Modern Physics: Homework 2

Due: 8 February 2021

1 Photon absorption

An atom initially at rest absorbs a photon with wavelength 500 nm and subsequently emits a photon with wavelength 660 nm.

a) Determine the energy gained by the atom in this process.

b) Suppose that the atom has mass 3.3×10^{-26} kg. Determine the speed of the atom after it has emitted the photon, assuming that any additional energy it acquired only contributed to its motion.

2 Down conversion

In "down conversion" a photon with wavelength λ_i is incident on a crystal. The crystal absorbs this and emits two photons without changing it's energy. The wavelengths of the emitted photons are λ_{e1} and λ_{e2} and these are not necessarily equal.

a) Which of the following is true?

   i) Both λ_{e1} and λ_{e2} are larger than λ_i.

   ii) Both λ_{e1} and λ_{e2} are smaller than λ_i.

   iii) One of λ_{e1} and λ_{e2} is larger than λ_i, the other is smaller.

   Explain your choice.

b) Determine an expression that relates $\lambda_{e1}, \lambda_{e2}$ and λ_i.

c) Light with wavelength 415 nm is incident on such a crystal and two photons with identical wavelength are emitted. Determine the wavelength of the emitted photons.

3 X-ray scattering

X-rays are incident on a crystal for which the spacing between crystal planes is 0.28 nm. Suppose that the first order reflection ($m = 1$) occurs at 15° above the plane of the crystal. Determine the energy of each X-ray photon.

4 Compton Effect

In the original Compton effect experiment the wavelength of the incident radiation used was 0.0711 nm.

a) Determine the wavelength shift for scattering from an electron at $20^\circ, 50^\circ$ and 90°.

b) Suppose that the same radiation was incident on a hydrogen atom. Would the wavelength shift be more difficult or less difficult to observe than for scattering from an electron? Explain your answer.

c) Suppose that visible light were used in an attempt to observe the Compton effect off an electron. Would this be more or less difficult to observe than for the 0.0711 nm radiation? Explain your answer.

5 Compton Effect and Mass of Scattering Object

The Compton effect is usually observed by bombarding electrons with X-rays or gamma rays. For example, Compton used X-rays with wavelength 0.0711 nm. The change in wavelength will depend on the mass of the object from which the radiation is scattered. This enters into the analysis via the Compton wavelength $\lambda_c = h/mc$ where m is the mass of the scattering object.

a) For a scattering object of mass m find an expression for the maximum change in wavelength (as the scattering angle varies) between the incident photon and scattered photon. Express this in terms of the Compton wavelength of the scattering object.

b) Find an expression for the maximum fractional change in the wavelength, i.e. $(\lambda_2 - \lambda_1)/\lambda_1$, as a function λ_1 and the Compton wavelength.

c) Calculate the Compton wavelength for an electron and determine the maximum fractional change in wavelength for the X-rays used by Compton.

d) Calculate the Compton wavelength for a proton and determine the maximum fractional change in wavelength for the X-rays used by Compton.

e) Calculate the Compton wavelength for an object of mass 1 kg and determine the maximum fractional change in wavelength for the X-rays used by Compton. What does this tell you about the possibility of detecting Compton scattering from everyday objects using X-rays?

6 Mathematical representation of waves

Consider a wave that is represented by

$$\Psi(x,t) = 4\cos{(kx - \omega t)}.$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi f$. The aim of the following exercises is to show that this expression captures many of the intuitive features of waves.

a) Consider a snapshot of the wave at $t = 0$. Use the expression to find the possible values of x at which the crests (maximum points) of the wave are located. By what distance are neighboring crests separated?

b) Show that the expression predicts that at any time t the value of ψ is the same for any two points separated by exactly one wavelength, i.e. show that

$$\Psi(x + \lambda, t) = \Psi(x,t).$$

c) As time passes one can follow a particular crest by focusing on the argument of the cosine. For example suppose that $kx - \omega t = 0$. This describes one particular crest of Ψ. Where is this crest located at time $t_i = 0$? Determine an expression for the location of this crest located at any later time t_f. How far has the crest traveled during the time interval from t_i to t_f? Use this to determine an expression (in terms of k and ω) for the speed with which this crest travels.

7 Sinusoidal waves: phase shifts

Consider a wave that is represented by

$$\Psi(x,t) = A\cos(kx - \omega t).$$

a) Suppose that the wave is shifted to the right by a quarter wavelength. Mathematically this can be accomplished by replacing x by $x - \lambda/4$. The new wave could be represented by an expression of the form

$$\Psi(x,t) = A\cos(kx - \omega t - \phi).$$

Determine the value of the phase ϕ in this case. *Hint: Recall the relationship between k and λ.*

b) Repeat the problem for a shift to the right by a half wavelength.

c) Repeat the problem for a shift to the right by one wavelength.

8 Superposition of two sinusoidal waves

Consider two waves

$$\begin{aligned}\Psi_1(x,t) &= A\cos(kx - \omega t - \phi_1)\\ \Psi_2(x,t) &= A\cos(kx - \omega t - \phi_2)\end{aligned}$$

where ϕ_1 and ϕ_2 are phase shifts.

a) For the case where $k = \pi, A = 1, \phi_1 = 0$ and $\phi_2 = \pi/2$, sketch as accurately as possible snapshots of these two waves at $t = 0$. Use the same axes. Based on your sketch would a superposition of these yield a wave with amplitude larger or smaller than A?

b) For the case where $k = \pi, A = 1, \phi_1 = 0$ and $\phi_2 = \pi$, sketch as accurately as possible snapshots of these two waves at $t = 0$. Use the same axes. Based on your sketch would a superposition of these yield a wave with amplitude larger or smaller than A?

c) For the case where $k = \pi, A = 1, \phi_1 = 0$ and $\phi_2 = 3\pi/2$, sketch as accurately as possible snapshots of these two waves at $t = 0$. Use the same axes. Based on your sketch would a superposition of these yield a wave with amplitude larger or smaller than A?

The superposition of these waves is

$$\Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t).$$

d) Show that

$$\Psi(x,t) = A' \cos\left(kx - \omega t - \phi'\right)$$

and determine an expression for A' in terms of A, ϕ_1 and ϕ_2. Similarly determine an expression for ϕ' in terms of ϕ_1 and ϕ_2. *Hint: the crucial trigonmetric identity is*

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right).$$

e) How must the two phases of the waves be related so that the superposition yields a wave with maximum amplitude?

f) How must the two phases of the waves be related so that the superposition yields a wave with minimum amplitude?