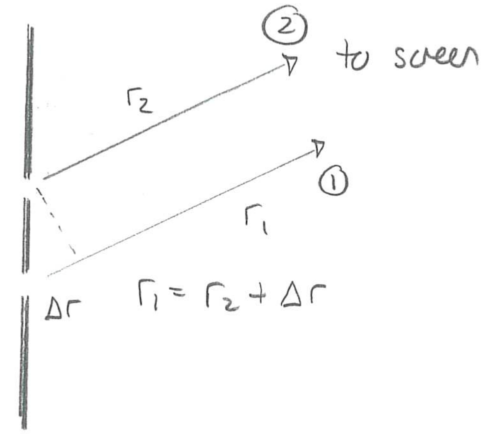
Lecture 9

Mon: HW by 5pm

Read 4.1, 4.2

Interference of light: wave picture

The process of interference of light can be described by adding waves. In terms of complex exponentials the two waves are



$$\Psi_1 = A e^{i(kr_1 - \omega t)}$$

$$\Psi_2 = A e^{i(kr_2 - \omega t)}$$

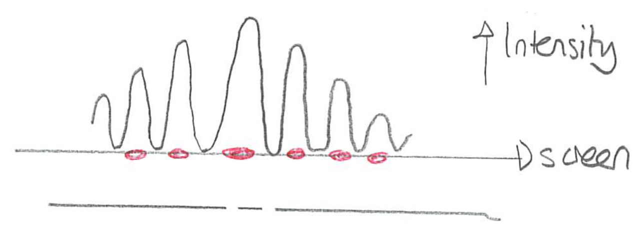
and we form

$$\Psi = \Psi_1 + \Psi_2$$

and use geometry and addition of complex exponentials to simplify this. The intensity will then be

$$I = \text{const} \times |\Psi|^2$$

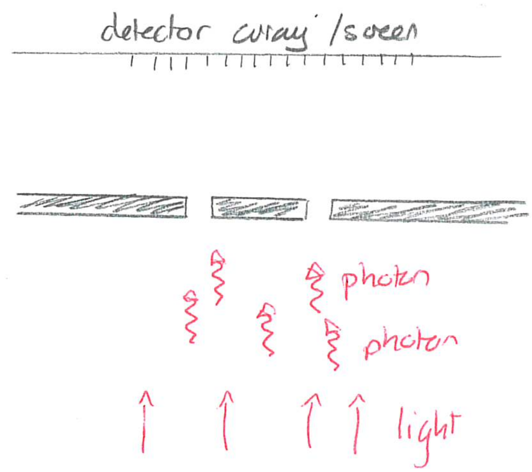
$$= \text{const} \times (\Psi \Psi^*)$$



and the result shows typical interference patterns and lets us determine locations of maximum intensity (bright) and minimum intensity (dark)

Interference experiments with photons

If the intensity of the light is sufficiently low then we can observe the arrival of single photons. The light incident on the slits is such that if the intensity were increased it would illuminate both slits. We would like to predict where individual photons arrive. Experiments and observations of this reveal that:



1) <u>each photon</u> that does pass the barrier arrives at exactly <u>one location</u> on the screen (i.e. at exactly one detector).

2) even if the photons are all prepared identically the arrival location of any single photon cannot be predicted with certainty. These arrival locations will vary statistically. This means that if the arrival location of one photon is known (after detection) the next photon will not necessarily arrive at that location.

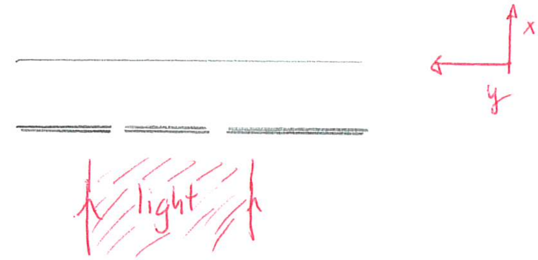
<u>Demo</u>: PhET Quantum Wave Interference
→ Photons Tab / Single Particles
→ Two slits – show individual photons

~~Demo 2 Day~~

Although we cannot predict the arrival of any single photon, we can predict the arrival of photons statistical. In general

> Quantum theory can predict the probabilities with which various measurable outcomes occur.

The process is :



Given light / photons with frequency f and wavelength $\lambda = c/f$

Associate a wave with the light / photons : Ψ

e.g. $\Psi(x,t) = A\cos(kx - \omega t - \phi)$

$= A\cos\left(\frac{2\pi}{\lambda}x - 2\pi f t - \phi\right)$

OR

$\Psi(x,t) = A e^{i(kx - \omega t - \phi)}$

Use the classical optics of waves to calculate how wave propagates

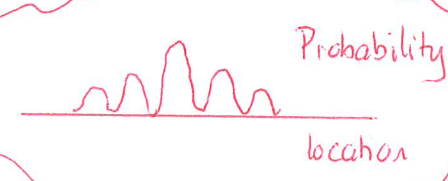
e.g.

