↓

Use wave propagation / superposition

↓

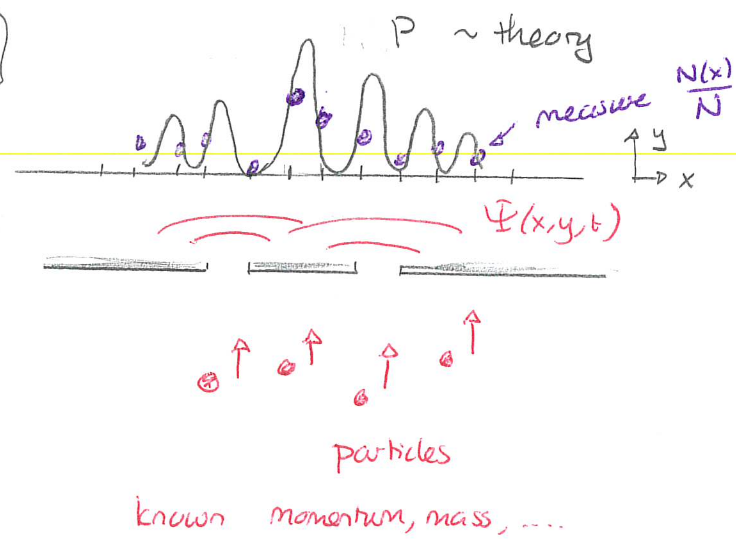
Probability density

$P(x,y,t) = |\Psi(x,y,t)|^2$

$\to$

Predicts fraction arriving at various locations

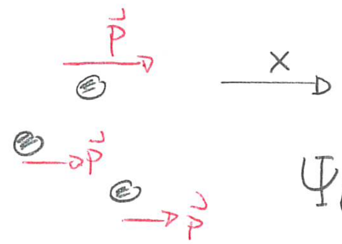
$P \to \frac{N(x)}{N}$



de Broglie Relation

In order to make such a scheme work we need to construct a complex wave from the particle information. We need:

1) a situation which we can describe using complex exponentials



$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

2) a wavelength and frequency for such waves.

We know that interference phenomena usually involve position differences and not time differences so we will need to focus on the wavelength aspects

The physical situation that we consider is a series of particles traveling along the +x direction (the values of y, z could be arbitrary) with well defined momentum p. Then

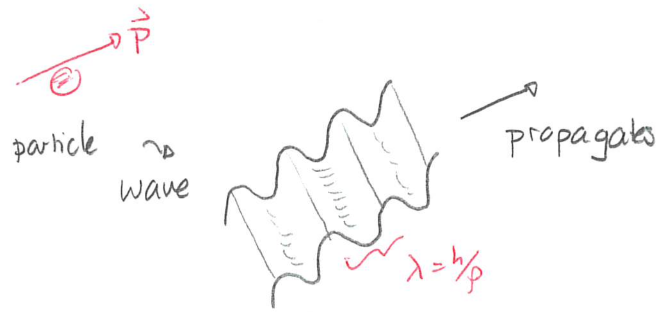
Particle traveling freely along x $\longrightarrow$ $\Psi(x,t) = Ae^{i(kx-\omega t)}$

Then the wavenumber is determined via the wavelength which is given by the deBroglie relation.

Consider a free particle with well defined momentum $\vec{p}$. Then the wave associated with this propagates in the direction of $\vec{p}$ and has wavelength

$$\lambda = \frac{h}{p}$$

deBroglie relation



We could apply this to particles fired through a single slit

In this case let

$a =$ width of slit

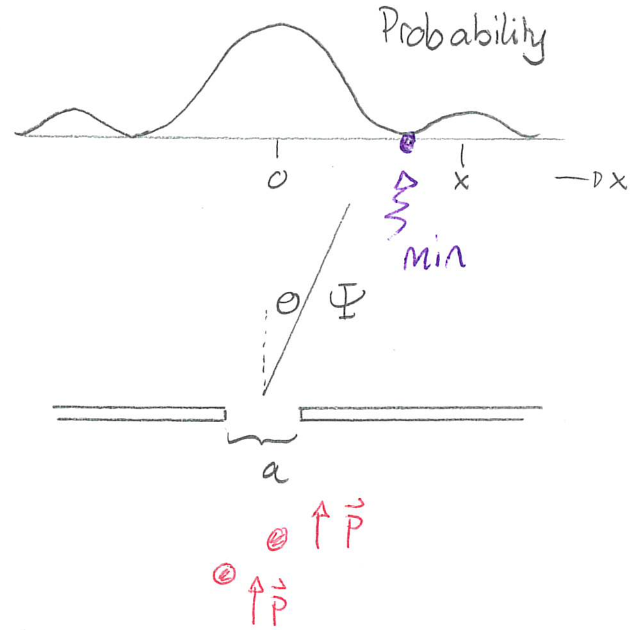
$p =$ momentum of particles

These describe the situation. The detection parameters are:

$\theta =$ angle from "straight through

$x =$ horizontal position along detector array.

$L =$ distance from barrier to detectors.



Then we associate:

wave

$$\Psi = Ae^{i(kr-\omega t)}$$

$$= Ae^{i\left(\frac{2\pi r}{\lambda}-\omega t\right)}$$

$\longrightarrow$

Superposition gives probability density at θ

$$P(\theta) = P_0\left[\frac{\sin\alpha}{\alpha}\right]^2$$

$$\alpha = \frac{\pi a}{\lambda}\sin\theta$$

We can discern aspects of the pattern by focussing on the first minimum.

Quiz 2

Quiz 3