Modern Physics: Class Exam I

26 February 2021

Name: ______________________________ **Total:** /50

Instructions

- There are 7 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$c = 3.0 \times 10^{8}\,\mathrm{m/s}$ $h = 6.63 \times 10^{-34}\,\mathrm{Js}$ $k_B = 1.38 \times 10^{-23}\,\mathrm{J/K}$ $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$

$m_{\mathrm{electron}} = 9.11 \times 10^{-31}\,\mathrm{kg}$ $m_{\mathrm{proton}} = 1.67 \times 10^{-27}\,\mathrm{kg}$ $m_{\mathrm{neutron}} = 1.67 \times 10^{-27}\,\mathrm{kg}$

Question 1

Two different lasers produce light with different colors, red and green. The red laser produces light with wavelength 650 nm and a green laser with wavelength 530 nm. Each produces the same number of photons every second. Which of the following (choose one) is true?

i) The power of the red laser is the same as the power of the green laser.
ii) The power of the red laser is larger than the power of the green laser.
iii) The power of the red laser is smaller than the power of the green laser.

Explain your answer.

$$P = \frac{\text{Energy}}{\text{Time}} = \frac{\text{Number of photons} \times \text{Energy per photon}}{\text{time}}$$

$$= \frac{\text{Number of photons}}{\text{Time}} \times \text{Energy per photon}$$

$$= \underbrace{\frac{\text{Number of photons}}{\text{Time}}}_{\text{same}} \times \frac{hc}{\lambda} \leftarrow \text{larger red} \Big\} \text{smaller red.}$$

/5

Question 2

The work function (minimum energy needed to remove an electron) for copper is 4.7 eV.

a) Determine the largest wavelength of light that can successfully remove an electron from copper.

Need $E_{ph} \geqslant \phi$

$$\frac{hc}{\lambda} \geqslant \phi \Rightarrow \lambda \leq \frac{hc}{\phi}$$ largest $$= \frac{6.63\times10^{-34}\,J\cdot s \times 3.0\times10^{8}\,m/s}{4.7\,eV \times 1.6\times10^{-19}\,J/eV}$$

$$= 2.6\times10^{-7}\,m$$

b) Suppose that light with this maximum wavelength is incident on the copper. The intensity of the light is reduced. Explain the effect that this will have on the electrons e.g, they will no longer be emitted, they will have less kinetic energy after emission, etc, ...

They will still be emitted. However the number of photons incident per second will be smaller. Each photon will emit an electron with no different KE than before

$\Rightarrow$ fewer electrons emitted per second

/7

Question 3

Consider $z_1 = -2 + 2i$ and $z_2 = e^{i\pi/3}$. Express $z = z_1 z_2$ in the form $z = u + iv$ where u and v are real.

$$z_2 = \cos \pi/3 + i \sin \pi/3 = \frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$z_1 z_2 = (-2+2i)\tfrac{1}{2}(1+\sqrt{3}i)$$

$$= (-1+i)(1+\sqrt{3}i)$$

$$= (-\sqrt{3}-1) + (1-\sqrt{3})i$$

/6

$$= -2.73 - 0.73i$$

Question 4

Two waves are represented by

$$\Psi_1 = Ae^{i(kx-\omega t+\phi_1)}$$
$$\Psi_2 = Ae^{i(kx-\omega t+\phi_2)}$$

where $A > 0, k > 0, \omega > 0, \phi_1$ and ϕ_2 are constants.

a) Describe the direction in which each wave travels.

Positive x since argument is $kx-\omega t$

b) Determine an expression for the superposition of these waves $\Psi = \Psi_1 + \Psi_2$ and use this to show that the intensity of the superposition, $I = |\Psi|^2$, is (either of)

$$I = 2A^2\left[1+\cos\left(\phi_1-\phi_2\right)\right] = 4A^2\cos^2\left(\frac{\phi_1-\phi_2}{2}\right).$$

$$\Psi = Ae^{i(kx-\omega t+\phi_1)} + Ae^{i(kx-\omega t+\phi_2)}$$

$$= Ae^{i(kx-\omega t)}\left[e^{i\phi_1}+e^{i\phi_2}\right]$$

$$|\Psi|^2 = |A|^2 \cancel{|e^{i(kx-\omega t)}|^2} \underbrace{\left|e^{i\phi_1}+e^{i\phi_2}\right|^2}$$

$$(e^{i\phi_1}+e^{i\phi_2})(e^{i\phi_1}+e^{i\phi_2})^*$$

$$= (e^{i\phi_1}+e^{i\phi_2})(e^{-i\phi_1}+e^{-i\phi_2})$$

$$= 2 + \underbrace{e^{i(\phi_1-\phi_2)}+e^{-i(\phi_1-\phi_2)}}_{2\cos(\phi_1-\phi_2)}$$

$$\Rightarrow |\Psi|^2 = 2A^2\left(1+\cos(\phi_1-\phi_2)\right)$$

Question 4 continued ...

Then $\cos\alpha = 2\cos^2 \alpha/2 - 1$ gives

$$|\Psi|^2 = 4A^2 \cos^2\left(\frac{\phi_1-\phi_2}{2}\right)$$

c) Use the previous result to explain how ϕ_1 and ϕ_2 must be related to give a minimum intensity.

