Modern Physics: Homework 8

Due: 23 March 2021

1 Emission spectrum for a one dimensional infinite well

An electron is trapped in an infinite well with width 8.0×10^{-10} m. *(231S21)*

a) Determine the five smallest energies available to the electron.

b) Determine the three largest wavelengths of light that this electron could emit. For each, describe the energy levels involved in the process.

2 Proton and alpha particle in an infinite well

An alpha particle consists of two protons and two electrons. Suppose that a proton or an alpha particle could each be trapped in an infinite well with the same width. Which of the following is true? Explain your answer. *(231S21)*

i) The lowest frequency of radiation which the proton could emit is the same as the lowest frequency which the alpha particle could emit.

ii) The lowest frequency of radiation which the proton could emit is larger than the lowest frequency which the alpha particle could emit.

iii) The lowest frequency of radiation which the proton could emit is smaller than the lowest frequency which the alpha particle could emit.

3 Infinite well emission: adjacent levels

A particle with mass m is trapped in an infinite well with width L. The particle is initially in the level n and drops to the level immediately beneath it. Show that the frequency of the radiation emitted is proportional to $2n - 1$. What does this imply for the upper limit on the frequency of the radiation that this could emit? *(231S21)*

4 Quantum dots

A quantum dot is a three dimensional version of an infinite square well in which an electron is trapped. To get an idea of the size of a useful quantum dot containing a single electron, consider, as a simple model, an electron in a one dimensional infinite square well potential of width L. Suppose that you would like energy spectrum of the quantum dot to be such that the dot emits visible light of wavelength 500 nm when the electron undergoes a transition from the second lowest energy level to the lowest energy level. *(231S21)*

a) Determine L such that the energy spectrum of the electron in the infinite square well results in emission of light as described as above.

b) Determine the second largest wavelength of light that can be emitted by this electron.

5 Infinite well: energy eigenstate wavefunctions and probability densities

Consider an infinite well with the range $0 \leqslant x \leqslant L$. *(231S21)*

a) Plot the energy eigenstates for $n = 5$ and $n = 6$.

b) Plot the position probability density for $n = 5$ and $n = 6$.

c) Suppose that a position measurement is performed. Based on the plots only is the outcome more likely to be in the left or right half of the well.

d) Suppose that many copies of the particle or provided and these are all in the same state (either $n = 5$ or $n = 6$). Use the plots to explain what the expected value of the average position measurement outcome is in each case.

6 Position measurements for a particle in an infinite square well

Consider a particle of mass m in the following infinite square well potential

$$U(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{otherwise.} \end{cases}$$

The energy eigenfunctions are given by

$$\psi_n(x) = \begin{cases} \sqrt{\dfrac{2}{L}} \sin\left(\dfrac{n\pi x}{L}\right) & 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$

where

$$E_n = n^2 \frac{\hbar^2 \pi^2}{2mL^2}.$$

In the following you are allowed to use integral tables, MAPLE, Wolfram Alpha, or your calculator to evaluate integrals. However, you must provide all the steps when you set the integrals up. *(231S21)*

a) Determine the expectation value for outcomes of position measurements, $\langle x \rangle$, for any energy eigenfunction.

b) Determine the uncertainty in the position measurement outcomes, Δx, for any energy eigenfunction.

c) For any state ψ_n, determine the probability with which a measurement will yield an outcome in the range $0 < x < L/2$. Determine the probability with which a measurement will yield an outcome in the range $L/2 < x < L$.

d) Suppose that a position measurement is performed which can only describe whether the particle is in the left third ($0 < x < L/3$), the middle third ($L/3 < x < 2L/3$), or right third of the well ($2L/3 < x < L$). It is known that the particle is either in the $n = 1$ or $n = 2$ state prior to this measurement. Given that there are very many copies of the well and the particle, each guarantee to be in the same state, would the position measurement be able to reveal whether the particles are all in the $n = 1$ or all in the $n = 2$ state? Explain your answer.

7 Harris, *Modern Physics, Second Edition*, Ch. 5 Prob. 26, page 187.