Modern Physics: Homework 9

Due: 30 March 2021

1 Emission spectrum

The emission spectrum of an unknown quantum system is analyzed and it is found that the three lowest frequencies that are emitted are 200 Hz, 400 Hz and 650 Hz. Explain whether the system could or could not be a harmonic oscillator. *(231S21)*

2 Molecular vibrations

A hydrogen bromide molecule consists of a single hydrogen atom and a single bromine atom. The vibrational spectrum of this would yield a series of frequencies each separated by 7.68×10^{13} Hz. Assume that when it vibrates, the bromine atom is fixed and the hydrogen atom oscillates. If the oscillator were modeled as a spring and mass system, what would the spring constant of the spring be? *(231S21)*

3 Harmonic oscillator: $n = 1$ energy eigenstate

Consider the following possible solution to the time-independent Schrödinger equation for the harmonic oscillator:

$$\psi(x) = Axe^{-bx^2/2}$$

where A and b are constants. The aim of this exercise is to show that this solves the time-independent Schrödinger equation for particular choices of b and the energy E. *(231S21)*

a) Show that
$$\frac{d\psi}{dx} = Ae^{-bx^2/2} - bx\,\psi(x).$$

b) By substitution into the time-independent Schrödinger equation show that ψ is a solution provided that $b = m\omega_0/\hbar$.

c) Determine the energy associated with this solution.

4 Harmonic oscillator: emission wavelengths

Suppose that the largest wavelength of light emitted by a harmonic oscillator is 600 nm. Determine the second and third largest wavelength of light emitted by this oscillator. *(231S21)*

5 Infinite well: momentum measurements

Consider a particle of mass m in an infinite square well of width L which is in the n^{th} energy eigenstate/stationary state.*(231S21)*

a) Determine the expectation value for the momentum, $\langle p \rangle$, for this energy eigenstate.
b) Determine the uncertainty in the momentum measurement outcomes, Δp, for this energy eigenstate.
c) The uncertainty in position measurement outcomes can be shown to be:

$$\Delta x = \frac{L}{2}\sqrt{\frac{1}{3} - \frac{2}{n^2\pi^2}}.$$

Use this and the result from the previous part to verify that the uncertainty principle is valid.

6 Infinite well: position and momentum measurements

A particle is in an infinite well for which $0 \leqslant x \leqslant L$. The wavefunction for the particle at one instant is

$$\psi(x) = \sqrt{\frac{1}{L}}\,\sin\left(\frac{\pi x}{L}\right) + \sqrt{\frac{1}{L}}\,\sin\left(\frac{2\pi x}{L}\right).$$

(231S21)

a) Verify that this wavefunction is normalized.
b) If the particular's position is measured, is the result more likely to be in the left or right half of the well? Explain your answer.
c) Determine the expectation value of the position.
d) Determine the expectation value of the momentum.
e) Based on the expectation values, what would you expect qualitatively for measurements of the particle's position at a slightly later time?

7 Harmonic oscillator: position and momentum expectation values

Consider a harmonic oscillator in its ground state ($n = 0$). The wavefunction that corresponds to this is

$$\psi_0(x) = \left(\frac{m\omega_0}{\hbar\pi}\right)^{1/4} e^{-m\omega_0 x^2/2\hbar}.$$

(231S21)

a) Determine $\langle x \rangle$.
b) Determine Δx.
c) Determine $\langle p \rangle$.
d) Determine Δp.
e) Determine $\Delta x \Delta p$. Does the result agree with the uncertainty principle?