Mon: HW by 5pm

Weds: Read

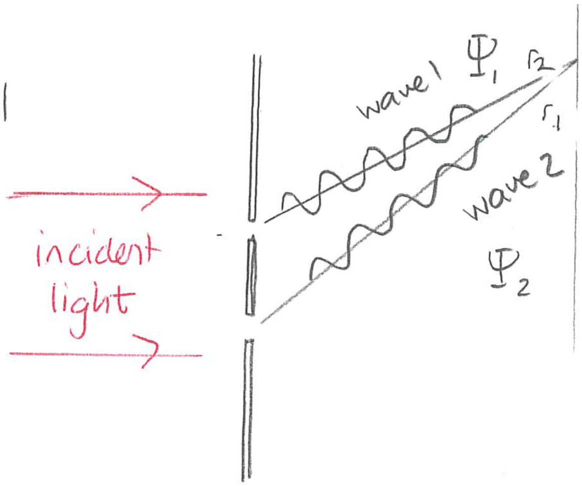
Interference of waves in double slit experiments

The process by which waves form superpositions describes the patterns seen in interference experiments.

Each slit produces a sinusoidal wave (they radiate spherically).

Then the wave that arrives at the screen is

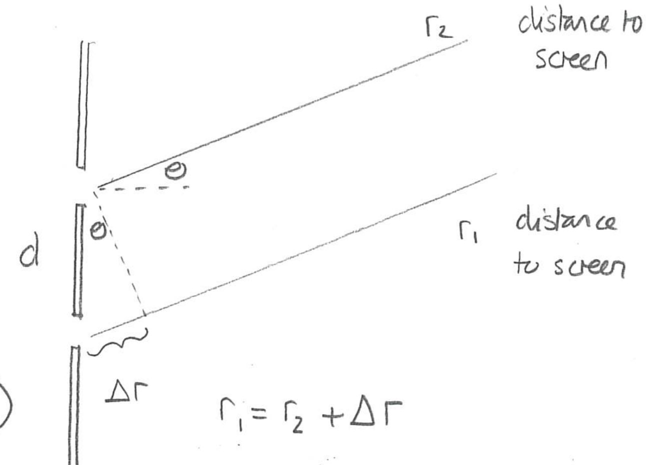
$$\Psi = \Psi_1 + \Psi_2$$



We need to form the superposition. If the two waves are "in phase" at the slits then at the screen:

$$\Psi_1 = A\cos(kr_1 - \omega t)$$

$$\Psi_2 = A\cos(kr_2 - \omega t)$$



$r_1 = r_2 + \Delta r$

$r_2 = r_1 - \Delta r$

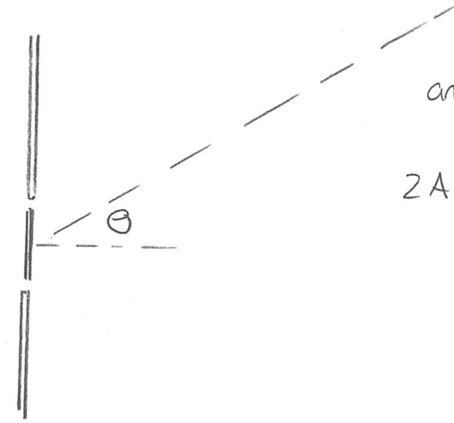
Then

$$\Psi = A\cos(kr_1 - \omega t) + A\cos(k(r_1 - \Delta r) - \omega t)$$

$$= A\cos(kr_1 - \omega t) + A\cos(kr - \omega t - k\Delta r)$$

$$\Rightarrow \Psi = \underbrace{2A\cos\left(\frac{k\Delta r}{2}\right)}_{\text{amplitude}} \cos(kr - \omega t)$$ By previous class.

So we get that the intensity of the pattern at angle θ is



amplitude

$2A\cos\left(\frac{k\Delta r}{2}\right)$

$$I = \text{const} \times \left[2A\cos\left(\frac{k\Delta r}{2}\right)\right]^2$$

$$= \underbrace{4A^2\text{const}}_{I_0} \times \cos^2\left(\frac{k\Delta r}{2}\right)$$

I_0 (intensity when $\Delta r = 0$ - in middle

$$I = I_0 \cos^2\left(\frac{k\Delta r}{2}\right)$$

Now approximately $\frac{\Delta r}{d} = \sin\theta$. Thus $\Delta r = d\sin\theta$

$$\frac{k\Delta r}{2} = \frac{2\pi}{\lambda}\frac{d\sin\theta}{2} = \frac{\pi d\sin\theta}{\lambda}$$

So we get:

Using waves gives a double slit intensity profile.

$$I = I_0\cos^2\beta$$

where $\beta = \frac{\pi d}{\lambda}\sin\theta$

Demo: Double Slit Slide

Quiz 1 30%

Demo: Other slides

In general we can assess any such interference phenomena by add such waves and evaluating the superposition, However, adding more than two sinusoidal functions becomes tedious. We will present a more efficient technique using complex exponentials.