$\Psi = \Psi_1 + \Psi_2$ superposition

Predict intensities using

$I = \text{const} \times |\Psi|^2$

<u>Probability</u> with which photon is detected at a given location is proportional to calculated intensity at that location



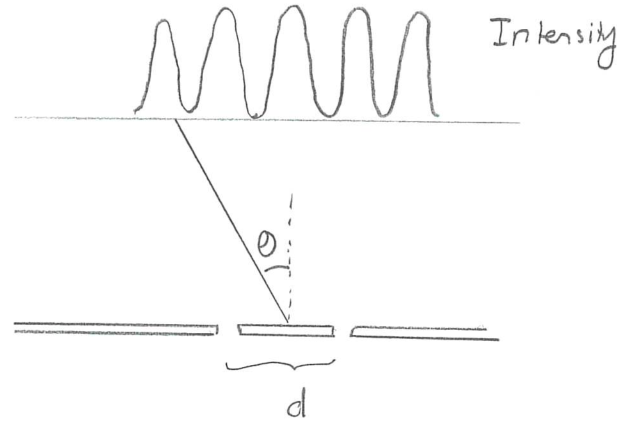
<u>Demo</u>: Physclips Vol3 → Light

→ Single Photons

So we will use waves to do all of the calculations. But the results of these calculations describe how photons - "particles" - behave

Quiz 1

Double slit



Consider light passing through two slits, each with negligible width.

We then need to add two waves and have seen that

$$\Psi = \Psi_1 + \Psi_2 \quad \text{and} \quad I = \text{const} \times |\Psi|^2$$

$$\Downarrow$$

$$I = I_0 \cos^2 \beta \qquad \text{where } \beta = \frac{\pi d}{\lambda} \sin\theta$$

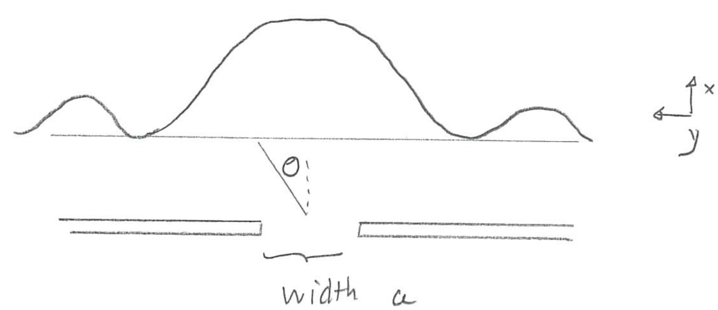
$$\Downarrow$$

$$\boxed{\text{Prob} = \text{const} \times \cos^2\left[\frac{\pi d \sin\theta}{\lambda}\right]}$$

We get maximum probabilities when $\frac{\pi d \sin\theta}{\lambda} = m\pi \Rightarrow \boxed{d\sin\theta = m\lambda}$

$m = 0, \pm 1, \pm 2, \ldots$

Single slit



Consider light passing through a single slit of width a. The arrival probability for photons can be calculated by using waves:

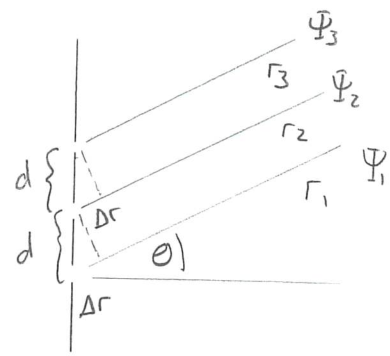
$$\Psi = \sum_{\substack{\text{each location in} \\ \text{opening ``y''}}} \Psi_y \Rightarrow I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{where } \alpha = \frac{\pi a}{\lambda} \sin\theta$$

$$\Rightarrow \boxed{\text{Prob} = \text{const} \times \left[\frac{\sin\alpha}{\alpha}\right] \quad \text{with } \alpha = \frac{\pi a}{\lambda} \sin\theta}$$

* Slide - Single slit

Quiz 2

Example: Suppose that light is incident on three slits with adjacent spacings, d. The light is monochromatic with wavenumber k and angular frequency ω. Determine the probability profile for photon arrival at any angles, θ.