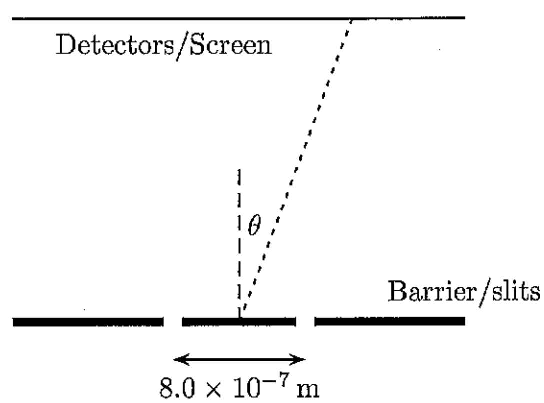
Need $\frac{\phi_1-\phi_2}{2} = \pm\pi/2, \pm\frac{3\pi}{2} \pm\frac{5\pi}{2}$

$\phi_1-\phi_2 = \pm\pi, \pm3\pi, \pm5\pi, \ldots \quad \pm(2n+1)\pi$

/12

Question 5

Neutrons are fired with speed 2.5×10^3 m/s toward a barrier that contains two narrow slits spaced 8.0×10^{-7} m apart. The neutrons that pass the barrier emerge at various angles θ measured as illustrated. Determine the smallest angle, $\theta \neq 0$ as illustrated (i.e. not straight through), at which neutrons are most likely emerge.



max when

$d\sin\theta = m\lambda$

Here $m=1$ $\qquad \lambda = \frac{h}{p} = \frac{h}{m_{neutron}v}$

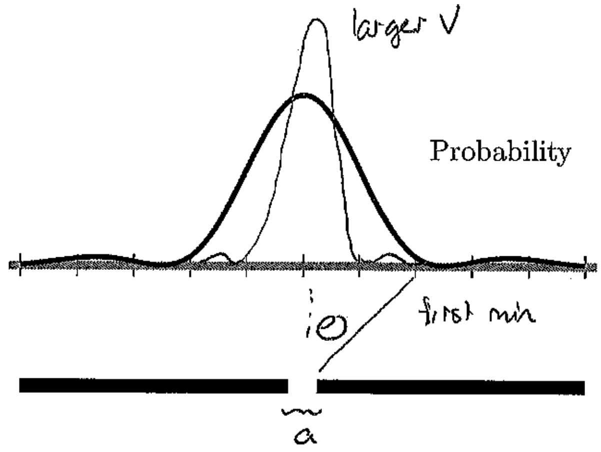
$$\Rightarrow \sin\theta = \frac{h}{d\, m_{neutron} v} = \frac{6.63\times10^{-34}\text{ J.s}}{8.0\times10^{-7}\text{m}\times1.67\times10^{-27}\text{kg}\times2.5\times10^3\text{m/s}}$$ /6

$= 0.000198$

$\Rightarrow \theta = \sin^{-1}(0.000198) = 0.011°$

Question 6

Particles are fired toward a single slit. The probability with which the particles arrive at a screen is illustrated.



a) Suppose that the speed with which the particles approach the slit was increased. Explain how this would affect the probability distribution.

First min occurs when

$$a\sin\theta = \cancel{m}\lambda$$

$$\Rightarrow \sin\theta = \frac{\lambda}{a} \qquad \text{But} \quad \lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\Rightarrow \sin\theta = \frac{h}{amv}$$

increase V $\Rightarrow$ decrease $\sin\theta$ $\Rightarrow$ decrease θ. Distribution is narrower

b) Suppose that the particles were fired with the same speed but fewer were fired per second. Explain how this would affect the probability distribution.

No effect. It would not change λ and thus not change the angles at which minima appear.

/6

Question 7

The energy levels of a hydrogen atom, according to the Bohr model, are

$$E_n = -13.6\,\text{eV}\,\frac{1}{n^2}.$$

a) Consider all possible transitions from a level where $n > 2$ down to the level where $n = 2$. Determine the *two longest wavelengths* of light that can be emitted for such transitions.

$|\Delta E_{atom}| = E_{ph} = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{hc}{|\Delta E_{atom}|}$

Need smallest $|\Delta E_{atom}|$

So $3 \to 2$

$4 \to 2$

will work

$3 \to 2$

$|\Delta E_{atom}| = |E_2 - E_3| = \left|-13.6\text{eV}\left(\frac{1}{4} - \frac{1}{9}\right)\right|$

$= 1.9\text{eV} = 1.9\text{eV} \times 1.9\times10^{-16}\,\text{J/eV}$

$= 3.0\times10^{-19}\,\text{J}.$

$\lambda = \frac{6.63\times10^{-34}\,\text{Js} \times 3.0\times10^{8}\,\text{m/s}}{3.0\times10^{-19}\,\text{J}} = 656\,\text{nm}$

$4 \to 2$

$|\Delta E_{atom}| = 13.6\text{eV}\left(\frac{1}{4} - \frac{1}{16}\right) = 2.55\text{eV}$

$= 4.1\times10^{-19}\,\text{J}$

$\lambda = \ldots\ldots = 482\,\text{nm}$

b) An observer claims that a line corresponding to electromagnetic radiation with wavelength 50 nm is observed in the spectrum of a gas that he suspects is hydrogen. Determine the change in energy that the hydrogen electron would have to undergo to emit this radiation and use this to decide whether the gas could be hydrogen.

The photon energy is: $E_{ph} = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,\text{J·s} \times 3.0\times10^{8}\,\text{m/s}}{50\times10^{-9}\,\text{m}}$

$= 3.97\times10^{-18}\,\text{J} = 25\text{eV}$

The atom would need to lose 25eV.

But highest energy $(n=\infty) \to 0\text{eV}$

lowest " $(n=1) \to -13.6\text{eV}$

max drop is 13.6eV

cannot be Hydrogen.

/8