Modern Physics: Homework 13

Due: 27 April 2021

1 Angular momentum

The rules that appear for angular momentum with the hydrogen atom apply to any angular momentum situation. The numbers that label states follow the usual rules: $l = 0, 1, 2, \ldots$ and $m_l, = -l, -l+1, \ldots, l-1, l$ with the same interpretations in terms of L_z and L. *(231S21)*

a) Suppose that $l = 3$. Determine the magnitude of the angular momentum and the maximum value of the z-component of the angular momentum.

b) Is it possible that the maximum value of L_z equals L? Explain your answer.

In the following, consider a ball with mass 40 g that swings in a circle with radius 30 cm. The period of the ball's orbit is 0.20 s.

c) Determine the angular momentum of the ball (recall that $L = mvr$ for classical situations). To what value of l would this correspond? If the value of l changed by ± 10 would the change in angular momentum of this object be noticeable?

d) What would the maximum value of L_z be? Would there be a noticeable difference between this and L?

2 Hydrogen atom states

Consider a hydrogen atom for which $n = 4$. *(231S21)*

a) List all possible states in which the atom could be.

b) Describe what physical quantities distinguish between these states. Describe the values that these quantities could take for these states.

3 Hydrogen atom wavefunctions: a useful integral

The following result will be helpful for calculating expectation values. *(231S21)*

a) Prove that

$$\int_0^\infty r^n e^{-rb}\, dr = \frac{n}{b} \int_0^\infty r^{n-1} e^{-rb}\, dr$$

where $b > 0$ and $n \geqslant 1$.

b) Use the result to show that

$$\int_0^\infty r^n e^{-rb}\, dr = \frac{n!}{b^{n+1}}.$$

4 Hydrogen atom radial probabilities

A hydrogen atom is in a state for which $n = 2$. *(231S21)*

a) Describe whether the radial probabilities will depend on l and m_l.

b) For each possible state determine the most probable value of r.

c) For each possible state determine the probability that $r < a_0$.

5 Hydrogen atom wavefunctions: solutions to the Schrödinger equation.

Consider the $n = 2$, $l = 1$ and $m_l = 0$ state of the hydrogen atom. The radial part of the wavefunction is

$$R(r) = \frac{1}{2a_0}^{3/2} \frac{r}{\sqrt{3}\, a_0} e^{-r/2a_0}$$

where a_0 is the Bohr radius. The angular parts are

$$\Theta(\theta) = \sqrt{\frac{3}{4\pi}} \cos\theta$$

and

$$\Phi(\phi) = 1.$$

(231S21)

a) Show that $R(r)$ and $\Theta(\theta)$ satisfy the relevant parts of the time-independent Schrödinger equation.

b) Show that the wavefunction is normalized.

c) Determine an expression for the most likely value of r at which the electron will be found.

d) Determine the expectation value $\langle r \rangle$ for the electron in this state.

6 Harris, *Modern Physics, Second Edition*, Ch. 7 Prob. 54, page 288.