Answer: $\Psi = \Psi_1 + \Psi_2 + \Psi_3$

use waves

$$\Psi_1 = A e^{i(kr_1 - \omega t)} \qquad r_1 = r_2 + \Delta r$$

$$\Psi_2 = A e^{i(kr_2 - \omega t)} \qquad r_3 = r_2 - \Delta r$$

$$\Psi_3 = A e^{i(kr_3 - \omega t)}$$

So

$$\Psi = A e^{i(k(r_2+\Delta r) - \omega t)} + A e^{i(kr_2 - \omega t)} + A e^{i(k(r_2-\Delta r) - \omega t)}$$

$$= A e^{i(kr_2 - \omega t + k\Delta r)} + A e^{i(kr_2 - \omega t)} + A e^{i(kr_2 - \omega t - k\Delta r)}$$

$$= A e^{i(kr_2 - \omega t)} e^{ik\Delta r} + A e^{i(kr_2 - \omega t)} + A e^{i(kr_2 - \omega t)} e^{-ik\Delta r}$$

$$= A e^{i(kr_2 - \omega t)} \left[e^{ik\Delta r} + 1 + e^{-ik\Delta r} \right]$$

$$= A e^{i(kr_2 - \omega t)} \left[1 + \underbrace{e^{ik\Delta r} + e^{-ik\Delta r}}_{2\cos k\Delta r} \right]$$

$$\Rightarrow \quad \Psi = A e^{i(kr_2 - \omega t)} \left[1 + 2\cos k\Delta r \right]$$

Intensity → $I = \text{const} \times |\Psi|^2$

and $|\Psi|^2 = |A|^2 |e^{i(kr_2 - \omega t)}|^2 \, |1 + 2\cos k\Delta r|^2$

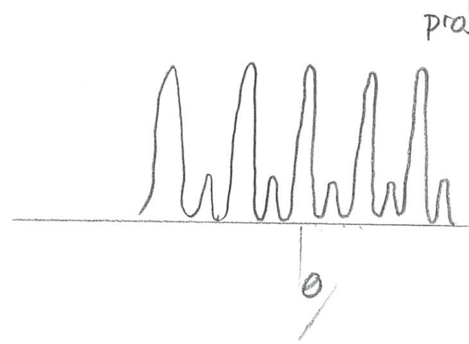
$$= A^2 (1 + 2\cos k\Delta r)^2$$

probability

Then $\quad \text{prob} = \text{const}\,(1 + 2\cos k\Delta r)^2$

and $\quad \Delta r = d\sin\theta.$

$$\text{prob} = \text{const}\left[1 + 2\cos(kd\sin\theta)\right]^2$$



We can establish when this yields maxima and minima. We just need to determine when

$$f(x) = \left[1 + 2\cos x\right]^2$$

has maxima or minima. We need

$$f'(x) = 0$$

This gives

$$x = m\pi \qquad (\text{local max})$$

$$\cos(x) = -\tfrac{1}{2} \qquad (\text{local min})$$

$$\Rightarrow \quad x = \pm\frac{2\pi}{3}, \pm\frac{8\pi}{3}$$

So $\quad$ max : $\quad x = 0, \pm 2\pi, \pm 4\pi \quad \rightarrow$ brightest

$\qquad\qquad\qquad x = \pm\pi, \pm 3\pi \quad \rightarrow$ intermediate

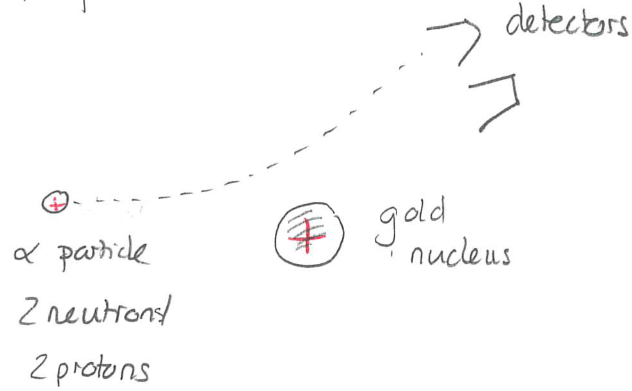
$\qquad\qquad\qquad x = \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}, \pm\frac{8\pi}{3} \quad \rightarrow$ dark.

Particle interference??

Given that light can be described in terms of photons and these behave in some ways like particles but often still need waves to describe their behavior, we ask whether the same is true for particles.

We can investigate such issues for small subatomic particles like electrons, protons or neutrons and even atomic nuclei. In scattering experiments we find that they could be described in particle terms

This sort of experiment was first performed successfully by Rutherford in 1909-1911



Demo: PhET Rutherford Scattering

The observed phenomena could be described by:

1) determining the path followed by each α-particle
2) using ordinary momentum $\vec{p} = m\vec{v}$
3) using kinetic energy $K = \frac{1}{2}mv^2$
4) " electrostatic potential energy $U = -k\frac{q_1 q_2}{r}$
5) using energy and momentum conservation.

This would then suggest:

> Matter behaves like particles with well defined position, energy momentum and trajectories

In order to observe any wavelike phenomena we would need some type of interference. How can this be done?