Modern Physics: Homework 11

Due: 13 April 2021

1 Particle incident on a step potential: energy less than step

Consider a particle with mass m and energy E incident on a step potential for which $E < U_0$. The wavefunction for the particle is

$$\begin{aligned} \psi_I(x) = &\quad Ae^{ikx} + Be^{-ikx} && \text{if } x < 0 \\ \psi_{II}(x) = &\quad Ce^{-\alpha x} && \text{if } x > 0 \end{aligned}$$

where

$$\begin{aligned} k &= \sqrt{2mE/\hbar^2} \\ \alpha &= \sqrt{2m(U_0 - E)/\hbar^2}. \end{aligned}$$

(231S21)

a) Using the matching conditions at $x = 0$, i.e. $\psi_I(0) = \psi_{II}(0)$ and $\left.\frac{d\psi_I}{dx}\right|_0 = \left.\frac{d\psi_{II}}{dx}\right|_0$, show that

$$\begin{aligned} B &= \frac{k - i\alpha}{k + i\alpha}\, A \\ C &= \frac{2k}{k + i\alpha}\, A \end{aligned}$$

b) Show that the reflection coefficient, $R = 1$.

c) Determine an expression for the probability with which a particle will be found at to the right of the step. Does this depend on the mass of the particle?

2 Particle mass and tunneling

This problem considers particles with various masses and energies that are incident on potential barriers. *(231S21)*

a) Consider particles with various masses that are incident on a barrier and whose energy is less than that of the barrier. Would a particle with greater mass be more or less likely to tunnel through the barrier.

Consider a classical object with mass 50 g moving with speed 20 m/s. This particle encounters a potential barrier with energy 11 J

b) If the width of the barrier is 5.0 cm determine the transmission coefficient.

c) Determine the width of the barrier such that the transmission coefficient is 0.010.

3 Tunneling electrons

Suppose that electrons are accelerated from rest through a region where there is a voltage difference of 200 V. These electrons then travel freely until they reach a 250 V barrier with width 1.5 nm. Beyond this barrier they again travel freely. Determine the fraction of electrons that pass through the barrier. *(231S21)*

4 Tunneling through a "wide" barrier

The probability for tunneling through a barrier involves the hyperbolic sine function

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

which appears in the tunneling coefficient

$$T = \frac{4\beta(1-\beta)}{\sinh^2[\sqrt{2m(U_0 - E)}\, L/\hbar] + 4\beta(1-\beta)}$$

where $\beta = E/U_0$. *(231S21)*

a) Show that if $x \gg 1$, then $\sinh(x) \approx e^x/2$.

b) Use this to show that if $\sqrt{2m(U_0 - E)}\, L/\hbar \gg 1$ then

$$T \approx 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) e^{-2L\sqrt{2m(U_0-E)}/\hbar}.$$

This is the "wide barrier" formula for transmission.

5 Scanning tunneling microscope

This problem considers tunneling in typical scanning tunneling microscopes. These use electrons to tunnel through a barrier. The energies are determined via voltages. Recall that in an electric potential V the energy of an electron is eV where e is the electron charge. Suppose that an electron is accelerated from rest through voltage V, giving it energy E. The electron then encounters a barrier with electric potential $V_0 > V$. Let $\Delta V = V_0 - V$.

(231S21)

a) Using the wide barrier approximation show that the transmission coefficient is

$$T = 16\frac{V}{V_0}\left(1 - \frac{V}{V_0}\right) e^{-\beta L\sqrt{\Delta V}}$$

where β is a constant. Determine this constant.

In the following, suppose that $V_0 = 100\,\text{mV}$ and $V = 40\,\text{mV}$. In this case the barrier is the gap between the tip of the microscope and the surface that it images.

b) Determine T if the barrier has width 4.0×10^{-10} m.

c) Determine T if the barrier has width 5.0×10^{-10} m.

d) Suppose that the gap between microscope tip and surface is initially 5.0×10^{-10} m. It then increases by 5%. By what percentage does the transmission coefficient change?