Modern Physics: Homework 10

Due: 7 April 2021

1 Superposition of energy eigenstates

Consider a particle of mass m in potential $V(x)$. Suppose that the energy eigenstates can be labeled by integers $n = 1, 2, 3, \ldots$ and denote these

$$\Psi_n(x,t) = \psi_n(x)e^{-iE_nt/\hbar}$$

where E_n is the energy eigenvalue. In general $P(x,t)$ is independent of time for energy eigenstates. But superpositions/combinations are also solutions. As an example, consider

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\Psi_1(x,t) + \frac{1}{\sqrt{2}}\Psi_2(x,t).$$

(231S21)

a) Show that the probability distribution, $P(x,t)$, varies with time and contains terms which oscillate. Find the frequency of oscillation of these terms. *Hint: the frequencies of oscillation appear in terms of the form $e^{-i\omega t}$. You just have to isolate any complex exponentials containing t in your expression to find these frequencies.*

In the following questions, consider a particle of mass m in an infinite square well of width $L = 1$ and suppose that it is in the state

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\Psi_1(x,t) + \frac{1}{\sqrt{2}}\Psi_2(x,t).$$

b) Plot $P(x,t)$ at $t = 0$ for $0 \leqslant x \leqslant 1$.

c) Plot $P(x,t)$ at $t = \pi\hbar/2\Delta E$ where $\Delta E = E_2 - E_1$ for $0 \leqslant x \leqslant 1$.

d) Plot $P(x,t)$ at $t = \pi\hbar/\Delta E$ where $\Delta E = E_2 - E_1$ for $0 \leqslant x \leqslant 1$.

e) Plot $P(x,t)$ at $t = 2\pi\hbar/\Delta E$ where $\Delta E = E_2 - E_1$ for $0 \leqslant x \leqslant 1$.

f) Does the expectation value of position appear to oscillate? If so with what frequency?

g) *Optional:* Show that, for this superposition,

$$\langle x \rangle = \frac{1}{2}\left[1 - \frac{32}{9\pi^2}\cos\left(\frac{\Delta E t}{\hbar}\right)\right].$$

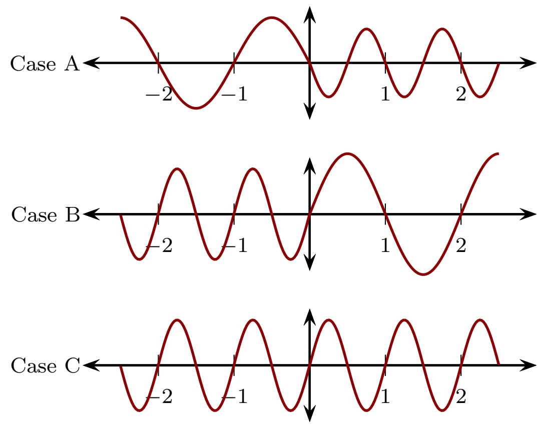
Hints: Use the result that, for an energy eigenstate, $\langle x \rangle = 1/2$.

2 Particle passing a step potential

A particle with energy $E > U_0$ is incident from the left on a step potential

$$U(x) = \begin{cases} 0 & \text{if } x < 0 \\ U_0 & \text{if } x > 0. \end{cases}$$

The particle passes the step and continues moving right. Which of the following most accurately represents the real parts of the particle's wavefunctions for this situation? Explain your answer. *(231S21)*



3 Particle incident on a step potential: energy greater than step

A particle with mass m is incident on a step potential

$$U(x) = \begin{cases} 0 & \text{if } x < 0 \\ U_0 & \text{if } x > 0. \end{cases}$$

The particle has energy $E > U_0$. The stationary state wavefunction for particle incident from the left is

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & \text{if } x < 0 \\ Ce^{-ik'x} & \text{if } x > 0 \end{cases}$$

where $k = \sqrt{2mE/\hbar^2}$ and $k' = \sqrt{2m(E-U_0)/\hbar^2}$. Show that

$$B = \frac{k-k'}{k+k'}\,A$$
$$C = \frac{2k}{k+k'}\,A.$$

(231S21)