Complex numbers

Complex numbers are constructed from ordinary numbers and the imaginary square root of -1. This is denoted

$$i = \sqrt{-1} \quad \Rightarrow \quad i^2 = -1$$

A complex number is a pair of real numbers (u,v) combined as:

$$z = \underbrace{u}_{\text{real part of } z \;\; Re[z]} + i \underbrace{v}_{\text{imaginary part of } z \;\; Im[z]}$$

complex number

We can define algebraic operations involving complex numbers.

Let $z_1 = u_1 + i v_1$

$z_2 = u_2 + i v_2$

Then

1) $z_1 = z_2 \iff u_1 = u_2$ AND $v_1 = v_2$ equality

2) $z_1 + z_2 = (u_1 + u_2) + i(v_1 + v_2)$ addition

3) $z_1 z_2 = (u_1 u_2 - v_1 v_2) + i(u_1 v_2 + u_2 v_1)$ multiplication

Note that multiplication is defined so that one can use the distributive rule:

$$(u_1+iv_1)(u_2+iv_2) = u_1(u_2+iv_2) + iv_1(u_2+iv_2)$$

$$= u_1u_2 + iu_1v_2 + iv_1u_2 + \underbrace{i^2}_{-1} v_1v_2$$

$$= u_1u_2 - v_1v_2 + i(u_1v_2+u_2v_1).$$

Quiz 2

The is an additional special algebraic operation for complex numbers called complex conjugation.

If	$z = u+iv$ then the complex conjugate of z is
	$z^* = u - iv$

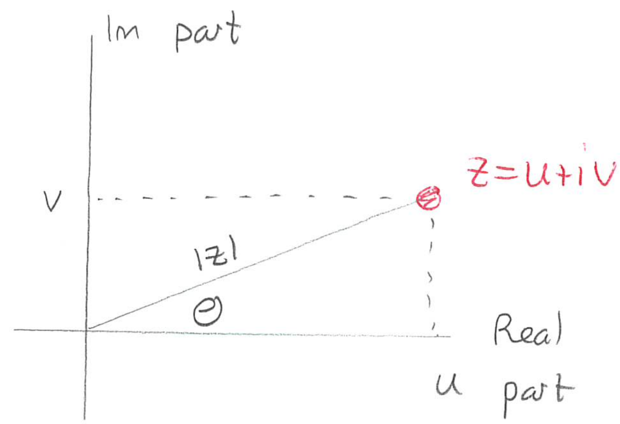
In complex conjugation one replaces $i \to -i$ or $-i$ by i. This satisfies:

1)	$(z_1+z_2)^* = z_1^* + z_2^*$	
2)	$(z_1z_2)^* = z_1^* z_2^*$	
3)	z is real $\Leftrightarrow$	$z^* = z$

Quiz 3

Complex plane and magnitude

Any complex number can be represented uniquely in a two dimensional plane as illustrated. This motivates the idea of the modulus or magnitude of a complex number.



$$\boxed{\text{The modulus of } z = u + iv \text{ is} \quad |z| = \sqrt{u^2 + v^2}}$$

One can prove:

1) $|z|^2 = z^* z$ ← important for calculations!!
2) $|z^*| = |z|$
3) $|z_1 z_2| = |z_1||z_2|$

Quiz 4 - only second round

Note that we can associate an angle θ with the complex number as illustrated. Then

$$u = |z| \cos\theta$$

$$v = |z| \sin\theta$$

Thus

$$z = |z| (\cos\theta + i \sin\theta)$$

We will define the complex exponential as:

$$\boxed{\text{For any } \underline{\text{real}}\ \theta \qquad e^{i\theta} := \cos\theta + i\sin\theta}$$

Euler Identity /

Euler Relation.

Thus we get for any complex number.

$$z = |z| e^{i\theta}$$

where $|z|$ is the magnitude of the angle

θ is angle c.c.w from positive real axis

Quiz 5 → Quiz 9